PAN
ARCHIVES OF ACOUSTICS
Vol. **39**, No. 4, pp. 639–644 (2014)

DOI: 10.2478/aoa-2014-0069

FEM Analysis of Active Reduction of Torsional Vibrations of Clamped-Free Beam by Piezoelectric Elements for Separated Modes

Elżbieta AUGUSTYN, Marek S. KOZIEŃ, Michał PRĄCIK

Institute of Applied Mechanics, Cracow University of Technology
Al. Jana Pawła II 37, 31-864 Kraków, Poland; e-mail: kozien@mech.pk.edu.pl

(*received October 9, 2014; accepted December 8, 2014*)

Beams with rectangular cross-section, with large length-to-width ratio, can be excited to torsional vibrations. If the piezoelectric elements are mounted to the beam in pairs at the same cross-section with two separated elements positioned on the same side of the beam, and the voltages applied to them are in the opposite phase, they produce twisting moments which can be applied to reduce the torsional vibrations. Results of FEM simulations are presented and analysed in the paper. All analyses are performed for a steel free-clamped beam. The piezoelectric elements made of PZT material are mounted in pairs on one side of the beam. The analyses are done for separated natural modes.

Keywords: piezoelectric elements, torsional vibrations.

1. Introduction

It can be observed that beams with rectangular cross-section, in which length-to-width ratio is high, can be excited to torsional vibrations. Such a model can be applied to the qualitative analysis of turbine blades regarding their torsional vibrations. The analysis of vibration of turbine blades can be found in (Rao, 1991). Possible methods of reducing such vibrations are proposed in (Hohl *et al.*, 1996; Pesek, Pust, 2014). In recent decades, the application of piezoelectric elements for the reduction of vibrations, especially of bending type, has been considered in many publications – e.g. (Brański, Szela, 2010; Filipek, Wiciak, 2008; Hohl *et al.*, 1996; Kozień, Wiciak, 2008; 2009; Leniowska, 2009). The other problem, not considered in the article, is choosing the proper control algorithm – see e.g. (Bismor, 2012; Mazur, Pawełczyk, 2013). The authors propose the idea of applying piezoelectric elements to reduce the torsional vibrations of beams (Augustyn, Kozień, 2014). If the piezoelectric elements are mounted to the beam in pairs at the same cross-section in such a way that two separated elements are lying on the same side of the beam and the voltages applied to them are in the opposite phase, they produce twisting moments which can be used to reduce torsional vibrations. A possibility for such an application of piezoelectric patches has been positively verified in the study (Augustyn, Kozień, 2014). The design questions usually posed regarding such an application are: the number of piezo elements, their position, and the value (amplitude for the harmonic type of cancellation) of the voltage fed to them. Due to its possible applications, in this paper, the beam with a monolithic (non thin-walled) cross-section is considered. Several other design choices need to be considered, such as the type of excitation, number of excited modes, shape of piezoelectric elements (square, rectangular), thickness of elements, their position along beam and across beam (for the chosen cross-section). The position of the elements can be optimized by considering different criteria: optimal placement for vibration control (Shouhstari, 1964), for shape control (Shoustari, 1964), or for sound radiation (Kozień, Wiciak, 2009). However, in general, for bending vibrations of beams and plates, the optimal placement is related to the areas of maximal bending moments in the structure (Brański, Szela, 2010; Brański, Lipiński, 2011; Shouhstari, 1964; Żołopa, Brański, 2014). The same idea was applied by the authors to analyse the reduction of torsional beam vibrations, with regards to the torsional moment. The choice of element shapes for the reduction of bending vibrations of plates were considered in (Wiciak, Trojanowski, 2014a; Wiciak, Trojanowski, 2014b). Due to the requirement of generating an antitwisting moment by pairs of piezoelectric elements, it seems essential to analyse the shape of the elements and, particularly, the distance between the pair of elements in the chosen cross-section. The piezoelectric elements are made of PZT material and mounted in pairs on one side of the beam.

Application of piezoelectric elements to reduction of torsional vibrations of a beam for the second natural mode is analysed in the paper. Three pairs of piezoelectric elements are used for reducing of vibrations. Positions of the elements and value of applied voltage amplitudes are considered. Analysis is performed for the steady-case by application of the finite element method (Ansys). The clamped-free beam of the rectangle cross-section is excited by harmonically variable concentrated force acting perpendicular to the beam in one of its free corners and generating torsional vibrations.

2. Positions of piezoelectric elements

2.1. Equation of motion

Let us consider a beam with a monolithic cross-section with two axes of symmetry. In such a case the equation of motion for torsion is independent from the equations of bending-type motion. Hence, the torsional vibrations can be analysed separately. The torsional vibrations can be described by Eq. (1), where $\varphi(x,t)$ is the angle of torsion of the cross-section, G is the shear modulus, ρ is the material density, J_s is the equivalent moment of inertia of the cross-section due torsion, J_0 is the polar moment of inertia of the thin-walled cross-section, $m_s(x,t)$ is the distributed torsional moment (external excitation), l is length of the beam.

$$\rho J_0 \frac{\partial^2 \varphi(x,t)}{\partial t^2} - GJ_s \frac{\partial^2 \varphi(x,t)}{\partial x^2} = m_s(x,t). \tag{1}$$

Finding the natural frequencies and corresponding modal shapes can be done for given boundary conditions. Moreover, $m_s(x,t) = 0$. When considering the clamped-free case, the equations take the form (2).

$$\varphi(0,t) = 0, \qquad \frac{\partial \varphi(l,t)}{\partial x} = 0. \tag{2}$$

Solution of the eigenproblem gives the following values of natural frequencies ω_n (3) and modal shapes $X_n(x)$ (4), $n = 1, 2, 3, \ldots$:

$$\omega_n = \frac{(2n-1)\pi}{2l}\sqrt{\frac{GJ_s}{\rho J_0}}, \tag{3}$$

$$X_n(x) = \sin\left(\frac{(2n-1)\pi}{2l}x\right). \tag{4}$$

The torsional moment $M_s(x)$ in a chosen cross-section can be found using the formula (5):

$$M_s(x,t) = GJ_s \frac{\partial \varphi(x,t)}{\partial x}. \tag{5}$$

2.2. Optimal position of piezoelectric elements

While considering the problem of the number and optimal positioning of piezoelectric elements the criterion of optimization needs to be defined. There are some possible solutions to this problem that can be found in literature. The first, most popular criterion is vibration control of the element (Shouhstari, 1964). The second possible criterion is shape optimization of the element (Shoushtari, 1964). The other criteria can be defined too, such as the optimal placement for minimizing of the sound radiated by the vibrating element (Kozień, Wiciak, 2009). In general, for bending vibrations of beams and plates, the optimal placement is connected with areas of maximal values of the bending moment in the structure (Brański, Szela, 2010; Brański, Lipiński, 2011; Shoustari, 1964; Wiciak, Trojanowski, 2014). The same idea can be applied when considering torsional vibrations of the beam. The analysis is performed for separated modes. The normalized mode shape functions (eigenfunctions) and corresponding function of the normalized torsional moment for the first two modes for clamped-free beam (shaft) are shown in Figs. 1, 2.

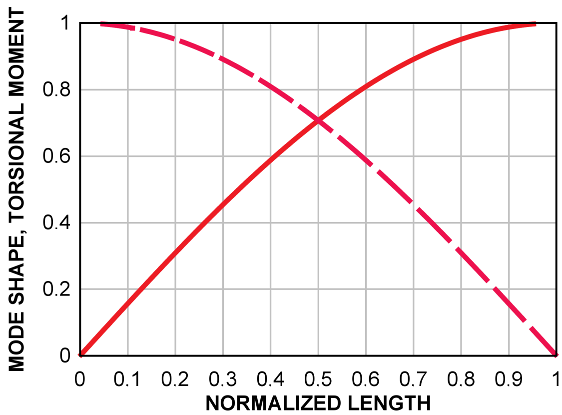

Fig. 1. Normalized mode shape function (solid line) and normalized torsional moment (broken line) for the first torsional mode.

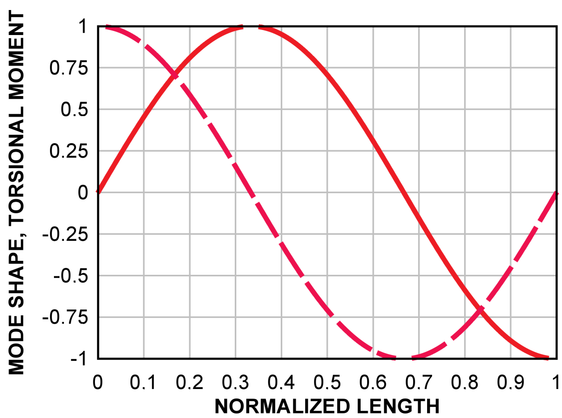

Fig. 2. Normalized mode shape function (solid line) and normalized torsional moment (broken line) for the second torsional mode.

It should be noted that the maximum values of the torsional moment are placed in regions of a zero torsion. Therefore, in such regions, pairs of piezoelectric elements should be placed in order to reduce the vibration of separated modes.

2.3. Configuration of pairs of elements

Piezoelectric elements, which are placed in pairs in the cross-section of the beam (shaft), especially in the case of rectangular cross-section, may generate the torsional moment if the voltage fed to the pairs of elements is in the opposite phase. This effect is observed due to generation by the pair of elements of the opposite type of strains across to the length of a beam, i.e. compression and elongation ones. Such an idea was applied in the experimental stand for multi-axial fatigue analysis in cycle regimes of high values (Straub *et al.*, 2011). An example of such pairs of elements for the static case in action is shown in Fig. 3 (finite element simulation).

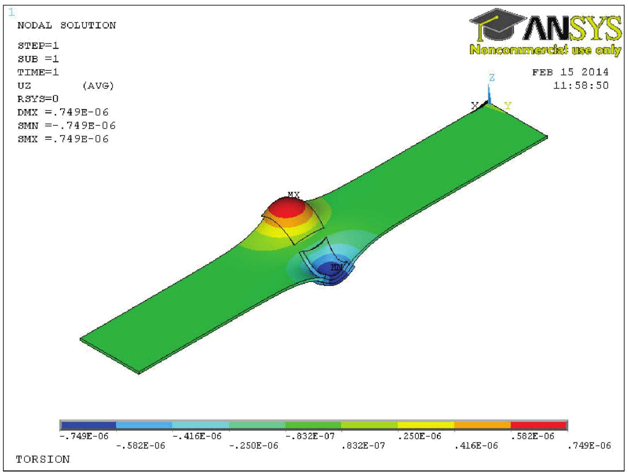

Fig. 3. Effect of action (in transversal displacements) of one pair of piezoelectric elements activated in opposite phases lying along one line.

The configuration of piezoelectric elements presented above can be applied for the reduction of the first torsional mode of the considered beam or for the higher modes but only when the pair of piezoelectric elements is positioned near the clamped part. If the elements are positioned in the areas of the zero angle of torsion in the middle of the beam, the change of the torsional moment sign is observed (see Fig. 2). Therefore, in order to generate the required antitorsional moment, two pairs of piezoelectric elements suitably activated in opposite phases should be used. An example of such configuration of elements is shown in Fig. 4.

It should be noted that due to different bending and rotational stiffness of the beam piezoelectric pair(s) of elements should be positioned in one line across the beam. If the elements are not collocated, the effect of bending vibrations may be observed. The effect is shown in displacements in Fig. 5. This effect is observed due to generation by the pair of elements of the opposite type of strains along the length of a beam, i.e. compression and elongation ones. If piezoelectric pair(s) of elements are positioned in one line across the beam the effect of opposite strains along the beam is cancelled.

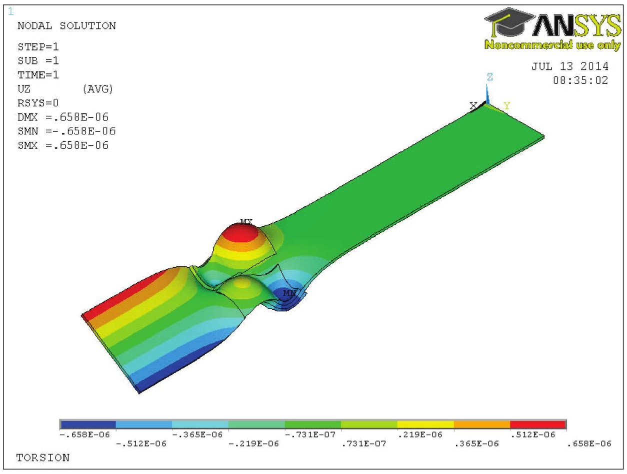

Fig. 4. Effect of action (in transversal displacements) of two collocated pairs of piezoelectric elements activated in opposite phases.

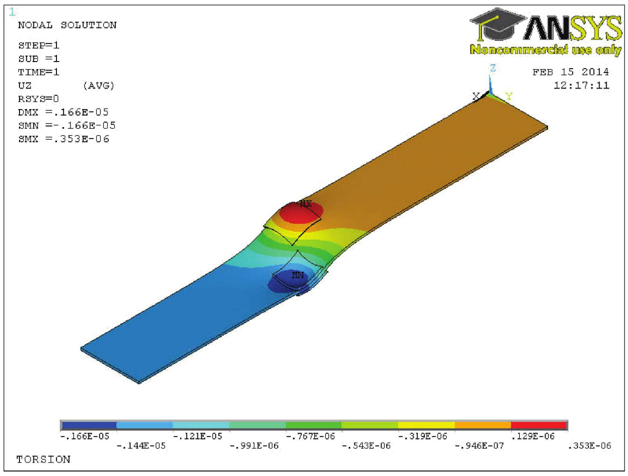

Fig. 5. Effect of action (in transversal displacements) of a non-collocated pair of piezoelectric elements activated in opposite phases.

3. FEM analysis

3.1. Description of analysis

The discussed analysis is done for the second mode of vibrations of the beam. In an earlier article (Augustyn, Kozien, 2014), the possibility of application of piezoelectric elements to reduce the torsional vibrations was verified based on the analysis of the first mode vibrations of the beam. A much more detailed analysis is performed for the second mode of vibrations. FEM simulations are done by using Ansys computer package for the clamped-free beam with the length of 0.154 m, width of 0.022 m, and height of 0.0008 m, made of steel (density $\rho = 7800\ \mathrm{kg/m^3}$, Young modulus $E = 210$ GPa, Poisson coefficient $\nu = 0.29$, damping ratio 0.005). The dimensions of piezoelectric actuators are $0.01 \times 0.01 \times 0.0006$ m. They are made of NEPEC, with the given piezoelectric properties: elastic coefficient matrix (6), piezoelectric coefficient matrix (7), and permittivity coefficient matrix (8).

$$\mathbf{c} = \begin{bmatrix} 13.2 & 7.1 & 7.3 & 0 & 0 & 0 \\ 0 & 13.2 & 7.3 & 0 & 0 & 0 \\ 0 & 0 & 11.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.6 \end{bmatrix} \cdot 10^{10} \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right], \tag{6}$$

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & -4.1 \\ 0 & 0 & -4.1 \\ 0 & 0 & 14.1 \\ 0 & 0 & 0 \\ 0 & 10.5 & 0 \\ 10.5 & 0 & 0 \end{bmatrix} \left[\frac{\mathrm{C}}{\mathrm{m}^2}\right], \tag{7}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} 804.6 & 0 & 0 \\ 0 & 804.6 & 0 \\ 0 & 0 & 569.7 \end{bmatrix} \left[\frac{\mathrm{pF}}{\mathrm{m}}\right]. \tag{8}$$

In this analysis, piezoelectric elements are mounted to the beam in pairs along its axis to the external surfaces of a beam in such a way that the distance between elements of the given pair and the fixed end is equal. Because the analysis is made for the second mode of vibrations, three pairs of elements are used. Having in mind the discussion about optimal position of elements (see 2.2), the pairs need to be placed in an area in which the values of the torsional moment for the second mode are maximal. There are two such areas for the second mode: in the location of the clamped end and at the distance of 2/3 of the total length of beam from the clamped end (see Fig. 2). Due to the finite dimensions of the elements, the influence of the clamped end is expected, therefore the position of the first pair is changed by a small variable parameter (one of the test made) in these analyses. The positions of the combined second and third pairs of elements are fixed in such a way that the line between the pairs is a nodal line of the second node (distanced $(2/3) \cdot 0.154\ \mathrm{m} \approx 0.1027$ m from the clamped end). The positions of the pairs of piezoelectric elements are shown in Fig. 6.

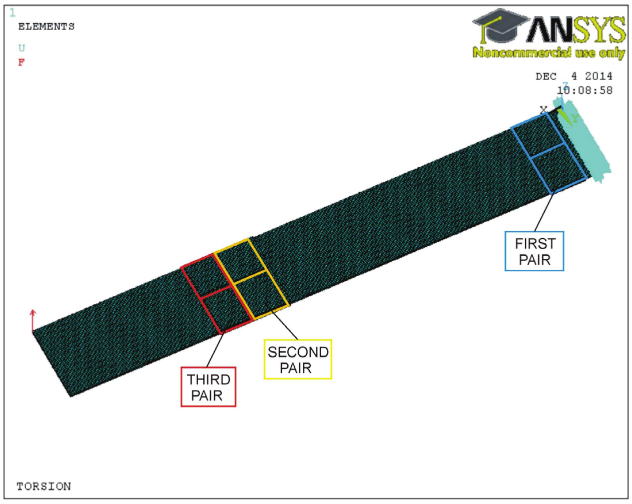

Fig. 6. FEM model of the beam and positions of pairs of piezoelectric elements.

To simulate the reduction of torsional vibration by piezoelectric elements, the steady-state case of excited vibrations is considered. Vibrations are excited by the concentrated harmonic force acting perpendicular to the surface of a beam in one of the free corners of a beam with frequency equal to the natural frequency for the considered second mode. The amplitude of the force was 0.1 N.

It is observed that during action of piezoelectric elements the bending modes of vibrations are not excited. Therefore, the results for only one control point are discussed. It is the point where the force is applied, located on the middle surface of the beam.

The following tests are performed:

1. Influence of the position of the first pair of piezoelectric elements.
2. Influence of the voltage amplitude for the first and group of the second and third pairs of piezoelectric elements.

3.2. Influence of position of the first pair of elements

It can be expected that due to the practical realization the position of the first pair of elements would influence the results. In the performed tests, three configuration options of the first pair of piezoelectric elements are considered. The positions of the edge of elements located near the clamped edge are chosen equal to: 0.001, 0.002, and 0.004 m. The positions of the combined second and third pairs are constant as described in 3.1. The applied voltages, set to the appropriate phase for each element, are of the same value for each element, and are changed through the parameter. The results of analyses are shown in Fig. 7, in the form of the amplitude of transversal displacement at the control point, for different configurations (different lines) and different voltage amplitudes.

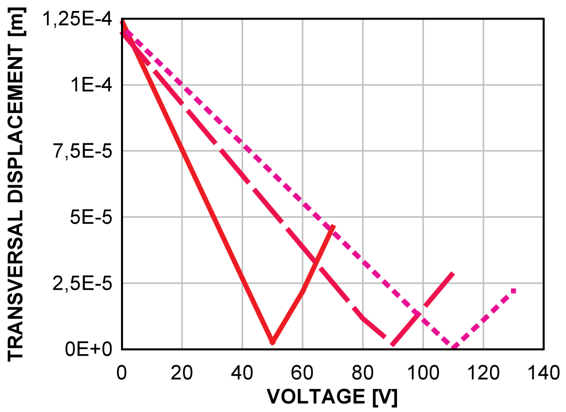

Fig. 7. Transversal displacement of the control point for different positions of the first pair of piezoelectric elements: distanced at 0.001 m (solid line), distanced at 0.002 m (broken line), distanced at 0.003 m (dotted line).

The slight shifting effect of the first pair of elements can be observed. The best results are for the shift of 0.004 m, but the applied voltage amplitude needs to be higher (110 V) in such a case.

3.3. Influence of the voltage amplitude for the first and combined second and third pairs of piezoelectric elements

In the second group of tests, the influence of different voltage amplitudes fed to the first pair and to combined second and third pairs of piezoelectric elements are analysed. Theoretically, the torsional moment has the same values in the considered areas (see Fig. 2), so the applied voltage should be the same. However, due to the practical realisation a small difference can be expected. The tests are performed for the same placement of the first pairs of elements as in the analyses discussed in 3.2. For each position, the chosen voltage amplitude is the optimal one as in the tests discussed in 3.2. Two groups of tests are performed:

1. Constant value of the amplitude of voltage fed to the first pair of piezoelectric elements (first set of elements) and variable voltage amplitude fed to the combined second and third pairs of piezoelectric elements (second set of elements).
2. Variable value of the amplitude of voltage fed to the first pair of piezoelectric elements (first set of elements) and constant voltage amplitude fed to the combined second and third pairs of piezoelectric elements (second set of elements).

The results of the analyses are shown in Figs. 8–10, with relation to the different value of shift of the first pair of elements. Figure 8 shows transversal displacement of the control point for a varied voltage amplitude for one set of elements and constant voltage amplitude for the other one (50 V) for shift of the first pair equal to 0.001 m. In Fig. 9 the same type of plots is ob-

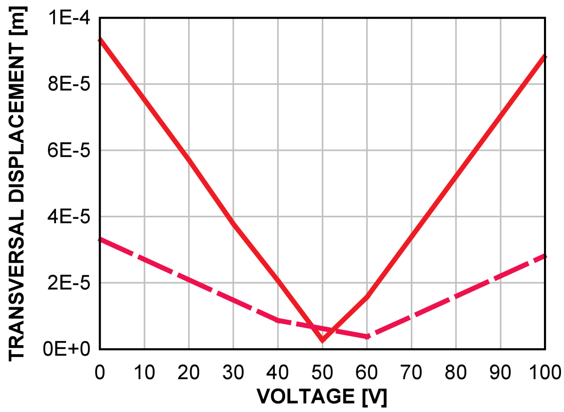

Fig. 8. Transversal displacement of the control point for the shift of the first pair equal to 0.001 m: voltage variable for the first set of elements, voltage applied to the second set of elements 50 V (solid line), voltage applied to the first set of elements 50 V, voltage variable for the second set of elements (broken line).

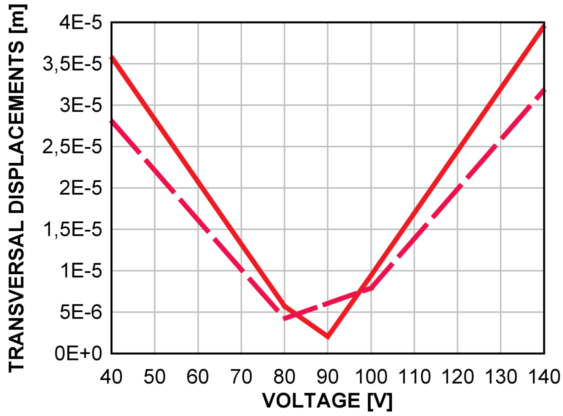

Fig. 9. Transversal displacement of the control point for the shift of the first pair equal to 0.002 m: voltage variable for the first set of elements, voltage applied to the second set of elements 90 V (solid line), voltage applied to the first set of elements 90 V, voltage variable for the second set of elements (broken line).

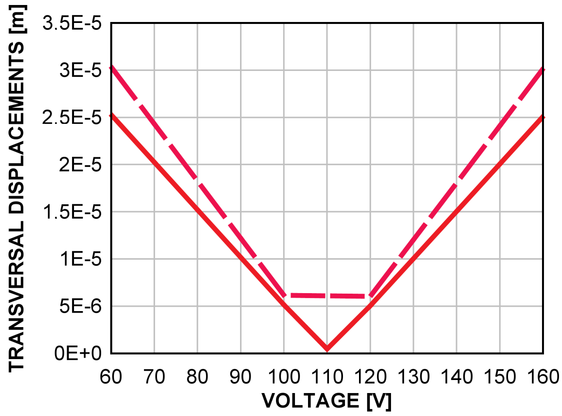

Fig. 10. Transversal displacement of the control point for the shift of the first pair equal to 0.004 m: voltage variable for the first set of elements, voltage applied to the second set of elements 110 V (solid line), voltage applied to the first set of elements 110 V, voltage variable for the second set of elements (broken line).

tained for the shift of the first pair equal to 0.002 m and constant voltage amplitude for one of the considered sets of elements equal to 90 V, and in Fig. 10 the plots obtained for the shift equal to 0.004 m, and constant voltage amplitude for one of the considered sets of elements equal to 110 V.

From the analyses it can be concluded that in practical applications the voltage applied to different groups of elements should have the same value.

4. Final remarks

The following remarks can be formulated based on the simulations.

- Bending-torsional vibrations can be identified in realistic structures.
- It is possible to apply piezoelectic elements to generate/reduction (actuator) and detection (sensor) of bending and torsional vibrations of beams.
- Due to phase of voltage put to elements it is possible to generate/reduce bending or torsional vibrations.
- A pair of piezoelectric elements which are to be placed near the clamped end of a beam may be positioned as close to the end as it is possible when considering the practical realization.
- The voltage applied to different groups of element should have the same value (and a suitable different phase) for reduction of separated modes of vibrations.

Optimisation of values of voltage amplitude, phase shift, number and position of actuators should be considered in the general application (irregular vibrations).

References

1. Augustyn E., Kozień M.S. (2014), *A Study on Possibility to Apply piezoelectric Actuators for Active Reduction of Torsional Beams Vibration*, Acta Physica Polonica A, **125** (4–A), 164–168.
2. Bismor D. (2012), *LMS algorithm step size adjustment for fast convergence*, Archives of Acoustics, **37**, 1, 31–40.
3. Brański A., Szela S. (2010), *Quasi-optimal PZT distribution in active vibration reduction of the triangular plate with P-F-F boundary conditions*, Archives of Control Sciences, **20**(LVI), 2, 209–226.
4. Brański A., Lipiński G. (2011), *Analytical Determination of the PZT's Distribution in Active Beam Vibration*, Acta Physica Polonica A, **119**, 936–941.
5. Filipek R., Wiciak J. (2008), *Active and passive structural acoustic control of the smart beam*, European Physical Journal, **154**, 57–63.
6. Hohl A., Neubauer M., Schwarzendahl S.M., Panning L., Wallaschek J. (1996), *Active and semi-active vibration damping of turbine blades with piezoceramics*, Proceedings of SPIE, 7288.
7. Kozień M.S., Wiciak J. (2008), *Reduction of structural noise inside crane cage by piezoelectric actuators – FEM simulation*, Archives of Acoustics, **33**, 4, 643–652.
8. Kozień M.S., Wiciak J. (2009), *Choosing of optimal voltage amplitude of four pairs square piezoelectric elements for minimization of acoustic radiation of vibrating plate*, Acta Physica Polonica A, **116**, 348–350.
9. Leniowska L. (2009), *Modelling and vibration control of planar systems by the use of piezoelectric actuators*, Archives of Acoustics, **34**, 4, 507–519.
10. Mazur K., Pawełczyk M. (2013), *Active noise control with a single-nonlinear control filter for a vibrating plate with multiple actuators*, Archives of Acoustics, **38**, 4, 537–545.
11. Rao J.S. (1991), *Turbomachine Blade Vibration*, Wiley, Chichester.
12. Pesek L., Pust L. (2014), *Blade couple connected by damping element with dry friction contacts*, Journal of Theoretical and Applied Mechanics, **52**, 3, 815–826.
13. Shoushtari N.D. (1964), *Optimal Active Control of Flexible Structures Applying Piezoelectric Actuators*, PhD Thesis, California Institute of Technology, Pasadena, California.
14. Straub T., Kennerknecht T., Robin P., Tort M., Kieffer G., Lapusta Y., Eberl C. (2011), *Small-scale multiaxial fatigue experiments in the very high cycle regime*, Proceedings of the 5th International Conference on Very High Cycle Fatigue, Berger C., Christ H.-J. [Eds.], Berlin, 473–478.
15. Wiciak J., Trojanowski R. (2014), *The effect of material composition of piezoelectric elements with chosen shapes on plate vibration reduction*, Acta Physica Polonica A, **125**, 4–A, 179–182.
16. Wiciak M., Trojanowski R. (2014), *Numerical analysis of the effectiveness of two-part piezoactuators in vibration reduction of plates*, Acta Physica Polonica A, **125**, 4–A, 183–189.
17. Żołopa E., Brański A. (2014), *Analytical determination of optimal actuators position for single mode active reduction of fixed-free beam vibration using the linear quadratic problem idea*, Acta Physica Polonica A, **125**, 4–A, 155–158.