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SUBSPACE PROJECTIONS WITH CENTRES DISPERSED ON SECOND-DEGREE FORMS

Bogusław JANUSZEWSKI

Rzeszow University of Technology, Department of Engineering Geometry and Graphics
2 Poznańska st., 35-084 Rzeszów, Poland, phone +48-17 865 13 07
email: banjanus@prz.rzeszow

Abstract. The concept of *subspace projections* has already been discussed in the subject literature and the rules of operation and properties of the projections whose centres and projecting forms for particular points of represented space are subspaces, were defined in relation to so-called *bundle projections* (with fixed centres) and to *subspace projections with bundle dispersed centres*. In this paper some general properties of *subspace projections with centres dispersed on second-degree forms* are discussed.

Key Words: subspace projection, bundle projection, subspace projection with centres dispersed on second-degree forms.

1. Introduction

The rules of operation and properties of *subspace projections*, i.e. the projections whose centres and projecting forms for particular points of represented space are subspaces, were already specified in relation to so-called *bundle projections* (with fixed centres) in [1], and to *subspace projections with bundle dispersed centres* [2,3]. The paper discusses general properties of *subspace projections with centres dispersed on second-degree forms*. This type of projection is referred to as **PQ** .

In order to define the apparatus of projection **PQ** for an **n** – dimensional projective $\boldsymbol{P}_\mathbf{n}$ space let us establish in this space three bundles < $\boldsymbol{P}_\mathbf{n}$, $\mathbf{K}$ >, < $\mathbf{F}_1$,$\mathbf{C}_1$ >, and < $\mathbf{F}_2$,$\mathbf{C}_2$ > of the same type (i.e. with the same factors of manifolds $\mathbf{f}_M$ = **n** – dim $\mathbf{K}$ +1), with the following properties:

1° dim $\mathbf{F}_1$ = dim $\mathbf{F}_2$, and dim $\mathbf{C}_1$ = dim $\mathbf{C}_2$,

2° $\mathbf{F}_1 = \mathbf{F}_2$, whereas $\mathbf{C}_1 \neq \mathbf{C}_2$, if the defining apparatus of the **PQ** projection belongs to the family of so-called *meet projections,* marked here as **PQM** (fig.1b),

3° $\mathbf{F}_1 \neq \mathbf{F}_2$, whereas $\mathbf{C}_1 = \mathbf{C}_2$, if the defining apparatus of the **PQ** projection belongs to the family of so-called *junction projections,* marked here as **PQJ** (fig.1a).

Moreover, let us introduce the following projective transformations:

- **Cl** being a collineation that transforms the bundle < $\boldsymbol{P}_\mathbf{n}$, $\mathbf{K}$ > onto the bundle < $\mathbf{F}_1$,$\mathbf{C}_1$ >,
- **Cr** being a correlation that transforms the bundle < $\boldsymbol{P}_\mathbf{n}$, $\mathbf{K}$ > onto the bundle < $\mathbf{F}_2$,$\mathbf{C}_2$ >.

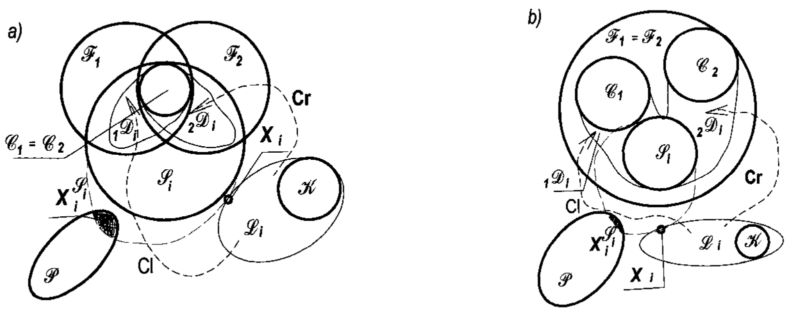

Figure1: Schematic diagram of the structures and the principles of operation of apparatuses: a - **PQJ** subspace junction projections, b - **PQM** subspace meet projections

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With such defined projective transformations, the bundles $< \mathbf{F}_1,\mathbf{C}_1 >$ and $< \mathbf{F}_2,\mathbf{C}_2 >$ correspond with each one to the other in the correlation $\mathbf{CR} = \mathbf{Cr}\ (\mathbf{Cl}^{-1})$.

The established bundles and defined projective transformations allow us to associate with every point $X_\mathbf{i} \in \boldsymbol{P}_\mathbf{n}$ the following three subspaces: 1° $\mathbf{L_i} = X_\mathbf{i}\ \mathrm{O}\ \ \mathbf{K}$,
2° ${}_1\mathbf{D}_{X\mathrm{i}} = \mathbf{Cl}\ (\mathbf{L_i}) \in < \mathbf{F}_1,\mathbf{C}_1 >$, 3° ${}_2\mathbf{D}_{X\mathrm{i}} = \mathbf{Cr}\ (\mathbf{L_i}) \in < \mathbf{F}_2,\mathbf{C}_2 >$.

Let us also assume that:

- in case of a so-called meet subspace projection **PQP**, the meet ${}_1\mathbf{D}_{X\mathrm{i}} \cap {}_2\mathbf{D}_{X\mathrm{i}} = \mathbf{S_i}$ with the dimension equal to dim $\mathbf{C}_1$ = dim $\mathbf{C}_2$,
- in case of a so-called junction subspace projection **PQJ**, the junction ${}_1\mathbf{D}_{X\mathrm{i}}\ \mathrm{O}\ {}_2\mathbf{D}_{X\mathrm{i}} = \mathbf{S_i}$ with the dimension equal to dim $\mathbf{F}_1$ = dim $\mathbf{F}_2$,

  is the centre of projection for a distinguished point $X_\mathbf{i} \in \boldsymbol{P}_\mathbf{n}$.

It is easy to prove, that:

- in the **PQM** projection, centres $\mathbf{S_i}$ of this projection, as meets of homologous elements of the correlated bundles $< \mathbf{F}_1,\mathbf{C}_1 >$ and $< \mathbf{F}_2,\mathbf{C}_2 >$ are subspace formers of a second-degree form $\Sigma_\mathbf{P}$,
- in the **PQJ** projection, centres $\mathbf{S_i}$ of this projection, as junctions of homologous elements of the correlated bundles $< \mathbf{F}_1,\mathbf{C}_1 >$ and $< \mathbf{F}_2,\mathbf{C}_2 >$ are subspace generators of a second-degree form $\Sigma_\mathbf{J}$.

Therefore, the defined sets of centres of the projections **PQM** and **PQJ** are subspaces dispersed on the second degree forms $\Sigma_\mathbf{P}$ and $\Sigma_\mathbf{J}$, respectively, and they satisfy our preliminary assumptions made in this analysis.

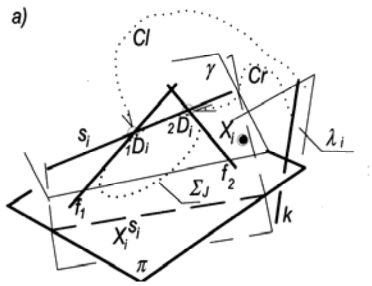
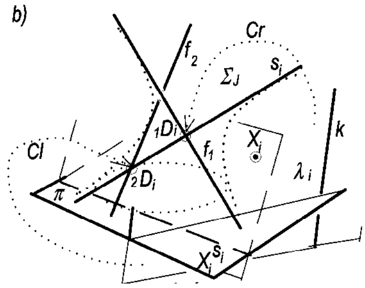
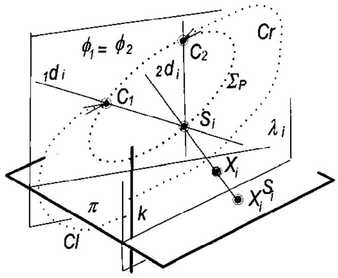
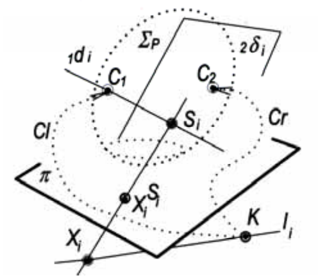
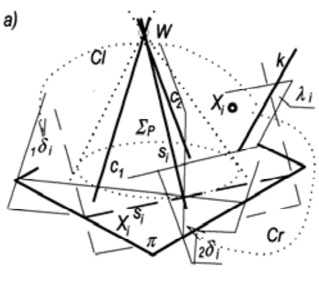
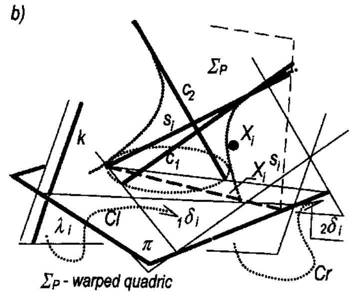
In order to complete the apparatuses of the projections **PQM** and **PQJ**, one should still establish their forms of projections in the $\boldsymbol{P}_\mathbf{n}$ space. Similarly, as in the case of any classical subspace projection also in the case of currently considered projections one should consider as the form of projection such a subspace $\mathbf{P}$ which determines the represented $\boldsymbol{P}_\mathbf{n}$ space together with almost every one of the centres $\mathbf{S_i}$,. Moreover, depending on whether the defined projection is supposed to be an *ordinary* or *generalized projection* [4], the distinguished subspace $\mathbf{P}$ should be either disjoint with almost all the projection centres, or it should cross every of these centres in a subspace nonnegative dimension not greater than dim$\mathbf{P}$ - 2.

The **PQ** projections can have practical application when they lead to a graphical representation of the represented $\boldsymbol{P}_\mathbf{n}$ space. This result is achieved when planes, traditionally referred to as π, are adopted as forms of projections of the **PQ** projections. In such a case, from our previous identifications it results that if the **PQ** projection is to be:

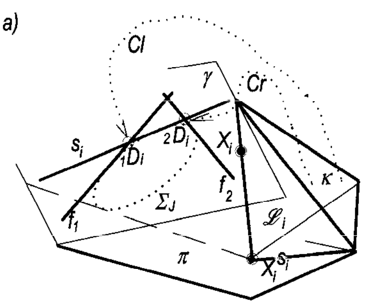
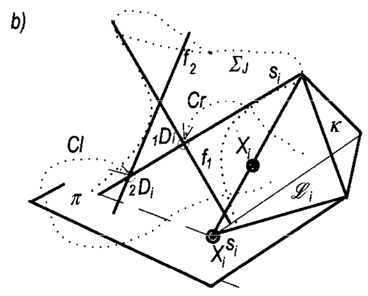
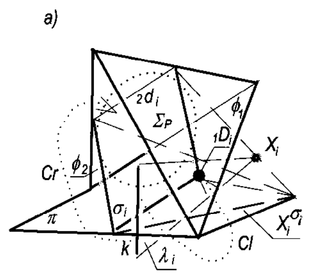
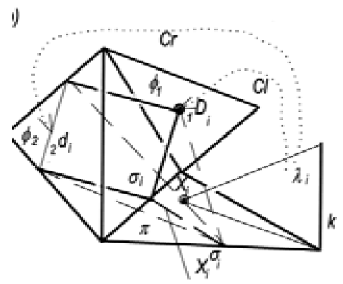
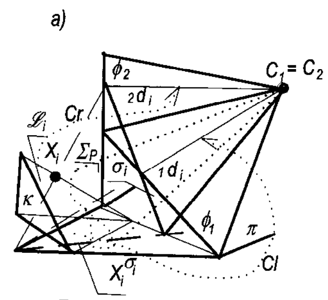
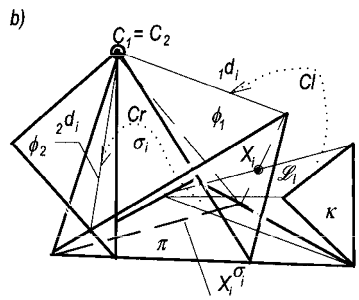
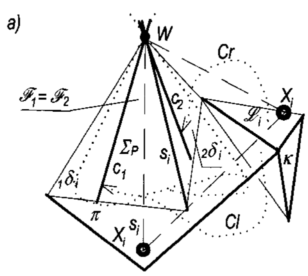
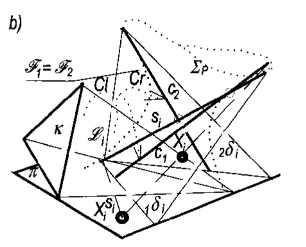
- an ordinary projection, then dim $\mathbf{C}_1$ = dim $\mathbf{C}_2$ = $\mathbf{n}$-3 = dim$\mathbf{S}_\mathrm{i}$, when the projection under consideration has characteristics of the **PQM** projection; and dim $\mathbf{F}_1$ = dim $\mathbf{F}_2$ = $\mathbf{n}$-3= dim$\mathbf{S}_\mathrm{i}$, when the **PQ** projection belongs to the **PQJ** family of projections;
- a generalized projection, then dim $\mathbf{C}_1$ = dim $\mathbf{C}_2$ = $\mathbf{n}$-2 = dim$\mathbf{S}_\mathrm{i}$, when the projection under consideration has characteristics of the **PQM** projection; and dim $\mathbf{F}_1$ = dim $\mathbf{F}_2$ = $\mathbf{n}$-2= dim$\mathbf{S}_\mathrm{i}$, when the **PQ** projection belongs to the **PQJ** family of projections,

The structures of **PQ** projection apparatuses that lead to mappings of the $\boldsymbol{P}_3$ and $\boldsymbol{P}_4$ spaces are presented in drawings in tables 1 and 2 respectively.

Tablet 1. Structure of apparatuses of the **PQ** mapping projections of $\boldsymbol{P}_3$ space

Type of the PQ projection	**dim C_i**	**dim F_i**	**Visual drawing of a projection apparatus**	
PQJ generalized	**-1**	**1**	 *Σ_J - envelope conic*	 *Σ_J – warped quadric*
PQM ordinary	**0**	**2**	 *Σ_P - conic*	
PQM ordinary	**0**	**3**	 *Σ_P - curvilinear quadric*	
PQM generalized	**1**	**3**	 *Σ_P – bundle serface*	 Σ_P - warped quadric

Tablet 2. Structure of apparatuses of the **PQ** mapping projections of $\boldsymbol{P_4}$ space

Type of the PQ projection	**dim C_i**	**dim F_i**	**Ideological schema of a structure of a projection apparatus**	
PQJ ordinary	**-1**	**1**	 ***Σ_J - envelope conic***	 ***Σ_J – warped quadric***
PQJ generalized	**-1**	**2**	 ***Σ_P– envelope quadric***	 ***Σ_P– warped hyper quadric***
PQJ generalized	**0**	**2**	 ***Σ_P - envelope bundle surface***	 ***Σ_P – bundle hyper surface with plane formers***
PQM ordinary	**1**	**3**	 ***Σ_P - bundle surface***	 ***Σ_P – warped quadric***

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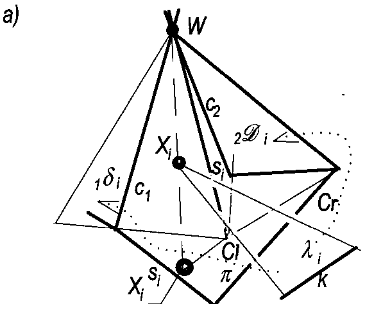
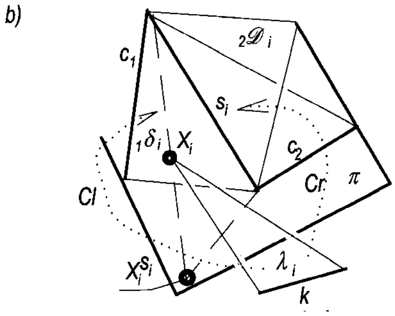
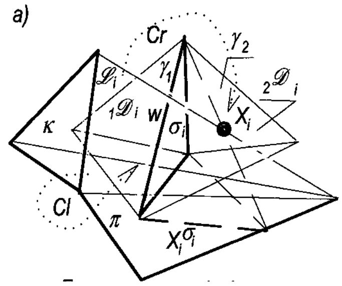
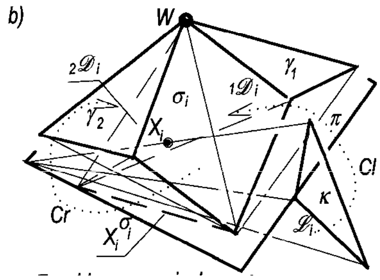
PQM ordinary	**1**	**4**	a)  *Σ_P - bundle hyper surface with straight line formers*	b)  *Σ_P – warped hyper quadric*
PQM generalized	**2**	**4**	a)  *Σ_P –bundle hyper surface with straight line vertex*	b)  *Σ_P –bundle hyper surface with plane formers*

It is also worth noting that like with subspace projections with bundle dispersed centres, some of the **PQ**-type projections under consideration are minimal reversible representations [4] of the $\boldsymbol{P}_\mathbf{n}$ space. It means that for every point $X_\mathbf{i}$ belonging to the $\boldsymbol{P}_\mathbf{n}$ space, which is generally situated versus the apparatus of the **PQ** projection, it is possible to reconstruct the position of this point towards the previously rebuilt apparatus of the **PQ** projection, using the $X_\mathbf{i}^{\mathbf{Si}}$ projection as a basis. For example, these are representations with such properties in the **PQ** family of graphical projections of the $\boldsymbol{P}_3$ and $\boldsymbol{P}_4$ spaces:

- an ordinary **PQM** projection of the $\boldsymbol{P}_3$ space with its apparatus described in the position 2 in table 1,
- an ordinary **PQM** projection of the $\boldsymbol{P}_3$ space with its apparatus described in the position 3 in table 1,
- an ordinary **PQM** projection of the $\boldsymbol{P}_4$ space with its apparatus described in the position 5 in table 2.

Generally speaking, it is possible to prove that the **PQ** subspace projection of the $\boldsymbol{P}_\mathbf{n}$ space onto the projection subspace **P** is a minimal reversible representation of this space, when it belongs to the **PQM**-type family, and elements of its apparatus have the following properties:

1° $\mathsf{S}_\mathbf{i} \; \mathrm{O} \; \mathbf{K} \neq \boldsymbol{P}_\mathbf{n}$,

2° $\mathsf{S}_\mathbf{i} \cap \mathbf{K} = \emptyset$,

3° $\dim \mathsf{S}_\mathbf{i} < \frac{1}{2}(\dim \mathbf{F}_\mathbf{i} - 1)$.

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RZUTOWANIA PODPRZESTRZENIOWE PRZESTRZENI RZUTOWYCH REALIZOWANE ZE ŚRODKÓW ROZPROSZONYCH NA UTWORACH DRUGIEGO STOPNIA

Zasady działania oraz właściwości *rzutowań podprzestrzeniowych*, tzn. rzutowań, których środki i utwory rzutujące poszczególne punkty odwzorowywanej przestrzeni są podprzestrzeniami, zostały już wcześniej sprecyzowane w stosunku do tzw. *rzutowań wiązkowych* (ze stałego środka) oraz dla *podprzestrzeniowych rzutowań ze środków rozproszonych wiązkowo.* W niniejszym opracowaniu podaje się ogólne informacje o właściwościach *rzutowań podprzestrzeniowych przestrzeni rzutowych realizowanych ze środków rozproszonych na utworach drugiego stopnia*.