EXPLORING SPATIAL RELATIONSHIPS: A STRATEGY FOR GUIDING TECHNOLOGICAL PROBLEM SOLVING

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Abstract:

The growing realization of the benefits to individual students and to state economies, of providing science learners with opportunities to expand their knowledge, skills and experience of knowledge-based technological design has led to seeking instructional strategies to facilitate the transition from traditional school settings to project based learning environments. This paper refers to engaging high school physics and computer-science majors in challenging design projects which seek to activate and implement the often inert formal content knowledge within the context of designing and constructing systems dealing with real world engineering challenges in robotics and electro-optics. In this paper we suggest that visualization of the problem space and guided exploration of its spatial relationships can promote the elicitation of relevant formal knowledge and lead to creative solution design. These methods are described in the context of designing and programming robot navigation and in the context of developing remote distance sensors.

Keywords: *project based learning; visualization; spatial relationships; guidance; robot navigation; non-contact distance sensor; inert knowledge*

1. Introduction

There is wide agreement that the science and technology education should develop learner abilities for problem solving, independent critical thinking, creative and inventive skills and productive team work. Project-based science and technology learning provides a natural environment for achieving such goals, thus enabling the shift from traditional, teacher centered instruction towards meaningful, student centered learning [1, 2, 3]. The recently published New Generation Science Standards has embraced these insights, and includes an explicit focus on Engineering Design [4].

Many programs promoting project-based-learning in the context of engineering design for high school students, have been initiated in different countries during the past decade [e.g. 5, 6, 7]. Many of these programs are based in institutions of higher education. This may be due to the realization that engineering is vital for national economies and that high school students should be encouraged to join departments of engineering in colleges and universities. Robotics courses naturally fall into this category of instructional activities. Some high school curricula include Robotics as a study domain [e.g. 8].

Abundant evidence has been gathered showing that students involved in electronics and robotics projects display high motivation to confront complex tasks and show originality and creativity [9, 10].

This paper refers to engaging high school physics and computer-science majors in challenging design projects which seek to activate and implement the often inert theoretical content knowledge [11] within the context of designing and constructing systems dealing with real world engineering challenges. The examples will be taken from two programs within the School of Contemporary Science run by the Davidson Institute for Science Education, in the Weizmann Institute of Science: Physics and Industry[1] (Appendix), and Computer Sciences, Academia and Industry[2].

2. The Challenges of Project Based Education

Traditional k-12 education provides very few opportunities for science learners to develop attitudes, skills and knowledge which are necessary for confronting technological problems and succeeding in project-based learning. Thus, even high ability learners lack the habits of mind and the practical skills for designing a technological system and progressing towards producing a working artifact. There is a growing realization that learning and creative thinking are complex processes. The linear project design prescriptions commonly found in the engineering and technology literature, are poor approximations of the ways these processes unfold in reality, or of the work methods of experts [12]. To this one needs to add the realization that problem-solving and thinking skills are context-bound. Thus, very little transfer can be expected between problems in different domains [13]. Swartz & Perkins [14] stress that learners need direct instruction regarding the cognitive skills required for problem solving in a specific domain. We should not expect that formal knowledge acquired in physics or geometry lessons will be automatically invoked in the context of programming robotic motion or designing remote-distance sensors. It is the instructors' responsibility to involve the learners in activities intended to elicit relevant formal knowledge.

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1. http://davidson.weizmann.ac.il/en/content/physics-and-industry<?>
2. http://davidson.weizmann.ac.il/en/content/computer-sciences-academia-and-industry

3. Visualization as a Problem-solving Strategy for Technological Projects

Visualization is one of the powerful strategies for problem solving in general, and creative technological problem solving, in particular. Visualization involves transforming a physical situation or event into an explicit verbal or graphical representation, in a form that helps define the problem and promotes progress to one or more solutions [15]. We intend to show how exploring and explicating spatial relationships can be instrumental in understanding a technological problem and designing creative solutions [16].

Spatial ability is a collection of specific skills related to the ability to receive, mentally process and spatially present information [17]. Spatial thinking includes the following stages in visual information processing: 1. Visual perception of object having spatial properties. 2. Mental rotation. 3. Physical or visual representation of spatial information - Visualization. In the following we shall refer to "spatial relationships" as the manner in which spatial properties are related to each other.

Spatial abilities are considered particularly relevant for learning mathematics, science and technology at school and a vital condition for vocational education and engineering careers [e.g. 18, 19]. Researchers believe that spatial abilities can be developed through drill and experience [20]. Educators have suggested advanced technological environments for developing spatial abilities in the fields such as mathematics, science, engineering and medicine [e.g. 21, 22].

Visual spatial abilities are often framed within the domain of 3D perception [e.g. 23, 24]. However, basic plane geometry offers a rich arena for developing an understanding of spatial relationships. Experiential visual perception allows children to identify circles and triangles, but visual experience by itself is unlikely to lead to the formulation of the many relationships that exist between lines and angles. Formal instruction in mathematics and geometry is responsible for teaching some basic spatial relationships such as Pythagoras' theorem; sum of the internal angles in a triangle; ratio of the circle circumference to its radius; ratio between the sides of similar triangles; ratio of the sides of a right angle triangle; definitions of geometric loci. Likewise, formal instruction in physics is responsible for teaching basic relationships related to motion, light propagation and electrical properties. All this formal knowledge is usually represented in concise "formulas", which high school students use extensively in text book problem solving. The challenge for project instructors is activating this stored knowledge in the context of designing technological problem solutions.

4. Exploring Spatial Relationships: The Case of Remote Distance Measurement

The need for reliable non-contact distance measurement exists in a variety of science and technology fields such as traffic control, robotic navigation, automated manufacturing, vehicle safety, helping the visually impaired, astronomy and astrophysics, land surveying, acoustic design and ballistics. Non-contact distance measurement is required in many of the students' projects. The instructional design involves applying structured thinking skills and activating formal knowledge from physics and mathematics, as the following sequence will show.

A challenge is presented to the students as a trigger for thinking about the problem of remote distance measurement.

"You are standing on the bank of a wide river, and you need to measure its width without physical contact with the opposite side. You are free to walk along the river bank and you can see a tree which is growing near the opposite side. You have a meter ruler and a protractor. "

1. The initial step is a two dimensional visualization of the key physical elements of the physical situation: the river, with the observer and the tree on opposite banks. To this we add a line representing the **unknown distance** "river width" (Fig. 1a).

2. The next step in the problem analysis involves explication of the meaning of "seeing" as receiving visual information (light) from a remote object. The concept of "line of sight" and the notion of light traveling along straight lines are invoked.

3. The virtual line of sight is added to the visualization, connecting the tree and the observer. The line forms an angle alpha with the river bank. Now, the triangle is visualized, indicating tangible and virtual properties. (Fig. 1b)

Fig. 1a

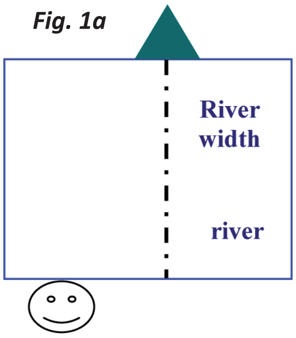


Fig. 1b

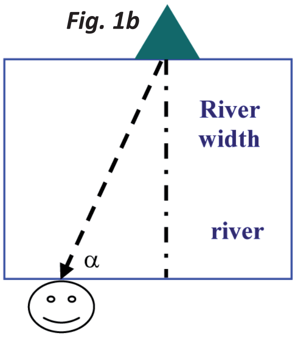


Fig. 1c

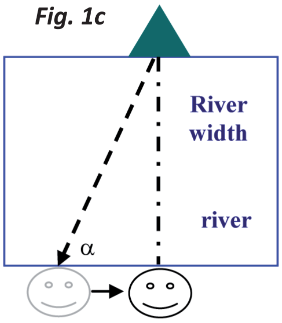
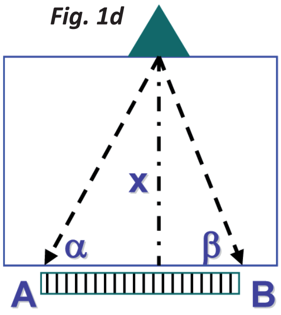


Fig. 1d



$$X=AB/(1/\tan\alpha+1/\tan\beta)$$

4. The following discussion is "How can we measure the angle alpha?" This will involve using the meter ruler to concretize the line of sight and the protractor to measure the angle. (Fig. 1b)

5. What additional information do we need to calculate the river width? The observer needs to move a known distance along the river bank, and repeat the previous process. The simplest option is to move to

a spot directly opposite the viewed object. (Fig. 1c)

6. The general solution for any two points of observation (Fig. 1d).

A. Sample implementation in Project Models

A laser beam is directed at a rotating plane mirror, placed at a known distance L (Fig. 2). The light is reflected, the angle of deflection being α. The reflected beam hits the target, and is reflected diffusely. A directional detector collects light at right angles to the original laser beam. When the detector sends a signal, **$x = L * \tan \alpha$**

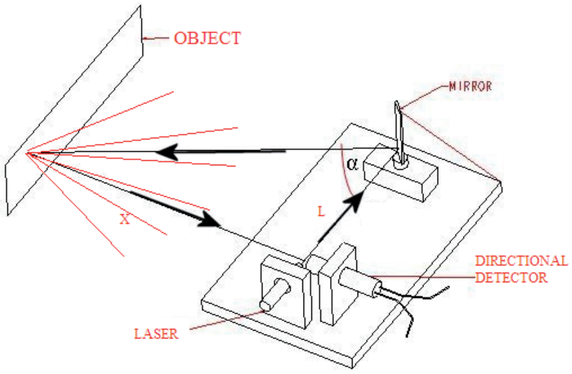


Fig. 2. Sample set up for remote distance measurement

1) Constructing a working sensor prototype

The system measures the distance (**x**) to an object to which there is a clear line of sight. In the given system, L is a constant determined by the system design requirements. Measuring different distances requires controlling the deflection angle of the laser beam (α). The laser beam is reflected from a small, front surfaced, plane mirror which is connected to the axis of a stepper motor which moves in a stepwise manner, advancing at a prefixed angle in response to electrical pulses[3]. This rotation should continue until a signal is received from the detector. The deflection angle (α) is twice the angle at which the stepper-motor has advanced from the initial reference position.

Progressing from the conceptual scientific solution to the working technological system necessitates adding a programmable interface which collects information from the designed sensors and produces visual displays and operational outputs which can drive external components (e.g. stepper motors, light sources, buzzers). In our projects we have incorporated the Arduino programmable processor board-[4]which can sense the environment by receiving input from a variety of sensors and can affect its surroundings by controlling lights, motors, and other actuators.

2) Measurement Algorithm

The stepper motor has a pre-set single step angle. It is controlled by a driver board (hardware and software) which receives the electrical pulses. The Arduino control board is programmed to supply electrical pulses to the stepper motor driver board, and count the number of supplied pulses. The detector system is connected as input to the Arduino board. Each time the Arduino receives a signal from the detector, the angle of deflection (α) and the related distance (x) are calculated. Repeated distance measurement of a moving object can be used for measuring velocity (speed and direction) and acceleration.

B. Different solutions based on similar triangles

Lens images can be used for determining the distance to a distant object with a known dimension, such as distance between car headlamps (Fig. 3). Geometric optics is taught in the 10th grade and students can be expected to "know" the relationship between the positions of objects and their images.

In a realistic situation, the end points of the object either emit light or reflect ambient light. A thin converging lens is placed so that the principal axis is at right angles to the object. The images of the extreme points are created on the focal plane. The object distance X is related to the distance between the extreme images (hi):

$$X = f * ho / hi$$

h_i can be measured manually or by using a detector array placed along the focal plane. By identifying the extreme detectors that respond at a given moment, it is possible to measure and record changes in the distance.

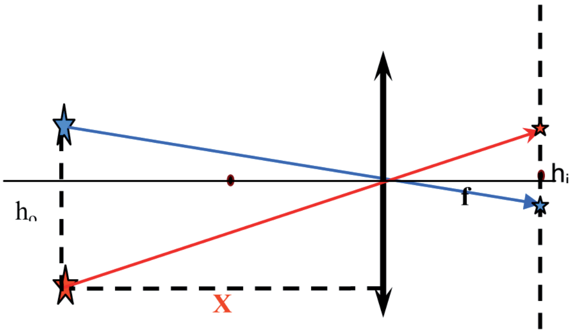


Fig. 3. Visualizing the spatial relationship between the size of the distant object and the size of the image on the focal plane

5. Exploring Spatial Relationships: The Case of Lego Robot Navigation

The Lego robot[5] advances using two wheels which are powered by separate motors (Fig. 4). The motors can be programmed to rotate at different rates in a forward or reverse direction. The robot's motion is determined by programming instructions related to traveled distance and the rotation rate of the driving motors, as well as by signals received from activated sensors.

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[3] http://www.solarbotics.net/library/pdflib/pdf/motorbas.pdf

[4] http://www.arduino.cc

[5] http://firstlegoleague.org/productEV3

Fig. 4. Sample Lego Robot

The system records the total rotation angle of each motor. Programming the robot motion and calculating distances, necessitates an explicit understanding of the spatial relationships related to circles and the physical relationships between translation and rotation. Novice robot programmers tend to adopt a “try it and see” trial and error strategy, to achieve their goals. Robot project instructors can use the following sequence to promote deeper thinking about the relationship between the motion of the wheels and that of the entire robot.

A. Advancing in a straight line

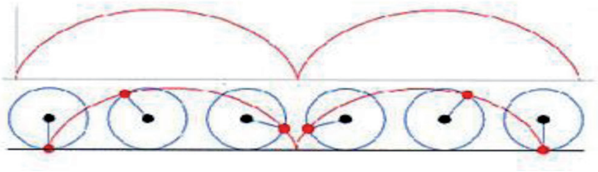

Fig. 5. Visualization of a wheel rotating and advancing

When a circle rotates around the point of contact with a line, the center advances a distance equal to the circumference for each completed rotation. This is called “rolling without slipping” (Fig. 5)[6].

To achieve motion along a straight path, both motors must rotate at the same rate. To calculate the traversed distance, the reading of the motor angle counter is read and the value is fed into the appropriate cell, divided by 360 and multiplied by the wheel circumference (2πr).

	Rotation angle	Wheel circumference (cm)	Distance covered
Both wheels	Θ°	2π*3.5	$\Theta^\circ \frac{7\pi}{360^\circ} = \Theta^\circ \cdot 0.061$

The instructional sequence starts with students observing the straight line motion of a pre-programmed robot. The next step is a graphical visualization of the motion of a wheel and a formal expression of the relation between the rotation angle and the distance traveled. After the students copy the instructor’s program into their robots, and write a procedure for displaying the distance – they realize that the actual path differs from the theoretical calculation due to friction and other imperfections.

B. Changing direction – making a 90° turn

Robot navigation requires changing direction and turning corners. There are two options for achieving a 90° turn: 1. Using one stopped wheel as a pivot, and the distance between the wheels as the radius.

	Distance covered	Rotation angle
Outer wheel	$\frac{2\pi R}{4}$	$\frac{360\cdot 0.5\cdot \pi\cdot R}{2\pi\cdot r} = \frac{90\cdot R}{r} = \frac{90\cdot 17}{3.5} = 437.14^\circ$
Inner wheel	0	0 motor stopped

2. Turning around an axis midway between wheel centers, by rotating one wheel in a forward direction and the other in a backward direction, at the same rates (Fig. 6).

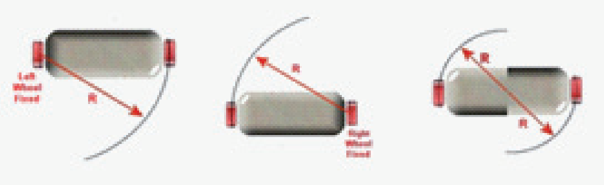

Fig. 6. Visualizing the spatial relationships in pivoting around one stopped wheel or around the robot center

	Distance covered	Rotation angle
Both wheels	$\frac{2\pi R}{4}$	$\frac{360\cdot 0.5\cdot \pi\cdot R}{2\pi\cdot r} = \frac{90\cdot R}{r} = \frac{90\cdot 8.5}{3.5} = 218.57^\circ$

The instructional sequence starts with the students manipulating the robot wheels, trying to achieve a turning motion. Students suggest the idea of making the wheels rotate in opposite directions, based on videos they have seen of the way tanks achieve turning motion. The students implement their understanding of straight motion and turns by programming the robot to travel along the sides of a square. This implementation leads to an increased awareness of the gap between theory and reality, and the need for the use of sensors to control robot navigation.

C. Navigating a circular path

Robot navigation often requires planning a path that avoids collisions with obstacles, thus requir-

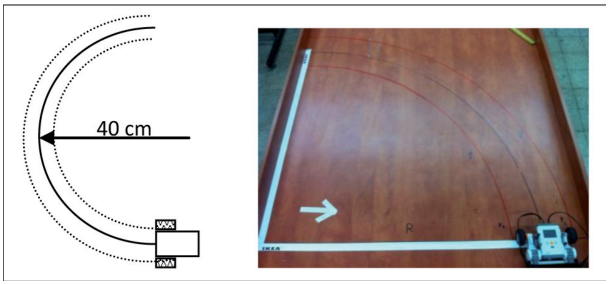

Fig. 7. Visualization of the different paths taken by the wheels and robot center

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[6] http://rgbhodoi.blogspot.co.il/2011/12/going-down-fast.html

ing deviation from a straight line. Due to the robot breadth, the inner and outer wheels traverse arcs of different radii. The difference in radii equals the distance between wheel centers (Fig. 7).

Assuming the wheels roll without slipping, each full rotation advances the wheel center by the wheel circumference. Since the wheels have equal radii, the inner wheel should complete fewer rotations than the outer wheel, as the robot advances along the curve. Thus, the rotation rates of the wheels need to be coordinated according to the difference in the length of the inner and outer arcs. For example, the distance between a Lego robot's wheel centers is 17.0 cm, and the radius of each wheel is 3.5 cm. For the robot to follow a semi-circular path with a median radius of 40 cm, an outer radius of 48.5 cm and an inner radius of 31.5 cm, the following spatial relationship will need to be established:

	Distance covered	Rotation angle	Motor power ratio
Inner wheel	$\frac{2\pi \cdot 31.5}{2}$	$\frac{360 \cdot \pi \cdot 31.5}{2\pi \cdot 3.5} = 1671.42°$	$\frac{31.5}{48.5} = 0.6495$
Outer wheel	$\frac{2\pi \cdot 48.5}{2}$	$\frac{360 \cdot \pi \cdot 48.5}{2\pi \cdot 3.5} = 2057.14°$	

D. Finding the diameter of an outlined circle

Fig. 8 shows the circle circumference outlined by a painted black line. The robot's light sensor recognizes the circle's outline. The robot starts out from a random point on the circle's edge and moves into the circle in a random direction. How should the robot be programmed to obtain the circle diameter?

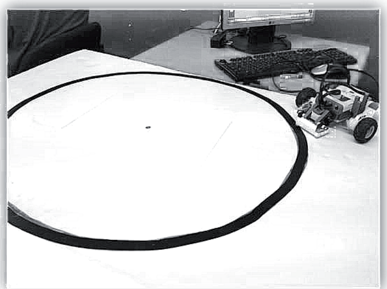

Fig. 8. The robot about to travel into the outlined circle

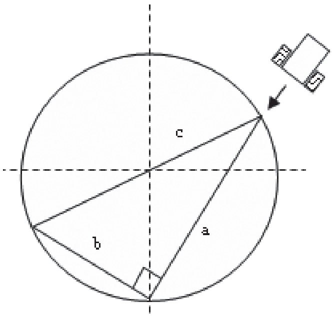

Fig. 9. Visualizing the spatial relationship between the diameter and the inscribed angle resting on it

Fig. 10. Programming the robot to move and measure side a

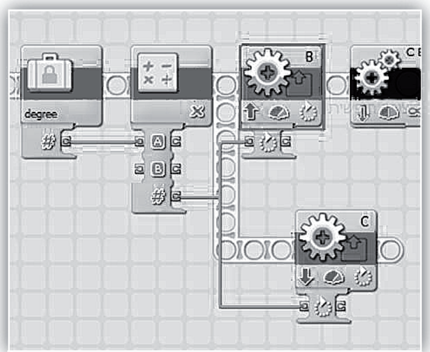

Fig. 11. Programming the robot to make a 90 degree turn around its center

The suggested solution invokes the spatial relationship between the diameter and inscribed angle (Fig. 9).

1. The robot moves into the circle and measures the distance between the first and second points on the circumference – side a (Fig. 10).

2. The robot makes a 90 degree turn inside the circle (Fig. 11).

3. The robot reverses out of the circle, then advances until it detects the circle outline. The program shown in Fig. 10 is now repeated and this time it measures the distance of side b.

4. The length of the hypothenuse is calculated using Pythagoras's theorem. Finally, the result is displayed.

The decision to construct a sequence for the "90 degree turn", rather than employ the available dark green block, was intended to promote better understanding of the spatial relationship between circle radius and rate of rotation for different wheels.

6. Summary

There is wide agreement throughout the science education community that science learners of all ages should be provided with opportunities to experience and practice the art and craft of engineering design. This can best be achieved by project-based learning, which can be carried out within the school curriculum or as extra-curricular elective programs, often hosted by the education or engineering departments of institutions of higher education.

In this paper we have focused on the technique of guiding students by exploring and explicating spatial relationships that can be found within their project space. High school science learners acquired a store of formulas in their science and math school lessons. Many of these formulas describe spatial relationships within systems. Students are accustomed to activate this stored knowledge in the context of end-of-chapter problem solving. We have provided several examples of the ways in which this stored and often "inert knowledge", can be activated by discussions, static and dynamic visualization and structured guidance.

The transition from traditional, teacher centered, text book based instruction to challenging problem solving and technological design is non-trivial, even for high ability science learners. Students must be provided with mental and technical tools and sufficient guidance to help them succeed in the unfamiliar learning environment. Students also need to become aware of the differences that exist between theoretical models and material reality. For example, the focus of a lens is not a mathematical point and the robot's wheels are not perfect circles. This is achieved by comparing experimental results with expected theoretical values, and refining the engineering design to compensate for these effects.

The activities we have described represent a small sample of activities that have been implemented in our work with high school science majors over the past decade. Initial testing indicated the absence of cognitive bridges between theoretical math and physics knowledge vs. material reality. We have collected ample evidence that exploring spatial relationships in real life problem contexts promoted the creation of such bridges. Students' new insights were expressed in the working models they created, solving a variety of seemingly different technological problems by implementing core ideas. For example, the triangulation method for remote distance measurement described in section IV in this paper, has been implemented in systems of transportation (collision avoidance and smart road signs), security (intruder detection and location) and assisting visually impaired persons.

Experience has taught us that achieving the desired transformation of the students' mental world view does not occur spontaneously. It requires an instructional design of revisiting the analysis of spatial relationships in a variety of contexts, using suitable pedagogical tools.

Appendix

The Physics and Industry (P&I) program is a 15-month, extracurricular, accredited program for 11th & 12th grade physics majors. The students meet on a bi-weekly basis at the Davidson Institute for Science Education, extending their physics knowledge into the field of electro-optics and constructing working models in response to authentic real world technological problems. Project topics include optical surveillance of restricted premises, assisting blind persons, preventing vehicle collisions, color recognition etc.

Detailed descriptions of the P&I instructional framework can be found in previous presentations [25]. For the purpose of the current paper we will focus on ideas for solving the problem of remote distance measurement which activate stored formal knowledge and contribute to an improved understanding of spatial relationships.

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