Studia Geotechnica et Mechanica, Vol. XXXVI, No. 2, 2014
DOI: 10.2478/sgem-2014-0021

EFFECT OF VARIATION OF TEMPERATURE FIELD ON THE PROCESS OF THERMAL CONSOLIDATION OF TAILINGS POND "ŻELAZNY MOST"

MONIKA BARTLEWSKA-URBAN

Wrocław University of Technology, Institute of Mining, Wrocław, Poland,
e-mail: monika.bartlewska@pwr.wroc.pl

TOMASZ STRZELECKI

Wrocław University of Technology, Institute of Geotechnics and Hydrotechnics, Wrocław, Poland,
e-mail: tomasz.strzelecki@pwr.wroc.pl

Abstract: The following study presents numerical calculations for establishing an impact of temperature changes on the process of distortion of bi-phase medium. The Biot consolidation equations with Kelvin–Voigt rheological skeleton were used for that purpose. The process was exemplified by thermal consolidation of post floatation dump "Żelazny Most". We analyzed the behavior of the landfill under the action of its own weight, forces of floating filtration and temperature gradient. Values of certain effective parameters of model were obtained during laboratory tests on material obtained from the landfill. The remaining data for mediums with similar characteristics were taken from literature. The results obtained from the stress state in the landfill allow the magnitude of plasticity potential to be specified based on known strength criteria. Change in the value sign of the plasticity potential clearly testifies to the emergence of an area of plasticity of material from landfill, however, this does not indicate the loss of stability of this hydrotechnical structure.

Key words: *thermal consolidation, poroelasticity theory, rheology, Kelvin–Voigt body, the theory of Biot*

1. INTRODUCTION

For the selected cross-section, the article by Strzelecki and Bartlewska [1] presents the problem of distortion of Żelazny Most tailings pond under the action of its own weight and forces of the floating of water which filters through the landfill. Subsequently, the article by Bartlewska and Strzelecki [2], based on the numerical 3D model of landfill, analyzes the sensitivity of the hydraulic engineering structure to changing value of thermal parameters and an impact of periodic temperature variation of landfill surroundings on the process of deformation, temperature and stress distribution. In this paper, the authors attempt to identify potential areas of plastification of waste dump material, based on the Drucker–Prager values of plasticity potential.

The "Żelazny Most", being the largest post floatation waste landfill and one of the biggest hydraulic objects in the world, plays an important role, both for KGHM POLSKA Miedź S.A. as well as general evaluation of the influence of such objects on the environment. Therefore, the usage of mine waste dump to increase the "crown" of the landfill starts to gain significance especially in the context of stability of the hydrotechnical construction discussed. "Żelazny Most" plays a key role in the water-sludge economy of three Copper Ore Enrichment Facilities which enrich the mining output of copper ore using floatation method. Undoubtedly, it is a very specific and unique project. The perimeter of the reservoir is around 15 km and the current production of the KGHM mines causes a constant expansion of the reservoir by overbuilding (raising) "the terraces" around the reservoir. In consequence, the most important elements of the object, that is, the intakes and drainages of water as well as the dams are constantly under geotechnical monitoring. The data from the measurements are analyzed on regular basis, and the decision about the next "crown" formation is made by a group of experts after thorough evaluation. That is why, analyzing and predicting stress and distortion states are crucial prevention elements within the range of landfill creep and its surrounding terrain. From this standpoint, it seems legitimate to create more precise models of the landfill behaviors under the influence of different boundary conditions, including the temperature varying from season to season. Whereas the ability to point out the potential areas of waste dump materials plasticizing, that is, its stability losses, may be an important element for predicting undesired incidents.

For describing rheological properties of the soils and rocks, authors of various publications use the rheological model of mechanics for one or bi-phase mediums. According to numerous publications dealing with this subject, accepting the model of multiphase medium as initial one, it is assumed that the solid phase has hydraulically connected pores. The links between pores, or micro slots, allowing the filtration flow of fluid and/or gas allow the consolidation process. Taking into consideration the temperature aspect in this process makes it possible to conduct precise analysis of the deformation process of the soil medium. A mathematical creeping model of the porous medium defined as a multiphase body was for the first time introduced by Maurice Biot in [3], [4]. This model has been the subject of numerous publications. Using the fundamental theorem of thermodynamics of irreversible processes Derski derived constitutive relations for bi-phase porous body for isothermal processes [5]. Using the method of asymptotic homogenization for periodic mediums (Bensoussan, Lions and Papanicolaou [6]) a collective system of equations of isothermal theory of consolidation was obtained in the works ofAuriault [7], Auriault and Sanchez Palencia [8], and the same was achieved with the use of statistical methods by Kröner [9], Rubinstein and Torquato [10]. A mathematical model for the case where gas fills the pores of the medium, based on asymptomatic homogenization method, was presented by Auriault et al. [11]. The experimental research conducted shows a significant role of appearance of a double layer of waters bound by the forces of electric field in the model soils. This fact combined with considerable variation in temperature causes the solid particles of the skeleton to actually connect to each other via these waters. Thus, in the description of the creep process of this type of soils, in addition to shear and volume compressibility, the viscosity of the skeleton needs to be considered as presented in the works of Bartlewska and Strzelecki [12] and Bartlewska [1]. The problem of the impact of the temperature field for adiabatic processes was researched by Caussy, who created a mathematical model of thermal consolidation [13], Kowalski et al. [14], Strzelecki et al. [15], and Bartlewska and Strzelecki [2]. This study is a continuation of the numerical research of thermal consolidation process of floatation waste dumps "Żelazny Most". The model proposed includes generalization of Biot's equations for different non-isothermal processes including rheological features of the skeleton. The crown deformations process of the "Żelazny Most" tailings pond was analyzed over time in the context of appearance of changes in the value of the Coulomb–Mohr plasticity potential, which may testify to the appearance of areas of landfill materials plasticizing.

The analysis was conducted in two variants: depending on temperature changeability based on the annual temperature distribution around the construction, presented using the sine function. Second is the benchmarking variant that assumed constant temperature of the environment. A comparison of the results from the two experiments allowed us to formulate conclusions about the impact of seasonal temperature changes on the process of creation of plasticity zones of the "Żelazny Most" materials.

1.1. GEOGRAPHICAL LOCATION OF "ŻELAZNY MOST" TAILINGS POND

Administratively, the tailings pond "Żelazny Most" is located in Lower Silesia in the Legnicko-Głogowski

Fig. 1. Satellite image of area occupied by the "Żelazny Most"

Copper Territory (LGOM), in two counties: Lubin and Polkowice. This floatation waste landfill was built in 1974, and its simultaneous operation and expansion has lasted since 1977. Currently the total length of dams surrounding the dump amounts to 14.3 km and the total area 1394 ha. The annual volume of waste deposited from the floatation ranges from 20 to 26 million tones, whereof almost 75% is used to overbuild the structure and only 25% undergoes the process of neutralization. Location and satellite view of the dump based on information from the KGHM Polish Copper SA (www.kghm.pl) is shown in Fig. 1.

2. NUMERICAL CALCULATION OF THERMAL CONSOLIDATION OF "ŻELAZNY MOST" TAILINGS POND

Unquestionable advantage of using the finite element method in the modeling process is the possibility of conducting comprehensive analysis of specific engineering issue in separate building stages. The comprehensive approach used in our work consists in simultaneous analysis of deformations and stresses in the landfill area and the ground as well as the boundary conditions, (i.e., the temperature impact on the distortion process) in one calculation process. However, it is important to remember that the quality and precision of each analysis is limited to the amount of initial information.

2.1. CRITICAL STATE CONDITIONS

General conditions of critical state were used for brittle materials and granules such as soil. The starting point of unrestricted malleable flow is defined by the critical state condition for the model of perfect elasticity. Depending on the prevailing conditions of hydrostatic pressure, the same material may behave as malleable or brittle. Thus, the dependence of the critical point surface of the first constant of the strain tensor is necessary. The computation model allows the process of distortion of viscoelastic skeleton to be analyzed in the context of theory of elasticity. A complete analysis would require a model which considers both elastic and plastic proporties. Such a scope of work, however, goes beyond the simulation covered in this publication. Nevertheless, calculations of stress state allow us to verify if the elastic limit has not been exceeded (using the knowledge of strength properties of the material obtained from landfills). In practice, obtaining parameters for viscoelastic-plastic models is difficult. In most cases, these models are calibrated based on approximate empirical correlations, direct results from laboratory tests, or based on field measurements of strain tests [16]. Literature concerning soil mechanics extensively describes different shearing resistance criteria. The present study is limited to the criteria of resistance, isotropic criteria – described using the constants of the stress state.

2.1.1. APPLIED STRENGTH CRITERIA

The Coulomb–Mohr criterion is the fundamental strength criterion for the shearing of the soils. This criterion is a starting point for the construction of more advanced constitutive models. This criterion is often used as a benchmarking criterion in alternative proposals of resistance criterion for the shearing through comparison of the contours on the deviatoric plane (p = const) as well as the uniaxial one ($\sigma_2 = \sigma_3$) in the space of main components of the effective stress, as presented in Fig. 2.

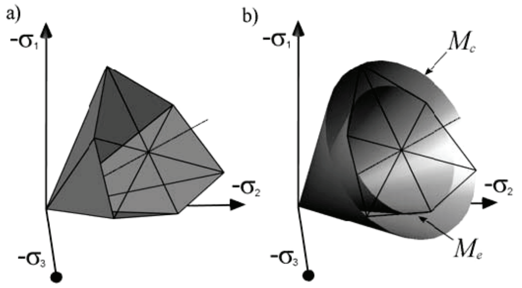


Fig. 2. (a) The Coulomb–Mohr criterion in the space of the main component of the effective stress, (b) comparison of the Coulomb–Mohr and Drucker–Prager criterion – two respective variants of compliance with the assumption of equal resistance to the uniaxial compression M_c and the resistance to the uniaxial tensile strength M_e [16]

The plasticity condition resulting from the Coulomb–Mohr criterion written using the main stress component becomes

$$F_{MC} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) + \frac{1}{2}(\sigma_{\max} + \sigma_{\min})\sin\phi - c\cos\phi = 0,$$

where $\sigma_{\min}$, $\sigma_{\max}$ – the smallest and the biggest strain, respectively, assuming a convention where the compressing strain takes negative values. This convention also applies to each formula taken from the publication [16]. Plasticity condition formulated in such a way describes six planes forming a specific pyramid in space of the main components of stress tensor, as presented in Fig. 2a. On the deviatoric plane the Coulomb–Mohr plasticity criterion represents a hexagonal contour connecting the points where the value of the internal friction angle is constant ($\varphi = \text{const}$) on which the ratios of side lengths and the specific angles of the hexagon are dependent. An important aspect of applying the Coulomb–Mohr criterion emphasized by the authors of the above mentioned publication is the undeniable difficulty in the implementation of the constitutive models. The main difficulty is caused by the presence of sharp edges of the surface plasticity in the space of the main stress components, where the direct result is difficulty in correct and unambiguous determination of the gradient of the plasticity function in those places. More about this issue can be found in paper [17]. Therefore, the Drucker–Prager strength criterion is often used for the solutions of the models where the plasticity function is a complicated higher-order function. The plasticity conditions for this criterion look as presented in the formula below

$$F_{DP} = q - Mp - c_q = 0, \qquad q = \sqrt{\frac{3}{2} s_{ij} s_{ij}},$$

where q is a constant of the stress deviator s, parameter M is the slope of plasticity surface $F_{DP} = \mathbf{0}$ plane, while $p–q$ is the limit constant of invariant q with $p = 0$ and is related indirectly with the soil cohesion.

In the case of the Drucker–Prager criterion, the plasticity plane is cone-shaped (in the plane of main components) however in the deviatoric plane, the criterion presents a sphere regardless of the value of parameter M connected to the internal friction angle. Therefore, the Coulomb–Mohr and Drucker–Prager criteria may be presented as two extreme cases to describe shearing resistance (understood as a constant value of the stress deviator invariant ($q = q_f$), Fig. 2b.

Although there is no clear correlation between the internal friction angle and resistance parameters, which according to Cudny and Binder [16] restrict the use of the Drucker–Prager criterion for practical solutions to basic research related to determination of potential areas of soil yielding described in this paper, this criterion was used because of the aforementioned ease of its implementation.

2.1. MATHEMATICAL MODEL OF THE BIOT THERMAL CONSOLIDATION WITH RHEOLOGICAL SKELETON

Mathematical equations of thermo-consolidation process for Biot's body with a rheological skeleton of Kelvin–Voigt are presented below. Those equations were derived based on fundamental laws of Newtonian mechanics for continuous mediums and thermodynamics of irreversible processes. The starting point is the initial assumption of the theory of bi-phase mediums composed of elastic-viscous skeleton and a viscous compressible fluid filling the pores of this medium. The assumptions as well as the course of the specific stages of building an analytical model are described in paper [2]. According to Bartlewska and Strzelecki [1], the physical relations for the Biot substance with a Kelvin–Voigt skeleton for any non-isothermal process can be expressed by the following equation

$$\begin{cases} \sigma_{ij} = 2N\varepsilon_{ij} + M\varepsilon\delta_{ij} + 2NT_a\dot{\varepsilon}_{ij} + (AT_b + NT_a)\dot{\varepsilon}\delta_{ij} \\ \qquad + \dfrac{Q}{R}\sigma\delta_{ij} + P_1(T - T_0)\delta_{ij}, \\ \sigma = Q\varepsilon + RQ + d(T - T_0). \end{cases} \tag{1}$$

In the Biot–Willis (1957) paper the constants appearing in constitutive relations were interpreted as follows:

- N is a the module of shape deformation of the skeleton.
- A is a volumetric distortion module of the skeleton filled with liquid.
- Q is a volumetric distortion coefficient of the liquid for the stress in the skeleton or on the contrary – a coefficient of the impact of the volumetric distortion of the skeleton for stress in the liquid.
- R is a volumetric distortion module of the liquid filling pores of the Biot substance.
- The parameter M is expressed by the formula

$$M = A - \frac{Q^2}{R}. \tag{2}$$

Constant d is expressed by the formula

$$d = -[3Qr^s + r^l R] \tag{3}$$

where r^s and r^l present the linear expansion of the skeleton and the volumetric expansion of the liquid, respectively, constant P_1 is calculated using the formula

$$P_1 = -\frac{T(3Kr^s + Qr^l)}{\lambda} \tag{4}$$

where λ is the soil heat transfer coefficient, T_a and T_b are the parameters of the skeleton expressed with the formulas

$$T_a = \frac{\eta^s}{N} \tag{5}$$

and

$$T_b = \frac{\lambda^s}{A}, \tag{6}$$

η^s, λ^s are the shear and volume viscosity of soil skeleton.

The system of equations of linear theory of thermal consolidation in the shifts of the skeleton and the function of stress in the liquid σ, the filtration flow equation and the heat conduction equation for the Biot substance with the Kelvin–Voigt rheological skeleton consists of five differential equations

$$\begin{cases} N\Psi_k \nabla^2 u_i + \left(A\Psi_L - \dfrac{Q^2}{R} + N\Psi_k \right) \varepsilon,_i \\ \qquad + \dfrac{H}{R}\sigma,_i - \rho g \delta_{i3} = -P_1 T,_i \\ \dfrac{kR}{f_o^2 \bar{\rho} g} \nabla^2 \sigma = T_0[\dot{\sigma} - H\dot{\varepsilon} + P_4 \dot{T}] \\ \lambda \nabla^2 T = T_0[P_2 \dot{\varepsilon} - P_3 \dot{\sigma} + P_5 \dot{T}] \end{cases} \tag{7}$$

where $\Psi_k = 1 + T_a \dfrac{\partial}{\partial t}$, $\Psi_L = 1 + T_b \dfrac{\partial}{\partial t}$ are differential operators, k is the coefficient of filtration of fluid through a porous medium, g – gravitational acceleration, and the coefficients P_2, P_3, P_4 and P_5 are given by the equations

$$P_2 = 3r^s \left(K - \frac{HQ}{R} \right) - Rr^l, \tag{8}$$

$$P_3 = 3r^s \frac{Q}{R} + r^l, \tag{9}$$

$$P_4 = RP_3, \tag{10}$$

$$P_5 = \frac{(3Qr^s + r^l R)^2}{R} + \frac{(\bar{\rho}_s + \bar{\rho}_w)c_v}{T}. \tag{11}$$

c_v is the specific heat at constant volume. The above set of equations is a starting point for the issue resolved in this study.

3. CONSTRUCTION OF NUMERICAL MODEL FOR THERMAL CONSOLIDATION OF THREE-DIMENSIONAL ISSUES

A three-dimensional numerical model of geological structure and the system of equations of thermal consolidation (7) constituted the basis for numerical experiments aimed at simulating the distortion of the post floatation reservoir "Żelazny Most", including periodic temperature changes in the environment of the landfill and presenting probable areas of landfill material plasticizing. Calculations were conducted using the Flex PDE V.6 Professional. Based on the analytical model of the Biot rheological skeleton, a three-dimensional numerical model of consolidation of the construction described was generated on the basis of the system of equations (7). The physical and strength parameters applied in the experiment were obtained in part from our own research, the scope of which included the standard analysis of soil, and partly from the related literature. Effective Biot's parameters were obtained based on statistical methods presented in paper [18]. The entire area subjected to computer simulation of the consolidation process consists of smaller areas with different parameters of the sediments stored within. For this reason the entire area was divided into smaller ones with well known parameters. Five sub-regions were distinguished as may be noticed in Fig. 4 showing initial network of finite elements. Changes of the value sign of potential plasticity and shift of each layer of the landfill under its own weight and temperature gradient between the surface of the bottom of the landfill and the surroundings were analyzed taking into account existing landfill drainage systems. The experiment covered the time span of 30 years and the results for the last two years of the simulation are being presented. The lower surface of the landfill was assumed to be heat permeable; T = 27 °C. For the side area boundaries it was assumed in the calculations that the temperature distribution depends on the process of heat flow in a landfill.

On the upper surface of hydro technical structures it was assumed that shifts are undefined, and the water pressure is equal to atmospheric pressure. The aim of the simulation was to analyse potential areas of plasticizing of the material under the influence of temperature which varies seasonally. By simple optimization a sine curve was matched based on minimizing the square error to the average monthly temperature data

(Fig. 3). Equation (12) describes temperature variation for the duration of the experiment.

$$T(t) = 11.42 * \sin(1992e - 7 * t - 0.825) + 9.18. \quad (12)$$

The results were generated for the first year of the experiment and then after 10, 20 and 30 years. Observed was the effect of temperature variation on the process of distortion of the landfill over time (Fig. 3).

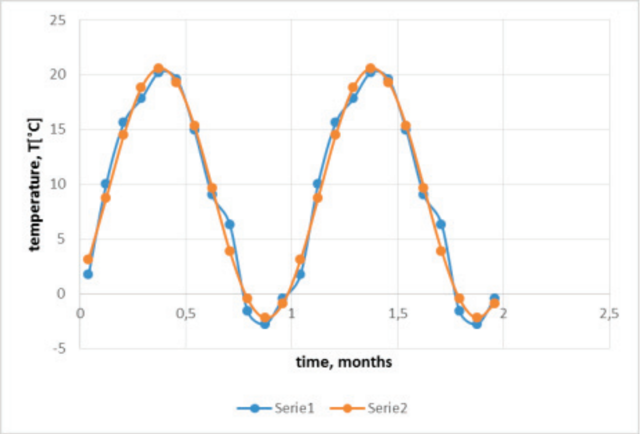


Fig. 3. Matching sine graph (series 2) to the average temperature (series 1)

4. CALCULATION RESULTS OF NUMERICAL THERMAL CONSOLIDATION

The results of numerical calculations performed using the finite element software Flex PDE 6 for the 3D model in the form of 2D graph charts in cross-sections plane are given below. The geometry of the reservoir along with the generated finite element grid is shown in Fig. 4.

Figure 5 is the cross section of tailing pond. The maximum subsidence magnitude obtained after a time equal to 30 years occurring in the central part of the waste dump is about 2.3 m. The graphs (Fig. 6a and 6b) confirm subsidence of each layer of the central part of the waste dump.

It should be noted, however, that individual layers subside with varying intensity. The largest displacement was observed on the surface of the dump (Fig. 6b), where we may also observe (with the applied strength parameters) sine dependence between subsidence of landfill and ambient temperature. This relation is represented by a sine graph. This relation is evident, however, on the edges of the landfill (Fig. 7a and 7b). Subsidence is highly dependent on the variable temperature during the year. It is greater during warmer months. In addition, a slight displacement of the edges of landfill to the opposite direction may be observed, which effect is caused by temperature and depends on the ambient temperature. The higher the temperature (summer months), the greater the effect of expansion. The graph illustrating shift area shows the direction of displacement of two selected layers of the landfill: surface area (Fig. 8a) and a layer at height z = 108 m (Fig. 8b). The direction of the shift is "out" of the landfill.

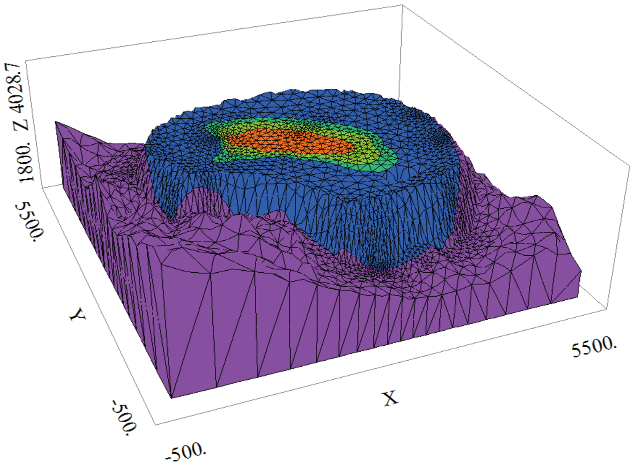


Fig. 4. Finite element grid generated by the software

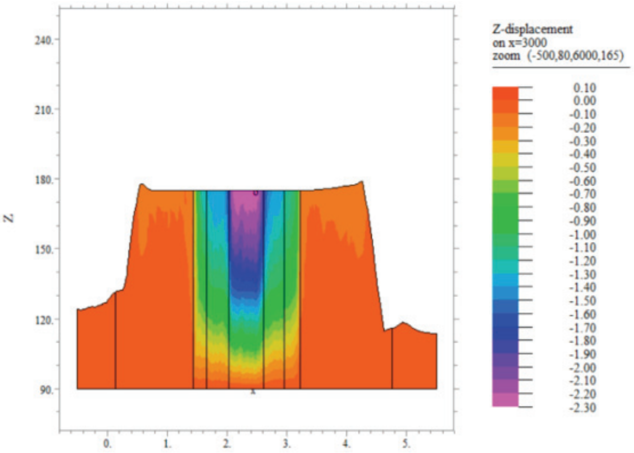
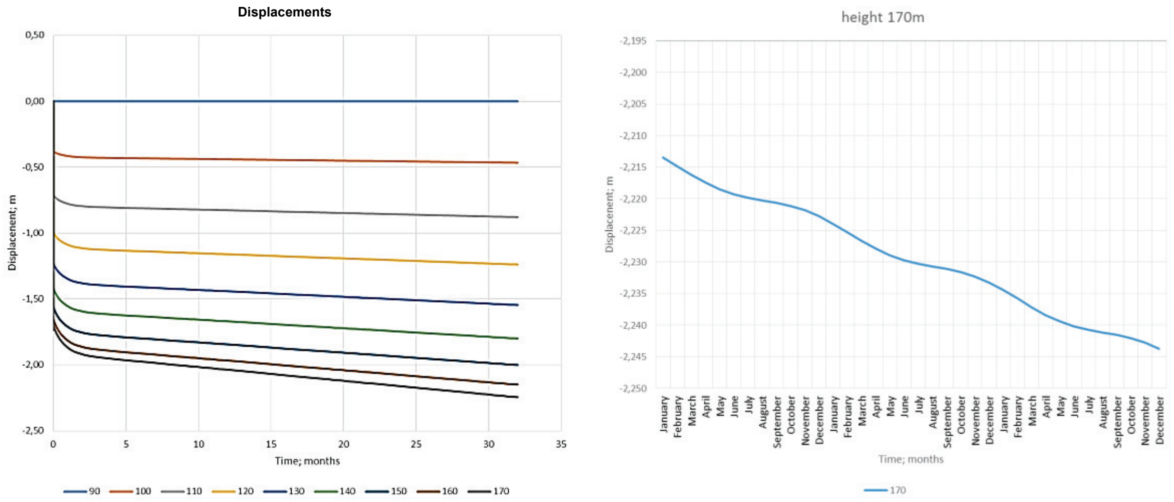


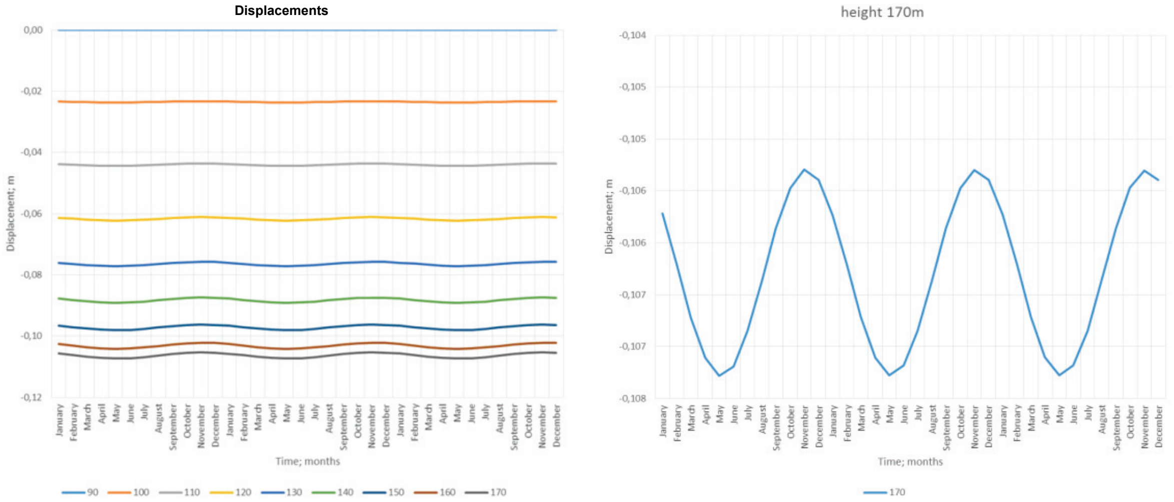
Fig. 5. Vertical displacement in m (side view *ZY*)

Figure 9 is a horizontal displacement graph after the time span of thermal consolidation equal to 30 years. We can read from the graph that the maximum horizontal displacement occurs in the middle of the dump on the border regions and in the surrounding waste dump bank and amount to 0.10 m.



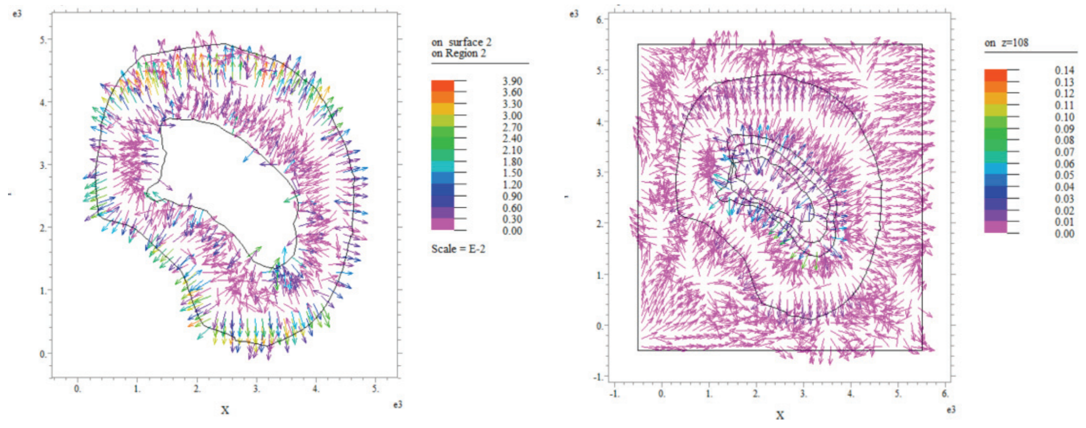
a) b)

Fig. 6. (a) The displacement changes at defined heights of the waste dump for the point $x = 2500$, $y = 2500$ in the last two years, (b) distortion of the upper layer of the waste dump for the point $x = 2500$, $y = 2500$



a) b)

Fig. 7. (a) The displacement changes at defined heights of the waste dump for the point $x = 2500$, $y = 4000$ in the last two years, (b) distortion of the upper layer of the waste dump for the point $x = 2500$, $y = 4000$



a) b)

Fig. 8. Displacement vector field a) the surface area b) tailing s pond at height $z = 108$

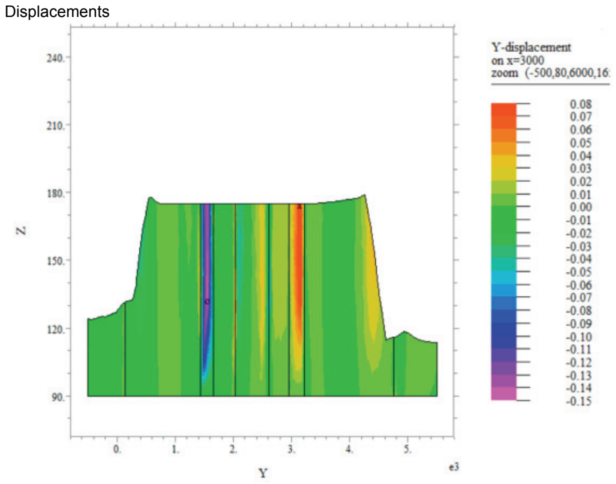
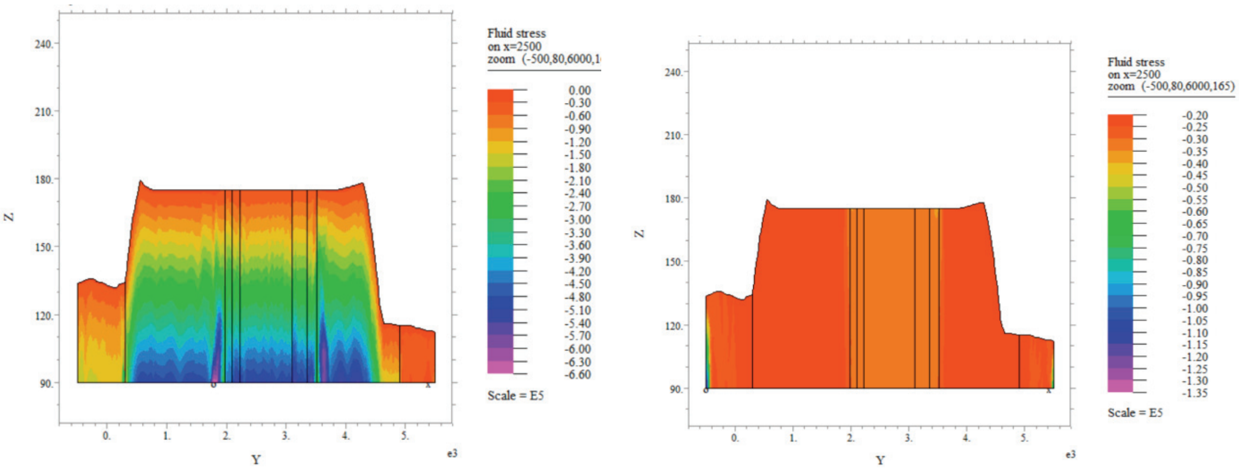


Fig. 9. Horizontal displacement in [m] (side view ZY)



a) b)

Fig. 10. The stresses in the liquid in Pa: (a) beginning of the experiment, (b) after 30 years

Figure 10 shows the stress distribution of liquid (pore stress) at the beginning (a) and at the end of the consolidation process (b). An increase of stress in the liquid is clearly visible. At the beginning of the experiment the highest compressive stresses were at the bottom of the dump, the lowest at the top. At the end of the experiment, the stress distribution was even. The stresses in the entire landfill are much lower than at the beginning of the experiment and slightly higher in the central part and the outer banks of the landfill.

Figure 11 shows the distribution of shearing stress in the dump. We may observe that the highest values of shearing stress are in the central section of the dump and overlap the areas of the greatest shifts in the horizontal direction.

Figure 12 shows Drucker's plastic potential. Negative value of the potential defines areas which are is in the range of elasticity. Where the potential is positive the value exceeds the shearing strength, within the accepted values of friction angle and cohesion. The graph shows that there are areas in which the medium may be in a state of plasticity.

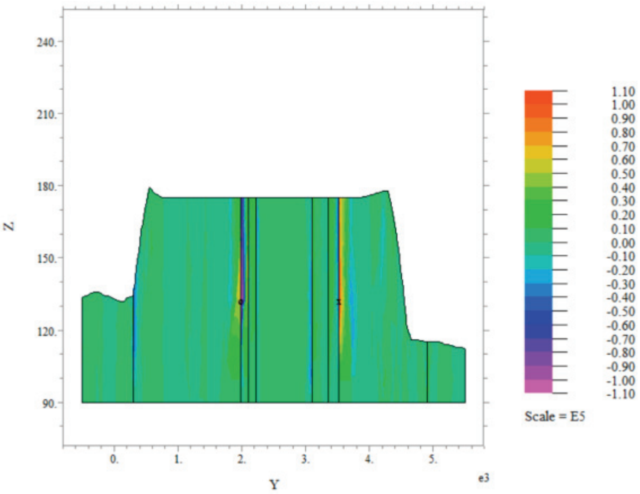


Fig. 11. Shear stress in Pa

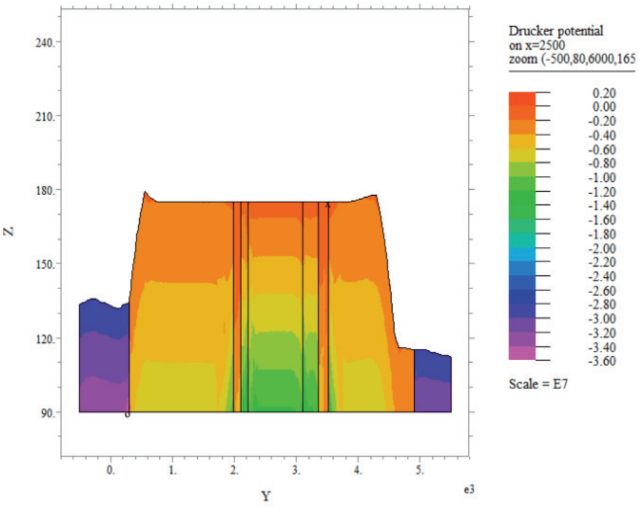


Fig. 12. Results for the 3D model – Drucker’s plastic potential

3. CONCLUSIONS

The example presented shows that the current state of knowledge in engineering allows us to use calculations of more complex models of stress distortion processes in porous mediums. The results concerning the stress state for the dump give an answer to whether or not the linear model of the viscoelastic medium leads to changes in the sign of Drucker’s plastic potential, which might indicate that the area of dump material plasticity appears. The calculation process includes a significant change in the mechanical and hydrogeological parameters of the landfill and its base as well as their anisotropy. Most of the results are consistent with intuitive predictions. The largest vertical shifts occur in the central part of the landfill and depend on the temperature of the environment. Also the effect of the displacement in the opposite direction is observed (expansion). This phenomenon is the effect of temperature and will be the subject of another publication. The largest horizontal shifts are in the central part of the landfill in the border zone, between the regions, at the same time creating a zone with the largest shear stresses. However, Fig. 12 indicates the existence of areas where the Drucker criterion was exceeded, which means that probable yielding zones exist in the dump subareas. This article is a continuation of the discussion about an impact of temperature on the consolidation process. The proposed model indicates the changing nature of the deformation process depending on temperature, but also on the plastic potential areas of the soil, which from the practical point of view could contribute to the prevention of the tailings pond stability.

REFERENCES

[1] BARTLEWSKA M., STRZELECKI T., *Equations of Biots consolidation with Kelvin–Voight rheological frame*, Studia Geotechnica et Mechanica, Vol. XXXI, 2009, No. 2, 3–15.

[2] BARTLEWSKA-URBAN M., STRZELECKI T., *Thermal consolidation of porous medium with a rheological Kelvin–Voigt skeleton*, Studia Geotechnica et Mechanica, 2012, 34, 3, 17–35.

[3] BIOT M.A., *General theory of three-dimensional consolidation*, J. Appl. Phys., 1941, 12, 155.

[4] BIOT M.A., *General Solutions of the Equations of Elasticity and Consolidation of a Porous Material*, J. Appl. Mech., 1956, 23.

[5] DERSKI W., *Some Contributions to the Theory of Flow of Fluids through porous Deformable Media*, Acta Mechanica, 1967, IV/1.

[6] BENSOUSSAN A., LIONS J.L., PAPANICOLAU G., *Asymptotic analysis for periodic structures*, North Holland Publishing Company, Amsterdam 1978.

[7] AURIAULT J.L., *Dynamic behaviour of porous media*, Transport Processes in Porous Media, Kluver Academic Publishers, 1991, 471–519.

[8] AURIAULT J.L., SANCHEZ PALENCIA E., *Etude de comportement macroscopique d’un milieu poreuxsature deformable*, Journal de Mecanique, 1977, 16 (4), 575–603.

[9] KRÖNER E., *Effective elastic moduli of periodic and random media: a unification*, Mechanics Research Communication, 1980, 7 (5), 323–327.

[10] RUBINSTEIN J., TORQUATO S., *Flow in random porous media: mathematical formulation, variational principles and rigorous bounds*, J. Fluid Mech., 1989, 206, 25–46.

[11] AURIAULT J.L., STRZELECKI T., BAUER J. HE S., *Porous deformable media by a very compressible fluid*, Eur. J. Mech. a. Solid, 1990, 9, 4, 373–392.

[12] BARTLEWSKA-URBAN M., STRZELECKI T., *One-dimensional consolidation of the porous medium with the rheological Kelvin–Voigt skeleton*, Studia Geotechnica et Mechanica, 2008, 30, 1/2, 115–122.

[13] COUSSY O., *Mechanics and physics of porous solids*, John Wiley & Sons, 2011.

[14] KOWALSKI S.J., MUSIELAK G., RYBICKI A., *Drying Processes in Context of the Theory of Fluid Saturated Porous Materials*, J. Theoretical and Applied Mechanics, 1998, 36, 3.

[15] STRZELECKI T., KOSTECKI S., ŻAK S., *Modelowanie przepływów przez ośrodki porowate*, Dolnośląskie Wydawnictwo Edukacyjne, Wrocław 2008.

[16] CUDNY M., BINDER K. *Kryteria wytrzymałości gruntu na ścinanie w zagadnieniach geotechniki* (*On shear strength criteria for soils in geotechnics*), Inżynieria Morska i Geotechnika, 2005, 6, 456–465 (PDF).

[17] NIEMUNIS A., *Über die Anwendung der Kontinuumstheorie auf bodenmechanische Probleme, Eine Vorlesung für Grundbau- und Tunnelbauvertiefer*, Uniwersytet w Bochum, on line: www.gub.ruhr-uni-bochum.de/mitarbeiter/andrzej_niemunis.htm, 2003.

[18] BARTLEWSKA M., The doctoral dissertation on the theme: *Określenie parametrów efektywnych modeli reologicznych gruntów spoistych*, Politechnika Wrocławska, Faculty of Geoengineering, Mining and Geology, Wrocław 2009.