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The valuation of exit option in a lignite mine using Monte Carlo simulation

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Keywords

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The valuation of exit option in a lignite mine using Monte Carlo simulation

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This study aims to demonstrate the application of simulation techniques to the valuation of real options. The nature of the paper is methodological and empirical. The purpose of the valuation of the option to close a lignite mine in Poland is to demonstrate the methodology and advantages of employing Monte Carlo simulation in the valuation of real options. Close to actual numerical data reveals a complex optimization problem in the context of strategy selection by decisionmakers. Numerous factors (extraction costs, reclamation costs, the write-off for the reclamation fund, etc.), their interpenetration and multilevel influence on the decision to close the mine early enables simulation methods to demonstrate their valuation capabilities. The valuation techniques used in the paper, particularly the simulation comparative valuation method, are described in detail and are rooted in the literature and theory of finance.

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1. Introduction

M anaging an investment or an organization that offers real options provides flexibility. Such planned opportunities are especially valuable in a highly volatile market environment and should be considered when making investment decisions and operating a business. The term real options was first used in the literature in a 1977 paper by Stewart Myers [[1](#page-16-0)]. He was one of the first to note that operating activity can create additional value, which, like the prospect of organizational learning or the flexibility of a company's operations, is not accounted for by discount valuation methods [\[2](#page-16-1)]. The literature defines real options in many ways. Primarily as a method for assessing the financial efficiency of an investment or operating activity, including the value of flexibility (particularly future development opportunities, but not exclusively), using the theory of pricing financial options [\[3](#page-16-2)]. Real options, on the other hand, permit a mode of thought and management that enables the analysis, formulation, and active modification of operating activity in light of the uncertainty and flexibility associated with the activity [[4\]](#page-16-3).

In the case of large, complex, multi-stage, highrisk investments in which a variety of implementation scenarios are available, real options are especially important for businesses. The literature is rich with examples of the application of real options in practice, particularly in R&D projects, such as in the pharmaceutical industry [[5\]](#page-16-4), large infrastructure investments [[6\]](#page-16-5) and the mining industry [\[7](#page-16-6),[8\]](#page-16-7), which will be the focus of this paper.

Due to the purpose of the paper, closing operationfocused options will be the most important. The discontinuation option is the right to cease operations when there is a clear and sustained deterioration in market conditions. The investor can then dissolve the company and exercise the liquidation value. Determining the optimal time for ending operations requires consideration of the mine's decommissioning value, current operating losses, and projected profitability. The discontinuation option is irreversible, so any potential future benefits that may arise after the investment has been closed could be lost. The business termination option reduces the negative impacts of uncertainty by assigning the project's worst possible outcome as its decommissioning value. The aerospace industry, infrastructure investments, new

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product launches, and the mining industry can all use opt-out options in high-capital investment projects.

Since the 1980s, the issue of a mine closure alternative has received considerable thought. The matter of mine shutdown has often been discussed as a component of various other alternatives in mine operational management. The significance of operational flexibility in enhancing the value of mines is widely acknowledged, thereby extending beyond the limitations of the net present value (NPV) framework, as per Guj's [[9\]](#page-16-8) findings. Savolainen [[10\]](#page-16-9) conducted a comprehensive literature review of 92 academic research articles on the mining investment literature published between 1995 and 2015. The study revealed a multitude of complex options available in the various stages of mining, including exploration, development, extraction, and abandonment. Research was also carried out related to the evaluation of mineral extraction contracts as well as the planning and evaluation of mines [\[11](#page-16-10)].

Brennan and Schwartz [[7\]](#page-16-6) were among the pioneering authors who investigated strategies for mine closure. Their study involved a thorough analysis of the options available to shut down or abandon a mine, taking into account the futures and spot prices of the relevant commodity. The study conducted by Moel and Tufano [[12\]](#page-16-11) analyzed the annual opening and closing decisions of 285 developed North American gold mines during the period 1988–1997. The findings of the study suggest that the mine operating decisions are in accordance with the principles of the real option theory. The researchers found that firm-specific managerial factors, which are typically not taken into account in a strict real option model, are actually related to the decision-making process of whether or not to shut down a mine. This finding is unexpected, given the principles of the theory of real options.

The valuation of mines and their embedded option to close the mine is a topic that has been explored in the real options literature, as reviewed by Colwell et al. [[13\]](#page-16-12). Valuation of real options is accomplished through the use of the least squares Monte Carlo (LSM) methodology. According to the authors, who utilized data from Brook Hunt Mining and Metal Industry Consultants in the UK spanning from 1992 to 1995, it was determined that the values of embedded options exhibit a high degree of sensitivity to estimation errors in the model's input parameters. The analysis indicates that the mean and median values of the shutdown options have considerable economic importance. However, there exists a wide range of variability in option values.

The findings of Jewartowski et al. [[14\]](#page-16-13) indicate that it may be financially viable to cease mining

operations prior to the complete exhaustion of the deposit, given the consideration of the fluctuation in extracted lignite prices and the associated expenses of land reclamation. The case study of the Polish lignite mine demonstrates the utility of incorporating price volatility into option analysis. The put option valuation in this study was conducted through the implementation of binomial trees.

Mine shutdown analyzes are often conducted in conjunction with the process of suspending and subsequently reopening a mine. According to Sabour's model [\[15](#page-16-14)], there is a positive correlation between the remaining life of a mine and the threshold price for reopening a suspended mine, as well as the threshold price for abandoning the mine. The aforementioned interdependence suggests that the factors utilized to make decisions about the alteration of the operational mode of a mine are subject to variation over time, with fixed costs, physical depreciation, and technological obsolescence serving as influential factors.

Guj and Chandra [[16\]](#page-16-15) present interesting findings. The researchers employed six distinct techniques to assess complex real options in the mining industry. These methods included an analytical approach, approaches based on binomial trees, different variants of decision trees, and Monte Carlo simulation. The application of basic option pricing techniques involved the utilization of a single aggregated volatility metric, as in the Marketed Asset Disclaimer (MAD) framework introduced by Copeland and Antikarov [[17\]](#page-16-16) (p. 247). The findings suggest that the use of fully disaggregated decision trees or Monte Carlo simulations, which incorporate the probability distributions of each distinct source of risk, results in a lower valuation of real options ranging from 15% to 35%. The authors claim that this approach is more realistic and aligns with the need for a conservative valuation. Previous research conducted by Brandao [\[18](#page-16-17)] suggests that the use of MAD valuation, which combines multiple risk factors into a single measure of volatility, results in a systemic positive bias.

The objective of our research is to assess the potential benefits of implementing a shutdown of the lignite mine earlier than the initial scheduled date. We employ a novel approach to option valuation that relies exclusively on the Monte Carlo method, which is applied to the financial model of the mine. The primary determinant of the feasibility of the mine shutdown option is the interaction between the expenses incurred in land reclamation subsequent to mine closure and the accumulated value of the land restoration fund that the mining company has accumulated during the operation of the mine. Moreover, we would like to check the influence of different factors on the value of real option to close the mine and confirm the influence of high impact of fixed costs and other parameters on the option value, like in studies of Sabour [[15\]](#page-16-14) and Colwell et al. [[13\]](#page-16-12).

2. Materials and methods

Real options are frequently valuated using methods based on financial option valuation techniques. Basic approaches like Black and Scholes [[19\]](#page-16-18) and Cox, Ross, and Rubinstein valuation model [[20\]](#page-16-19) can be employed for real option valuation. The Monte Carlo method are frequently utilised as it allows to valuate more complex options such as barrier, vanilla, and Bermuda options [\[8](#page-16-7)]. A relatively new method of real options valuation that maps the volatility of the environment is the Datar-Mathews method [[21](#page-16-20)]. It is based on the Monte Carlo simulation but combines the results of simulation with the Black-Scholes model [[22\]](#page-16-21).

In our study, we perform a simulation-based valuation of the shutdown option (exit option) for an opencast lignite mine, as described in subsequent sections. The proposed methodology of simulation comparative valuation (SCV) is based on a dual Monte Carlo simulation (2 MC). The 2 MC framework, which refers to the assessment of real options, was formulated by Wisniewski in 2008 [\[23](#page-16-22)]. The determination of the real option's worth in this approach is based on the disparity between the project's value with the option and the project's value without the option. The two values are determined by computing the mean of the outcomes derived from the Monte Carlo simulation. The valuation of a project, with and without an option, is dependent on the input parameters utilized in the financial models that characterize these investments. The input variables may exhibit determinism and embody the variability that is inherent in the project and its surroundings. The modelling of input variables offers significant freedom, as each input parameter can be stochastically characterized and linked to correlation or autocorrelation within this methodology. Thismethod facilitates consolidation by aggregating all risk factors based on the MAD method introduced by Copeland and Antikarov [\[17](#page-16-16)].

The simulation comparative valuation (SCV) methodology is based on a widely recognized set of analytical tools used to evaluate the efficiency of an investment, such as net present value. As such, it is imperative that the SCV approach aligns with the fundamental assumptions of the NPV framework. Using suitable algorithms, particularly decision tree analysis and simulation techniques, it becomes

feasible to perform a real option valuation by expanding the analysis of the NPV financial model.

A simulated comparative valuation is a real option valuation method that consists of a conditional comparison of the expected value of the project in which the real option is exercised (NPVext) and the investment value in which the option is not exercised (NPVbase) according to the following formula:

$$
ROV = E[max(NPVext - NPVbase; 0)]
$$
 (1)

$$
NPV_{ext} = f(FCF_{base}, t, r)
$$
 (2)

$$
FCF_{base} = f\begin{pmatrix} p_1 & p_2 & \dots & p_n \\ p_1 & 1 & \dots & \dots \\ p_2 & \dots & 1 & \dots \\ \dots & \dots & \dots & 1 & \dots \\ p_n & \dots & \dots & \dots & 1 \end{pmatrix}
$$
 (3)

$$
NPV_{ext} = f(FCF_{ext}, t, r)
$$
\n(4)

$$
FCF_{ext} = f(FCF_{base}, t, r, I, B)
$$
 (5)

$$
FCF_{ext} = f \begin{pmatrix} p_1 & p_2 & \dots & p_n & I & B \\ p_1 & 1 & \dots & \dots & \dots & \dots \\ p_2 & \dots & 1 & \dots & \dots & \dots \\ \dots & \dots & \dots & 1 & \dots & \dots \\ p_n & \dots & \dots & \dots & 1 & \dots \\ I & \dots & \dots & \dots & \dots & 1 \\ B & \dots & \dots & \dots & \dots & 1 \end{pmatrix}
$$
 (6)

 t – project duration,

 r – capital cost,

 FCF – free cash flows,

 B – benefits resulting from the realization of the real option,

 I – expenditures necessary to exercise the option.

It should be emphasized that the methodology each time requires individualization in the valuation process. Base and extended models should be built from scratch, adjusting their structure and functions to the characteristics of the operating activity and the option built on it. The SCV algorithm is so universal that it can be used to determine the value of any real option, as well as complex options. With the use of financial modelling techniques, you can recreate the characteristics of each investment and program-related opportunities.

In the SCV method, it is not necessary to distinguish between the types of options and classify them. Each type of real option can be individually and in detail mapped in the simulation NPV. Thus, the characteristics of the valued option are described in the extended model, while the general algorithm of the SCV method remains unchanged.

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Determining the value of the real option is carried out by a conditional comparison (ROV greater than or equal to zero) of the value obtained from the model with and without options.

We presented the real option valuation methodology described in the previous chapter as an example of the valuation of the shutdown option, which is possible to implement in a lignite mine. The data for the presented case study, including in particular the volume of coal extraction, the level of prices, fixed and variable costs, etc., refer to a real lignite mine located in Poland. This valuation refers to the paper of Jewartowski, Mizerka and Mróz [[14](#page-16-13)]. The data from the above-mentioned paper were adapted for the construction of a simulation model of real option pricing, and additional assumptions were added regarding the parameters of the distribution of input stochastic variables (e.g. minimum price levels and production volume were defined). The variables are characterized in detail in the next part of the description.

The extraction of lignite at the mine, because of the properties of this fossil fuel, is closely related to its direct use in the nearby coal-fired power plant. Therefore, the demand for the power plant, which is the result of the state's energy policy and the state of the energy market, determines the extraction volume. Mining plans, in their basic version, have been developed for 26 years and assume a gradual decrease in production for the first 17 years, followed by an acceleration of mine closure processes, resulting in reduced production.

2.1. Model description

The financial model used to analyze the option to close the lignite mine includes all the relevant relationships affecting the economics of the mine's operations (cf. [formula 8\)](#page-5-0). In particular, four main relationships are mapped affecting the performance and value of the real option: the lignite mining plan, the operating leverage, the cost of reclaiming the open-pit mine, and the reclamation fund accumulated to rehabilitate the site after mining. Some of the parameters of the financial model at the beginning of the first year are the result of past actions taken by the company operating the mine and are the result of past accumulation of certain items

(such as the post-mining reclamation fund collected to date), or are the result of the previous state of the market and the organization of work, such as the found level of prices and costs, as well as past mining and the resulting estimate of site reclamation costs current at the beginning of the modelling.

$$
FCF_t = Revenue_t - FixedCosts_t - VariableCosts_t - \nYearlyMLF_t
$$
\n(7)

$$
Revenue_t = Production_t \times Price_t \tag{8}
$$

The model reproduces the technical mine exploitation plan and expresses it through the planned mine production in each year in the Pro $duction_t$ vector (cf. [formula 9\)](#page-6-0). Deviations from this plan can result from differences between the technical exploitation of the deposit and the actual mineral content of the deposit. Differences can also result from economic considerations and temporary deviations from the demand expressed in the technical production plan. These deviations were expressed in the model by introducing variability into the Production_t vector. Variability of output was described by a normal distribution $N(Production_{tr})$ $0.25 \times$ *Production*_t) up to the 16th year of the mine operation and N(Production_t, $0.1 \times$ Production_t) from the 17th to the 26th year. Additionally, it was assumed that, regardless of other factors, a certain minimum demand for lignite would be maintained due to the need to ensure the continued operation of the power plant, which is the main consumer of the product. This level was estimated at *Production*, $0.25 \times$ *Production*_t, that is, $x - \sigma$, and was introduced into the distribution as the limiting condition of the normal distribution. The time distribution of planned production over the life of the mine is shown, while an example of the statistical distribution of production in year 1 of operation is shown. The second element of variability on the revenue side is the selling price per ton of lignite (cf. [formula 9\)](#page-6-0), which is also described by a normal distribution with parameters $N(Price_t, 0.1 \times Price_t)$ and, like production, is limited on the left to one standard deviation (cf. [Table 1\)](#page-5-1).

Operating leverage results from the relationship between the level of fixed costs and variable costs and has been mapped in the model through a separate fixed cost plan and a link between variable

Table 1. Distribution of planned lignite production over time (million T).

Year										10			
Average lignite production	42.5	42.5	37.3	37.7	37.6	37.1	35.8	36.1	36.3	36.3	35.8	37.8	36.4
Year		15	16	17	18	19	20	21	22	23	24	25	26
Average lignite production	36.9	36.9	35.0	35.0	29.7	22.6	20.0	15.0	10.0	7.1		5.0	5.0

costs and sales volumes (see [formula 8\)](#page-5-0). Fixed costs are an important factor shaping the economics of operations, and for open-pit mines, they are quite significant and are about 70%; [\[24](#page-16-23)]. In our model, these costs account for about 63.7% of costs in year 1 and increase by the inflation rate in subsequent years. At the end of the mining period, from the year defined as $t_{decr.start} = 18$, fixed costs begin to decrease due to depletion of the deposit (cf. [formula 10\)](#page-6-1). As production decreases, fixed costs will decrease in proportion to the decrease in production (parameter $DecrProdRatio_t$, but at a rate slower than this decrease as determined by the parameter FCdecr-*Ratio_t* which is 0.4 over the entire period of the mine's reduction in production.

$$
UnitDR costs_t = UnitDR costs_{t-1} \times (1 + Inflation_t) \qquad (14)
$$

The value of the mine at the end of the mining period is described by [formula \(16\)](#page-6-5) and consists of the sum of discounted cash flows over the entire mining period plus the present value of the reclamation fund in the last year of mining (year n) and minus the discounted reclamation costs in that year. Similarly, we determine the value of the mine in each year of operation of the EV_k deposit ($k = 1, 2, ...,$ n), with the sum of discounted cash flows calculated for each year k in the range $t = 1, 2, ..., k$, while the present value of the reclamation fund and the present value of the reclamation costs are calculated at the end of the specified year k (cf. [formula 17\)](#page-6-6).

$$
FixedCosts_t = \begin{cases} FixedCosts_{t-1} * (1 + Inflation_t), t < t_{decr.start} \\ FixedCosts_{t-1} * (1 + Inflation_t) * (1 + DecrProducts_{t} * FCdecreaseRatio_t), t \geq t_{decr.start} \end{cases}
$$

 (13)

VariableCosts_t = Revenue_t \times VariableCostsRatio_t (10)

In the case of open-pit mines, an extremely important parameter affecting the economics of operations is the cost of reclamation of the open-pit mine, which occurs after the mineral is extracted. These costs are usually very high. Consequently, an important factor affecting the efficiency of an openpit mine is the level of write-offs from annual financial surpluses for subsequent reclamation costs. These write-offs occur annually (cf. [formula](#page-6-2) [12,](#page-6-2) variable YearlyMLF_t) and, in our model, represent a certain fixed percentage of revenues annually (variable YearlyMLFratio_t). These funds are accumulated in a mine liquidation fund (variable $MLfund_t$) with a reinvestment rate equal to ReinvestmentRate_t (cf. [formula 13](#page-6-3)).

$$
YearlyMLF_t = Revenue_t \times YearlyMLFratio_t \qquad (11)
$$

$$
MLfund_t = YearlyMLE_t + MLfund_{t-1} \times (1+ReinvestmentRate_t)
$$
\n
$$
(1+ReinvestmentRate_t)
$$

Cumulative reclamation costs (variable $DRcosts_t$) increase with inflation in the same way as unit reclamation costs (variable $UnitDRcosts_t$). The annual increase in reclamation costs is proportional to production in each year (see [formula 14\)](#page-6-4).

$$
DRcosts_t = DRcosts_{t-1} \times (1 + Inflation_t) + Production_t \times
$$

UnitDRcosts_t

$$
PV_{mine} = \sum_{t=1}^{n} \frac{FCF_t}{(1+r)^t} + \frac{MLfund_n}{(1+r)^n} - \frac{DRcosts_n}{(1+r)^n}
$$
(15)

 (9)

$$
EV_{k=1,...,n} = \sum_{t=1}^{k} \frac{FCF_t}{(1+r)^t} + \frac{MLfund_k}{(1+r)^k} - \frac{DRcosts_k}{(1+r)^k} =
$$

= $PV(FCF_{t=1,...,k}) + PV(MLfund_k) - PV(DRcosts_k)$ (16)

We calculate the value of the option to close the mine according to the concept of the value of the put options. Classically, the only condition for exercising an option for early closure of a lignite mine is the higher value of the mine in the case of early closure. This is described by [formula \(18\)](#page-7-0), and we denote this option by ROV_{c0} .

$$
ROV_{c0} = \max_{t=1,...,n} \left\{ \begin{array}{l} EV_t - PV_{mine} \mid (EV_t - PV_{mine}) > 0\\ 0 \mid (EV_t - PV_{mine}) \le 0 \end{array} \right. \tag{17}
$$

However, we have the additional characteristics of a lignite mine that are the additional conditions that must occur for the closure of the mine to be possible. These additional conditions are the collection of funds within the reclamation fund that are sufficient to carry out the reclamation of the mine site after closure. If this condition were not met, the cost of reclamation of the mine would be passed on to others, including, in particular, the local community, and perhaps $-$ due to the scale of the project $-$ to all taxpayers. In doing so, we recognize two cases. The first is ROV_{c1} (cf. [formula](#page-7-1) [19](#page-7-1)), defined as the exercise of the option to close the

mine only if the funds raised and cash flows from the mine in the year of option exercise combined are greater than or equal to the reclamation costs for a given stage of the open-pit mine operation. The second is ROV_{c2} (cf. formula 20), defined as exercising the option to close the mine only if the funds raised for reclamation are higher than or equal to the cost of reclamation at that stage of mine operation.

$$
PV_{mine} = \sum_{t=1}^{n} \frac{FCF_t}{(1+r)^t} + \frac{MLfund_n}{(1+r)^n} - \frac{DRcosts_n}{(1+r)^n}
$$
(18)

$$
EV_{k=1,...,n} = \sum_{t=1}^{k} \frac{FCF_t}{(1+r)^t} + \frac{MLfund_k}{(1+r)^k} - \frac{DRcosts_k}{(1+r)^k} =
$$

= $PV(FCF_{t=1,...,k}) + PV(MLfund_k) - PV(DRcosts_k)$ (19)

The option to close the mine will be exercised most often with a classic put option without additional conditions (ROV_{c0}). Also, the value of this option will be the highest. The other two cases of put options with additional conditions will be exercised less frequently. Condition 1 is less restrictive, and hence, the value of the ROV_{c1} option will be higher than the value of the ROV_{c2} option.

3. Results

The results of the simulation show that the operating activity of the mine consisting of the extraction and sale of lignite is profitable. The average NPV of the lignite mine is slightly greater than PLN 1.77 billion, which, with a median of PLN 1.74 billion, indicates an almost symmetrical distribution (there is a slight right skewness of the distribution; see [Fig. 1](#page-7-2)). The standard deviation is PLN 0.8 billion, which means that the coefficient of variation reached the value of 0.45. Therefore, the variability of the project is at a moderate level.

In this context, closing the mine before the planned 15-year operation period seems unjustified. However, the analysis of the value of the project exit option conducted in this paper indicates that in some circumstances, it pays to close a lignite mine earlier (see [Fig. 2\)](#page-7-3).

Fig. 1. Distribution of the NPV of the lignite mine in the base scenario from Monte Carlo simulation.

Fig. 2. Distribution of real option value (ROV_{c0}) from the Monte Carlo simulation.

The closure of the mine is imminent, and it is crucial to choose the right economic moment for the termination of operations. The option is designed in such a way that it is always exercised when there are adequate market conditions and when early closing is profitable. Therefore, the determined value of the option shows the optimal execution of the option due to the changing market conditions (price variability and variability in lignite extraction).

The value of the early termination option ROV_{c0} [\(Fig. 2\)](#page-7-3) is PLN 0.57 billion, which is 32% of the whole mine value in the base case. The valuation was made by comparing, in each iteration of the simulation model, the value of the base project and the alternative of using closing options under favourable market circumstances. The distribution is close to normal and has the right skewness. The standard deviation is small; its value oscillates around PLN 59 million.

The option is exercised in each iteration of the simulation, which means that each time it is profitable to close the mine earlier than at the end of the last period. The time distribution of production [\(Table 1\)](#page-5-1), which in later periods reaches very low values (close to 0), has a key impact on such frequent exercise of options. The option is exercised mainly in the period from 9 to 12 years of the project.

For ROV_{c0} , the exercise of the option may occur regardless of the available funds necessary to close the mine. [Figures 3 through 5](#page-8-0) show the number of simulation runs out of a total of 100,000 iterations in which the various mine closure options (c0, c1 and $c2$) are exercised in a particular year. In [Figure 3,](#page-8-0) the basic mine closure option not limited by any additional condition is exercised most often in the 17th year of analysis in about 34% of the runs. In practice, the exercise of the option depends on the raising of appropriate funds, which changes the characteristics of the time distribution of mine closures. The changes are small but significant. As in the case of ROV_{c0} , the option (ROV_{c1}) is almost always exercised (see [Fig. 4](#page-8-1)). Also when liquidation funds are collected in the form of write-offs and present value of current year cash flow $-$ in 99.4% of cases the mine operation is terminated early. This option is

the most often performed in the simulation in the 16th year of analysis in about 26% of the runs. The main difference is that, in the case of ROV_{c1} , the option is exercised earlier (larger values in periods from 6 to 16 inclusive). This is due to the discounting and reinvestment of cash accumulated in the mine liquidation fund.

In the case of ROV_{c2} , exercising the option in each iteration is limited due to the condition of acquiring enough funds for mine recultivation ([Fig. 5\)](#page-9-0). For this reason, the exercise of options is shifted in time, mainly to the last periods (from 23 to 25). At the same time, the number of cases (14.8%) in which the mine conducts its operations in accordance with the baseline plan is growing. Most simulation runs end with the c2 option being exercised in year 25 (almost 50% of iterations), a year before the mine closes as planned.

Analysis of the simulation results of the presented lignite mine model shows that with a certain configuration and level of fixed and variable costs, the option of early closure of a lignite mine is valuable. As expected, a reduction in the efficiency of the mine operation increases the real option value of closing the mine. Very important is the level of fixed costs, which are high for the mine and increase the value of the option to close the mine if demand falls or variable costs rise. We have shown that higher production volatility and higher product price volatility increase the profitability of closing a mine earlier. A decrease in production due to depletion of the lignite deposit, as well as an increase in variable costs $-$ for example, related to an increase in taxes for environmental use $-$ can also make earlier mine closure more profitable.

It should be added that the solution we presented assumes optimal option exercise in each simulation run in accordance with the theory of real options. The decision to exercise the option to close the mine takes into account all economic (price, costs, decommissioning fund) and technical (extraction of the deposit) factors together. It is made in each simulation run by comparing the value of the mine in operation until the planned end and the value of the mine closed earlier. In contrast, the manager at the time of the decision to close the mine is not sure of the value of the mine. At the time of the decision to close the mine, its final value is still a stochastic variable and extends within certain limits of variability. The manager, on the other hand, is only certain of the mine's liquidation value at that point. Based on the conducted analysis of the mine closure option, the probability of the correct exercise of the option at a certain point in time (see Charts 3 through 5) or at a certain price or cost level can be estimated. We will return to our assessment of the managerial view of the option situation in our conclusions.

4. Discussion

In general, an increase in the volatility of key input factors increases the value of the option. The put option associated with mine closure behaves similarly. This is clearly visible in the case of the volatility of the mine's production (see [Fig. 6](#page-10-0)). In all analyses, we examine three exit option values. The value of ROV_{c0} captures the value of the exit option without any restrictions and is identical to the theoretical value of the put option in the situation of abandonment. The value of ROV_{c1} introduces, in addition to the exercise condition of the option, an additional restriction that limits the exercise of the option only to situations in which the value of the reclamation fund and the cash flow in the year under study is sufficient to cover the cost of reclaiming the site after mine closure. This condition is occasionally not met even when the general

Fig. 6. Sensitivity of exit option value to changes in production volatility.

option execution condition is met. However, it occurs relatively often, leading to only a slightly lower option value for ROV_{c1} than for ROV_{c0} . The differences in the value of these two options are greater at low volatility compared to higher volatility, for which the difference is virtually constant. The ROV_{c2} option introduces even more restrictive assumptions on the exercise condition of the option by introducing an additional requirement that the cost of mine land reclamation can be covered only by the capital accumulated in the land reclamation fund. Since this fund, with the write-downs used in the base case, is sufficient at the end of the mine's life, the value of the mine's shutdown option in this case is much lower than for the previous two options, and the increase in the value of the option as volatility increases is also much slower than for the other two options.

The same is true for the volatility of lignite prices (see [Fig. 7\)](#page-10-1). However, the increase in the value of the mine closing option is less intense and occurs above price volatility of 25%. Up to this level, the increase in volatility causes a slight decrease in the value of the option resulting from the interaction of this

parameter with other input parameters. The differences between the normal mine closure option (ROV_{c0}) and the option with the condition of covering reclamation costs from the site reclamation fund supported by cash flow from the year of potential mine closure (ROV_{c1}) are greatest at low volatility and fade to virtually zero above price volatility of 45%. The most restrictive closure option, ROV_{c2} is again as before much smaller than the other two, but its value increases more rapidly with increasing lignite price volatility.

In summarizing the analysis of the impact of volatility on the value of the option to close the mine, it should be noted that, in the case of an open-pit lignite mine, the value of the option to close the mine early is relatively low due to low volatility and that of both output and prices. Extraction is driven by the technical plans for operating the mine, and the minimum size requirements got to the main (and often only) customer, which is the power plant. The same applies to prices, which are set in the contract between the lignite mine and the power plant. Usually, these contracts take into account the level of extraction

Fig. 7. Sensitivity of exit option value to changes in the volatility of the lignite price.

costs and do not introduce high volatility (if any at all) in prices for the final product.

Variable costs include various cost items that affect the sum of costs in proportion to the mine's operations. In our model, the proportionality of variable cost behavior is achieved through the percentage ratio of variable costs to revenues. In this way, variable costs are determined taking into account both the current intensity of operations and the current inflation rate.

The increase in variable costs, expressed in terms of the ratio of the share of these costs in revenue, results in an increase in the value of the closing option, with the value of the ROV_{c1} option being, as in the volatility analysis, slightly lower than that of the basic ROV_{c0} option without additional exercise conditions (see [Fig. 8](#page-11-0)). The ROV_{c2} option with the most restrictive exercise condition is, as in the volatility analysis, significantly lower than the other two options; however, it also increases as the level of variable costs increases. An interesting phenomenon with a high variable cost ratio is the sudden decrease in the value of the option for early closure of the mine ROV_{c1} , which is due to the achievement of negative profitability above about 38% of variable costs in the revenue structure. This has an impact on the more frequent negative value of cash flows in the year of earlier mine closure, so some iterations of the simulation that previously met the condition for exercising the ROV_{c1} option do not meet it, which in effect reduces the value of this option compared to ROV_{c0} .

Therefore, the general trend with regard to variable costs is that the value of the option to close a lignite mine early increases as these costs increase. However, it is important to remember that a downward shift in the break-even point will result in a decrease in the value of the option to close the mine based on its own resources. This is important

because an increase in these costs can be caused, for example, by tax changes that additionally burden the mine in proportion to production. Such can be, for example, the nature of additional environmental fees. When they exceed reasonable amounts (i.e., bearable by the mine), the costs of such changes will be passed on to the public since, from a certain level, the mine will not be able to cover these costs from its own revenues.

The conducted analysis shows that the lower the fixed costs of a mine, the longer it can maintain a positive operating profitability. So, if the operation of the mine at a low level of fixed costs is profitable, why close it? For this reason, the possibility of early closure loses its importance and value. The described dependencies are illustrated in [Figure 9.](#page-12-0)

When there are high fixed costs, which is the norm in lignite mines (fixed costs usually account for about 70% of total costs; see [\[24](#page-16-23)]), the option of closing the mine earlier becomes important. Depending on the condition of exercising the option (method of collecting funds for the early closure of the mine), the value of the option will change more or less dynamically. Assuming that we examine whether it is profitable to close a lignite mine earlier without the need to set aside cash from current operations for mine closure and reclamation (ROV_{c0}) , the value of the option changes almost exponentially.

However, when we make the execution of the option conditional on raising the appropriate fund supported by the current year cash flow (ROV_{c1}) , it can be seen that the value of the exit option has its maximum value close to PLN 940 million. It is observed that the value of fixed costs is around PLN 1700 million. The maximum ROV_{c1} is directly related to the distribution of the lignite extraction in the mine and the preparation of the mine for planned closure (fixed costs decreasing proportionally to the decrease in extraction starting from 2031).

Fig. 8. Sensitivity of exit option value to changes in volatility of the lignite price.

Fig. 9. Sensitivity of exit option value to changes in fixed cost.

Where the condition is that the mine liquidation fund must exclusively cover all costs of decommissioning and restoration (D & R) the option ROV_{c2} has little value compared to the previous two cases. Mine liquidation fund write-offs, in many cases, are not enough to cover $D \& R \text{ costs}$, so there are cases where a mine should be closed earlier for economic reasons, but there are not enough funds to do so. In this case, the described dependencies clearly limit the value of the output option.

Regulations on the recultivation of open-cast lignite mines affect the possibility of early closure of the mine. In Poland, there are legal provisions regulating the minimum amount of write-offs, which are to serve the efficient and safe closure of the mine and its subsequent reclamation in accordance with the natural environment and the needs of local communities.

According to the Polish geological and mining law, an entrepreneur who has obtained a concession for activities related to: 1) extraction of minerals from deposits, 2) exploration and exploration of hydrocarbon deposits and extraction of hydrocarbons from deposits, 3) underground nonreservoir storage of substances, 4) underground storage of waste must set up a mining plant liquidation fund and accumulate funds on it. The fund's resources are accumulated in a separate bank account in the form of cash. The fund's resources may also be collected in the form of treasury bills or bonds issued or guaranteed by the state treasury. Open-pit mines, in accordance with the law, should allocate to the fund the equivalent of not less than 10% of the mining fee (tax) due. In simple terms, the mining fee is the product of the tonnage of the extracted mineral and the rate per extracted ton. The fee rates are updated every year, and the minister responsible for the environment announces them in the Monitor Polski (official journal of the Republic of Poland). For 2022, this rate is symbolic of PLN 2.22 per tonne of extracted lignite. Taking into account the

production of the mine in the first year, that is, 42.5 million tons, it can be estimated that the mining fees would amount to approximately PLN 95 million. 10% of this value, i.e., PLN 9.5 million, means the minimum write-down for the reclamation of the mine. This is far too little, especially since the estimated costs of decommissioning and recultivation for the first mining period alone will amount to approximately PLN 65 million. In practice, when a decision to close a mine is made, the write-downs are adjusted to the closing date, or the owners cover closing costs with current surpluses. The minimum, prescribed by law, write-offs for the costs related to the liquidation of the mine and the reclamation of the post-mine land are not sufficient. They are a theoretical value; in reality, the management boards of mines decide on higher levels of write-downs. In the model for the purpose of creating a liquidation and reclamation fund, the level of 2% of revenues from coal sales was assumed as the base level.

From a financial point of view, in most cases, a mine can be closed earlier regardless of the percentage of decommissioning and reclamation charges. However, in cases where the execution of the option to close a mine depends on the amount of the decommissioning fund collected and the writedowns are too low, the closure may not be possible. This is presented in the graph [\(Fig. 10\)](#page-13-0).

Given the need to raise adequate funds $(MLF + NPV(t) > D & R$ and $MLF > D & R$) and low percentages of annual decommissioning and reclamation charges, the value of the option to close the mine is lower than it could be (see ROV_{c0}). If the writedowns increase to approximately 3% of the option value in the three variants, it is essentially the same.

The value of the mine closure option depends on the financial conditions of the mining project (discount rate, reinvestment rate). When examining the efficiency of a lignite mine, it is very important to use appropriate capital cost rates and reinvestment rates for calculations. This is especially important

Fig. 10. Sensitivity of the exit option value to changes in the decommissioning and reclamation provision (% of annual sales revenue).

for long-term investments, where financial math can create and destroy value. In the case of a valued mine closure option, the high discount rate reduces the value of the option (see [Fig. 11](#page-13-1)). At the cost of capital, not only positive cash flows related to operating activities are discounted, but also expenses necessary to close the mine.

The discount rate in the model, due to the nature of cash flows (no possibility to include debt), was

assumed to be the discount rate at the level of 8.75%. As can be seen in the chart above, as expected, the higher the discount rate, the lower the option values.

The reinvestment rate works in two directions (see [Fig. 12](#page-13-2)). The first of them enables the option to be exercised under more restrictive conditions related to the collection of funds for the liquidation of the mine. The second reduces the value of the option: the higher the reinvestment rate, the more

Fig. 12. Sensitivity of the exit option value to changes in the reinvestment rate.

Fig. 13. Value of the real option to shutdown a lignite mine according to the reinvestment rate and the discount rate.

effectively the money accumulated over the years works, which makes closing the mine less profitable. The level of the reinvestment rate used to measure provisions for decommissioning and re-cultivation in open-cast mines in Poland ranges from 2.8% to 3.8% in the years $2016-2021$. The yield of long-term treasury bonds is most commonly used.

The cost of capital rate and the reinvestment rate should not be considered separately. They are tied together. For this reason, it is worth paying attention to the chart below (see [Fig. 13\)](#page-14-0), which presents a two-dimensional analysis of option value sensitivity (ROV_{c0}) . It fully confirms the observations described above.

The combination of high capital cost and reinvestment rates makes the value of the option almost worthless regardless of the exercise condition (how the money is raised to decommission and restore the mine). Lower reinvestment rates and lower capital costs make it more profitable to close a mine faster, as shown in the graph above.

5. Conclusions

The value of an option to close a lignite mine early is a strategic decision that depends only partially on the financial performance of the company. Regardless of the financial factors that affect the value of the option, the primary factors determining its closure are: 1) cooperation with a power plant that uses mined coal, the closure of a coal-fired power plant results in the closure of the mine; 2) depletion of the deposit; 3) the need for reclamation and environmental considerations; 4) the climate policy of Poland and the EU. Only after the boundary conditions defined by the above-mentioned

strategic factors have been adopted and the policymakers have determined the indicative timeframe in which the closure will take place is the efficiency calculation and selection of the optimal timing of the mine closure possible to estimate.

The manner and amount of write-offs to the mine liquidation fund are of great importance for the value of the exit option from the lignite mine. The statutory percentage of write-downs is insufficient to compensate for the damage caused during mining. It is, therefore, necessary to adjust the appropriate rate of write-offs so that it is possible to collect a sufficient amount of capital for recultivation. Regardless of whether the closure of the mine will be financed by the owner, only from the liquidation fund, or additionally from current cash surpluses from operating activities, ensuring adequate cash is the basic condition for the existence of a positive value of the real option.

The value of the option to close a lignite mine early is sensitive to changes in the value of financial rates, in particular, the cost of capital rate, reinvestment rate, and inflation rate. Each of them works differently $-$ increasing the cost of capital increases the value of the option, increasing the reinvestment rate decreases its value, and increasing inflation makes the option gain in value.

Other variables affecting the option value are relatively easy to interpret. Assuming that the mine can be closed at any time, all parameters that reduce the base value of the mine's NVP, e.g. increasing fixed and variable costs, decreasing production, and lower prices, will result in a higher value of the option. Factors that increase the financial efficiency of a lignite mine reduce the value of the option and mean that it will either not be exercised or will be exercised in the last years of the mine's operation. The tightening of the

climate policy causes an increase in the variable costs of operations, which results in a higher value of the option to close the mine earlier.

We utilize a new method of option valuation based solely on the Monte Carlo approach applied to the financial model of the mine. The advantage of this approach is not only the possibility of combining a few sources of uncertainty or introducing changes in volatility during the option life (like in the MAD approach of Copeland and Antikarov, [\[17](#page-16-16)]) but also to check out many practical characteristics of a real option situation affecting the viability of option execution. As always, the estimation of input parameters in the option pricing model is kind of problem. However, because the SCV method involves only the variables contained in a typical financial model describing the decision situation, they are easier to estimate than the parameters found in other methods of real option pricing. A typical problem in the valuation of real options using traditional methods is the estimation of the volatility of the value of the investment project, which in the proposed SCV approach is solved by implementing the volatility in the financial model and comparing the simulation results in two variants. Therefore, the estimation of parameters and volatility is more intuitive than in traditional methods of valuing real options. Of course, estimating the values of the parameters describing the financial model still remains a practical problem, as in any valuation method based on future cash flows. As with any valuation method, it is helped by formalization and consistent procedures to estimate parameters and their statistical distributions.

It should also be noted that the method we have described for valuing real options makes exactly the same assumptions as all other methods for valuing them. One of the main ones is knowing the value of the project (in our case, the value of the mine) at the time the option is exercised. In the real world, however, this value is stochastic. In reality, the manager is unable to predict the future and can only make the decision to exercise the option with a certain probability of success. The manager does not know the strike price of the option to close the mine because, from his/her point of view, at any time when he would make such a decision, its value is uncertain (at most, it can be described by a stochastic distribution). This translates into the stochastic nature of other parameters that could act as a proxy variable for the value of the project and facilitate the decision to close the mine. Such a variable could be, for example, the price of lignite or the annual profits achieved. Our analyzes indicate

that the proxy variables are stochastic in nature, and setting a certain level at which the option exercises (exactly as the manager would make that decision) based on their distributions in each year yields the right result only with a certain probability. The same applies to the year in which the option is exercised and the impact of this factor on the probability of correct decision. In order to apply this concept in practice, it would be necessary to reverse the model we are presenting so that the option is exercised at certain levels of the proxy (decision) variable and not by comparing the value of the project with and without the option included. The valuation results obtained in this article provide input for estimating the acceptable level of the decision variable (e.g., the price of lignite) at which the option exercise based on a proxy variable is closest to the optimal real option exercise according to the assumptions of option theory. The method to verify such an approach is a double Monte Carlo simulation based on the simulation over a decision tree, but its description is beyond the aim and scope of this article. Based on the presented valuation of real options made using simulated comparative valuation method, it can be seen that this method allows:

- obtaining an answer to the question in which year the early mine closure option should be exercised, taking into account the current financial parameters and their statistical description and how much it is worth;
- searching for the optimal option value;
- valuation with a large, multithreaded investment and option exercise even with complex conditions (Bermuda options, variable exercise condition, and variable option strike price);
- obtaining information on option value distributions and exercise probabilities;
- analysis of the sensitivity of the option value to stochastic input parameters and to the characteristics of the exercise condition of the option.

The SCV method is not without disadvantages. The main ones include:

- The need to build an often complex simulation financial model. The method is susceptible to errors in model construction. There is no universal, repeatable method of constructing a financial model due to the individual nature of investments and real options.
- A stochastic or deterministic description of many parameters describing the decision-making situation is necessary. The need to establish assumptions is both an advantage (managers'

knowledge can be used) and a drawback (complexity and the need to establish assumptions about the model's input variables).

- The need to implement a real option (often with the complicated condition of option exercise) into the base financial model.
- The need to perform numerical operations on additional software (e.g. Crystal Ball, @Risk, DPL, etc.).

Our approach to valuing real options using the SCV method can be applied not only to options for early termination of operations $-$ as in the case of a lignite mine $-$ but also to estimate the value of other types of decision-making flexibility in large investments. For example, the SCV method can be used to estimate options for phased implementation of investment, options for expanding operations as a result of technical and economic changes in the environment, switching operations between different options for the use of fixed assets, as well as for valuing options for flexible switching on or off the operation as a result of responses to changing economic conditions of the project's operation.

Ethical statement

The authors state that the research was conducted according to ethical standards.

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Conflict of interest

The authors declare no conflict of interest.

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