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Maintenance cost study for deteriorating systems: age-replacement policy vs. condition-based maintenance policy

Keywords

maintenance cost, degradation model, age-replacement, condition-based maintenance

Abstract

This paper deals with the maintenance cost of deteriorating systems. Two maintenance strategies are studied: age-replacement and condition-based maintenance. To compare these two policies, degradation models are used: these models characterize the degradation level but also the system time-to-failure. In order to compute the optimal condition-based maintenance cost, we suppose that the main influencing sources are the preventive threshold, the inspection frequency and the inspection cost. Numerical examples illustrate the maintenance cost computation and compare the optimal costs of both policies.

1. Introduction

Manufacturing systems are subject to different stresses which depend on their operating environment and time. Systems wear increases with usage and age until failure happens. For example, the lubrication performance of the engine oil decreases faster for a construction truck than for a long-haul truck.
System failures can incur high cost: if the lubrication property of the engine oil reduces, the engine will be bad lubricated. Then the vehicle will be immobilized due to an engine breakdown. Preventive maintenance aims at detecting the loss of system or component performance and at deciding if the system or component needs to be replaced before the failure appears.
Many preventive maintenance strategies exist: we will focus in this paper on age-replacement [3], [4], [9], [10], [18], [19] and condition-based maintenance policies [6], [7], [8], [10], [13], [14], [15], [16]. The first policy takes only account of the system age whereas the second is based on the degradation level, which is more representative of the current system status.
The system or component wear depends on the operating environment and the conditions of use. The degradation accumulation can be modeled using stochastic processes which are determined by the degradation type. We will only focus on the gradually continuous degradations that are classically modeled with gamma processes [1], [13], [17]. However, systems are more and more subject to an environment that often varies [5], [17]. The system use changes over the time and the degradation accumulation increases with respect to this usage. For instance, the road topology influences the use of the brake pads. In a hilly environment, brake pads warm up more often than on a flat road. To model this kind of degradation, an effective tool is the Markov additive process which can drive the degradation accumulation according to the usage.
This paper compares the age-replacement cost and the condition-based maintenance cost for a simple degradation model using gamma process and for a

complex degradation model with Markov Additive process. Our objective is to evaluate the additional cost that results in using a condition-based maintenance instead of an age-replacement policy.
This paper is organized as follows. Section 2 describes the degradation models. Section 3 deals with the maintenance models while section 4 analyses the sources that influence the cost of condition-based maintenance. Section 5 gives a comparison between the costs of both policies from simulation using the degradation models.

2. Degradation models

General concepts on reliability models are first presented. Then we focus on two particular models based on system degradation.

2.1. Reliability models

Meeker and Escobar [11] notice two approaches to develop a reliability model:

(i) survival reliability,
(ii) degradation-based reliability.

The first methodology aims at computing the lifetime distribution, which quantifies whether the system is functioning or not. It is also possible to calculate the failure rate that can not be computed for a particular component. Singpurwalla [17] pointed out that it derives from a population of components.
The second approach develops degradation models which represent the behavior of the system degradation accumulation. The time-to-failure of a particular system can be determined due to a specified failure threshold.
We will focus on the degradation-based reliability because it provides information on a specific system. The following describes two models of the degradation reliability that have been mainly used in the maintenance area [[6], [7], [8], [10], [13], [14], [15], [16].

2.2. Gamma process as degradation model

Consider a system or a component subject to a wear accumulation in time which can be represented by the degradation variable *X*. The degradation is supposed to be gradual and monotone. Abdel-Hammed [1] proposed to use the gamma process as a proper model for deterioration occurring at random time-instants. The best advantage with this tool is that the required mathematical calculations are relatively straightforward.

$X(t)$, $t > 0$ is a gamma process with shape parameter α and scale parameter β:

(i) $X(0) = 0$,
(ii) $X(t)$ has independent increments,
(iii) For $t > 0$ and $h > 0$, $X(t+h) - X(t)$ is a gamma distribution :

$$f(x) = \frac{\beta^{-\alpha h}}{\Gamma(\alpha h)} x^{\alpha h - 1} e^{-x/\beta} \quad . \tag{1}$$

with Γ the gamma function where

$$\Gamma(y) = \int_0^\infty z^{y-1} e^{-z} dz \,, \ \forall y > 0 \quad . \tag{2}$$

Suppose X a gamma process and $z_0 > 0$ a fixed failure threshold. Then the probability distribution function F of the first time to reach T is [7]:

$$F(t) = P(T < t) \,. \tag{3}$$

$$F(t) = \frac{\int_{z_0/\beta}^{+\infty} x^{\alpha t - 1} \exp(-x) dx}{\Gamma(\alpha h)} \,. \tag{4}$$

F is also known as the hitting time distribution.
Due to the monotony property it is possible to model many physical degradation processes. Moreover, gamma process is a jump process which can represent the accumulation of an infinite number of small shocks. The independence and stationarity of increments of this process means that this model supposes that future degradation is independent of the current level of degradation but depends only on the period over which the system will be allowed to be deteriorated.
The gamma process is suitable to model gradual damage monotonically accumulation over time in a sequence of tiny increments: Van Noortwijk [13] modeled dykes erosion due to crest-level decline with the gamma process.

2.3. Markov additive process as degradation model

Consider now a system whose degradation accumulation depends on the operating environment or/and the conditions of use. These external variables or external covariates can influence the type or/and the wear rate. Çinlar [5] developed the Markov additive process which is a flexible modelling tool to represent this kind of degradation according to the external covariates.
Figure 1 shows an example of Markov additive process degradation model with one external covariate. The degradation level grows according to the covariate state. The degradation accumulation increases faster

when the covariate state equals 2 than when the covariate state equals 1.
This model takes into account the effect of other factors on the failure mechanism, and as a consequence gives better precision on the system behavior.

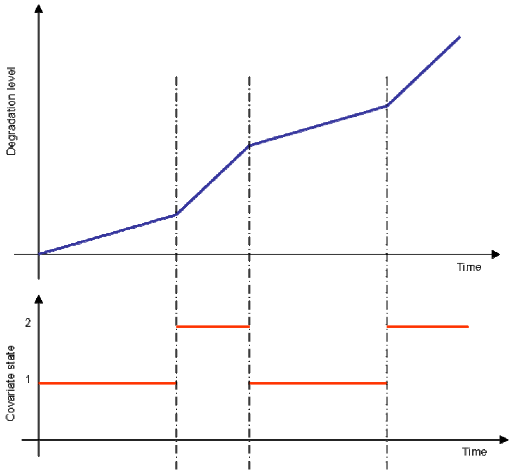

Figure 1. Example of a Markov Additive Process degradation model

3. Maintenance policies model

The previous reliability models that describe the system degradation may be used to compute the maintenance cost for different strategies.
This section begins with the definition of the different maintenance operation costs. Then two different maintenance policies are presented. The first is the age-replacement policy, which is based on the age of the system. The second policy is the condition-based maintenance, which refers to the degradation level of the system.

3.1. Maintenance cost

To compare the global maintenance cost of different policies, we need to introduce some maintenance operation costs. They cover the hardware and man-hour costs:

(i) C_i is the inspection cost. The inspection consists only in checking the system state and store the system degradation level.
(ii) C_p, the preventive cost. The preventive replacement aims at replacing the system before its breakdown.
(iii) C_c, the corrective cost. The system has failed and needs to be replaced. The new system is considered as good as new.

with:

$$C_i < C_p < C_c . \tag{5}$$

The comparison is based on the mean maintenance cost per time unit. The following describes maintenance policies and the methods to compute the expected mean cost per time unit for each policy.

3.2. Age replacement

Age replacement policy consists in replacing a component upon failure or at an operational age T_0, which ever comes first. We suppose that after the replacement, the component is "as good as new" and that the time required to replace the failed system is negligible.
When the component has reached the age T_0, the preventive replacement cost is C_p, and the cost of replacing a failed item (before age T_0) is C_c.

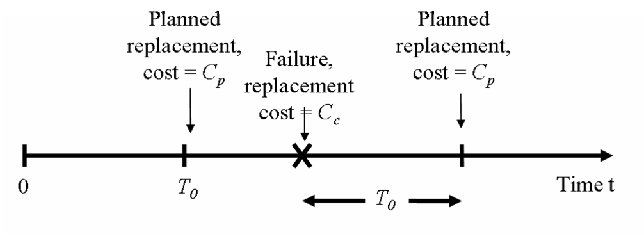

Figure 2. Age replacement policy and costs [14]

The time between two consecutive replacements is called a replacement period. The mean time between renewals with replacement age T_0 is:

$$MTBR(T_0) = \int_0^{T_0} t.f(t)dt + T_0.P(T \geq T_0) . \tag{6}$$

$$MTBR(T_0) = \int_0^{T_0} (1 - F(t))dt . \tag{7}$$

The mean number of replacements, $E_{T_0}(N(t))$, in a long time interval of length t may be therefore approximate as

$$E_{T_0}(N(t)) \approx \frac{t}{MTBR(T_0)} = \frac{t}{\int_0^{T_0}(1-F(t))dt} . \tag{8}$$

The total cost per replacement period is equal to the replacement cost C_p plus the extra cost (C_c - C_p) whenever a failure occurs.

$$C_p + (C_c - C_p)P(T < T_0) = C_c.F(T_0) + C_p.R(T_0) \tag{9}$$

The total mean cost per unit time $C_A(T_0)$ with replacement age T_0 is determined by:

$$C_A(T_0).MTBR(T_0) = C_c.F(T_0) + C_p.R(T_0) \tag{10}$$

Thus,

$$C_A(T_0) = \frac{C_c.F(T_0) + C_p.R(T_0)}{\int_0^{T_0} R(t)dt}. \quad (11)$$

Where F is the probability distribution function and R is the reliability function of the system which verifies:

$$R(t) = 1 - F(t). \quad (12)$$

Usually the operation age T_0 is specified by the manufacturer. Nevertheless this specification is the results of durability test and deals with the component performance and not with the cost optimization.
Figure 3 shows the basic shape of the mean maintenance cost per time unit function. The function decreases quickly to reach a minimum (optimum) value and then increases slowly to reach a constant value.

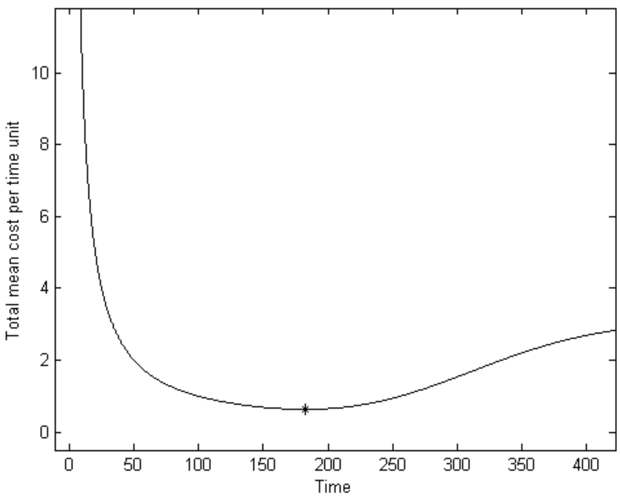

Figure 3. Function of maintenance cost per unit time of age-replacement policy $C_A(t)$

The optimum of the average maintenance cost per time unit $C_A(T_0)$ is such that:

$$\frac{dC_A}{dt}(T_0) = 0. \quad (13)$$

T_0 represents is the optimal replacement age.

3.3. Condition-based maintenance

Condition-based maintenance policy is based on the degradation level Z of the item. It consists of deciding whether or not a system may be maintained according to its state using condition monitoring techniques. First a critical threshold is fixed by the manufacturer. A second threshold is variable and determines the time to change preventively the system.

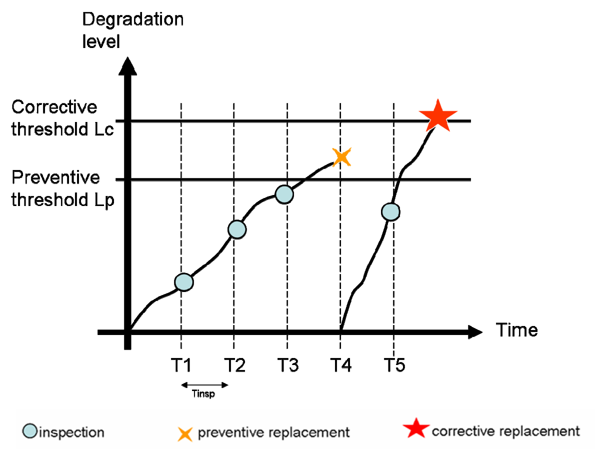

Figure 4. Condition-based maintenance policy

The system is checked at each regular inspection time interval and its degradation level Z determines the type of maintenance operation.

(i) $Z < L_p$, the system is functioning. An inspection cost C_i is imputed.
(ii) $L_p \leq Z < L_c$, the item has to be preventively replaced with a preventive cost C_p.

When the degradation level Z has reached the corrective threshold L_c, the component is considered as failed and is immediately changed with a cost C_c.

The cumulative maintenance cost of this policy is:

$$C_{CB}(t) = N_i(t).C_i + N_p(t).C_p + N_c(t).C_c + C_d.d(t) \quad (14)$$

where N_i is the total number of inspections, N_p is the number of the preventive replacements, N_c is the number of the corrective replacements.
C_d is the cost of "inactivity of the system" per time unit and $d(t)$ is the time spent in a failed state in $[0, t]$. Since the failed component is immediately changed, we suppose that these two variables equal 0.

We focus on $\overline{C_{CB\infty}}$ the expected mean cost per time unit over an infinite horizon.

$$\overline{C_{CB\infty}} = \lim_{t\to\infty} \frac{C_{CB}(t)}{t} = \lim_{t\to\infty} \frac{\mathrm{E}(C_{CB}(t))}{t} \quad (15)$$

Using the renewal theory [2], it is well known that the limit at infinity (15) can be changed into a ratio of expectations on a single renewal cycle T_{cycle}. Then,

$$\overline{C_{CB\infty}} = \frac{E(C(T_{cycle}))}{E(T_{cycle})} \quad (16)$$

For a given degradation process, the number N_i, N_p, N_c and the cycle T_{cycle} depend on the preventive threshold and the inspection frequency. We will focus on the influence of these parameters in the next section.

4. Numerical experiment and influence analysis of condition-based maintenance

This section presents the results of numerical experiments on the computed cost of condition-based maintenance. We suppose that the main influencing causes are the choice of the preventive threshold and the choice of the inspection time interval. Consider that the cost values are expressed in arbitrary unit:
$C_i = 5$
$C_p = 100$
$C_c = 500$
The system is considered as failed when the degradation level reaches the critical threshold $L_C = 100$.

4.1. Influence of the preventive threshold

The preventive threshold determines the time to change the system before the failure; it has to be fixed between the initial degradation level and the critical threshold. Higher this threshold is, more the chance to have a failed component is accepted. The mean maintenance cost per time unit depends on this tolerance.

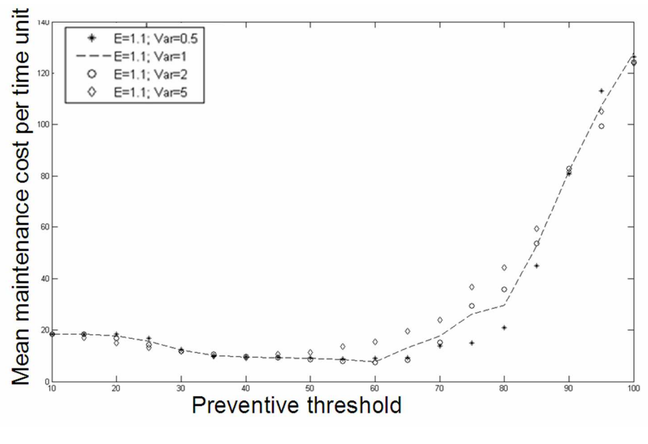

Figure 5. Influence of the preventive threshold on the mean maintenance cost (Ci/Cc = 0.05, Cp/Cc = 0.2, Cc = 500)

Figure 5 shows the influence of the preventive threshold on the mean maintenance cost per time unit. An optimal preventive threshold can be determined. It represents the limit between the cost of preventive actions and the cost of corrective actions. The optimal cost would be the minimal value of this function, however other variables influence the maintenance cost like the inspection time interval.

4.2. Influence of the inspection time interval

More the number of inspection increases, more the system is maintaining. The system or component could be replaced before its breakdown and then money will be saved even if the maintenance cost increases.

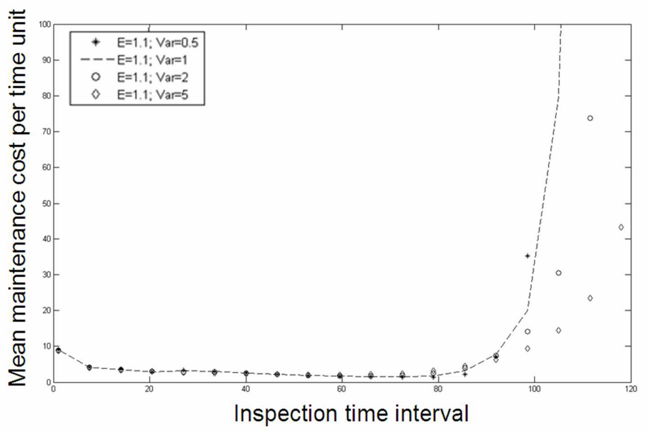

Figure 6. Influence of the inspection time interval on the maintenance cost (Ci/Cc = 0.05, Cp/Cc = 0.2, Cc = 500)

Figure 6 shows the influence of the inspection time interval on the mean maintenance cost per time unit. As the influence of the preventive threshold, the minimum maintenance cost per time unit is reached with an optimal inspection time interval. If the inspection time interval is less than the optimum, the maintenance cost increases due to the occurrence of the inspections. In the contrary case, it increases due to the corrective actions.

5. Maintenance cost comparison

To find the optimum cost of the condition-based maintenance, we need to link these two influencing sources to the cost and to find a compromise between both variables. The following deals with two degradation cases and shows the evolution of the total mean cost per unit time over the inspection time interval and the preventive threshold.

5.1. Model 1: Gamma process degradation model

The first case is a simple degradation modeled by a gamma process with mean $E = \alpha/\beta = 1.1$ and variance $Var = \alpha/\beta^2 = 5$.
Figure 7 shows the degradation accumulation and its probability distribution function, which is computed with equation 4.

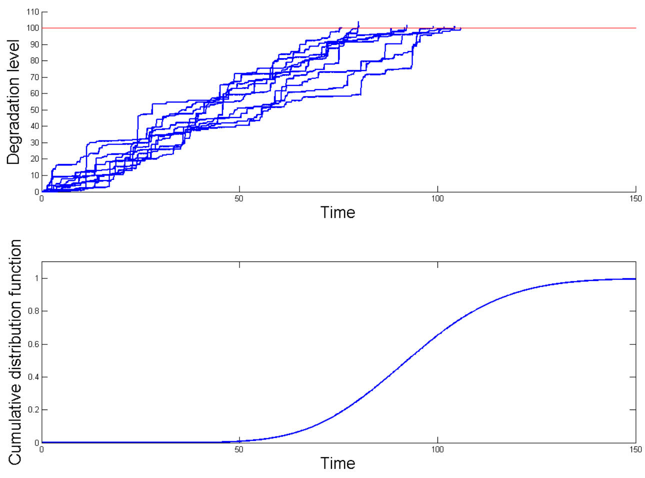

Figure 7. Degradation level and probability distribution function of time to failure.

Figure 8 compares the condition-based maintenance cost and the optimal age-replacement cost (computed with the equations 11 and 13) with a fixed inspection cost (C_i = 5). The condition-based maintenance cost increases due to the failure when the preventive threshold is higher and the inspection time intervals are longer. On the contrary, the optimal age-replacement cost is constant. The area A1 represents the couples of preventive thresholds and inspection time intervals when the condition-based maintenance is less expensive than the age-replacement policy.

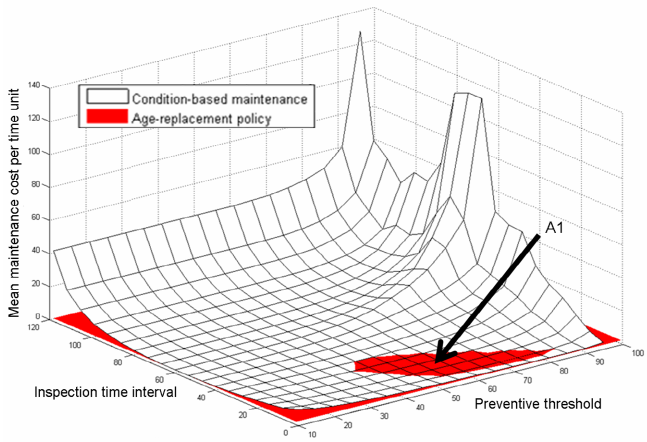

Figure 8. Mean maintenance cost per unit time of age-replacement and condition-based policies. (Ci/Cc = 0.05, Cp/Cc = 0.2, Cc = 500)

The condition-based maintenance cost depends on the inspection frequency and the tolerance to have a failure but also on the inspection cost. We suppose that the inspection cost is now variable but always lower than the preventive cost. *Figure 9* shows the optimal maintenance cost per time unit (according to the preventive threshold and to the inspection frequency) as a function of the inspection cost for both policies. We can see clearly that the optimal condition-based maintenance cost is lower than the optimal age-replacement cost if the inspection cost is less than 11. Otherwise the age-replacement policy is less expensive for this type of degradation model.

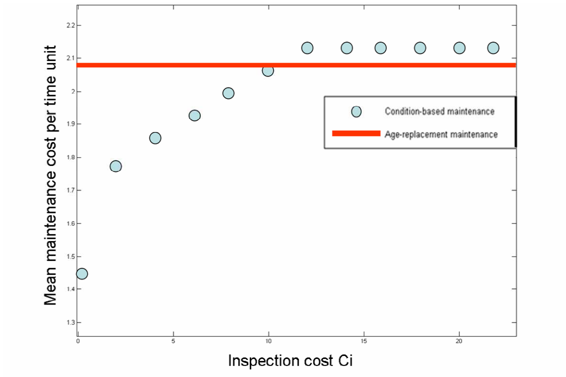

Figure 9. Model 1: Mean maintenance cost per unit time as a function of the inspection cost.

5.2. Model 2: Markov additive process degradation model

This model represents a system which is used in two different operating environments using Markov Additive process. The accumulation of the degradation rate is supposed to be the double in severe conditions.

(i) $0 \leq t < 20$ days: severe stress
(ii) $20 \leq t < 40$ days: normal stress
(iii) $40 \leq t < 50$ days: severe stress
(iv) $t \geq 50$ days: normal stress

The degradation processes are represented by gamma processes with $E_1 = \alpha_1/\beta_1 = 1.1$ and $Var = \alpha_1/\beta_1^2 = 5$ if the conditions are normal and $E_2 = \alpha_2/\beta_2 = 2.2$ and $Var = \alpha_2/\beta_2^2 = 10$ if the conditions are severe.

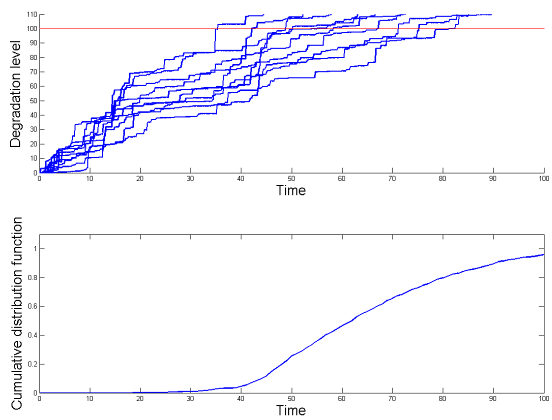

Figure 10. Degradation level and cumulative function of time to failure

Figure 11 shows the maintenance costs of condition-based (with fixed inspection cost C_i = 5) and of age-replacement policy. The condition-based maintenance cost increases with the occurrence of inspection and the tolerance to have a failure. The optimal age-replacement cost is always constant over these variables. The area A2 represents the couples of preventive thresholds and inspection time interval values where condition-based maintenance is less expensive than age-replacement policy.

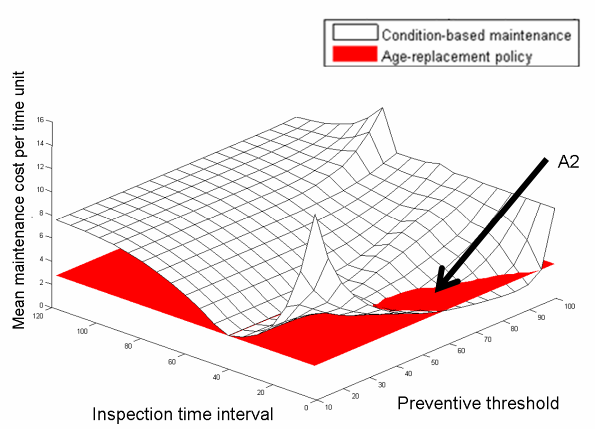

Figure 11. Mean maintenance cost per unit time of age-replacement and condition-based policies. ($C_i/C_c = 0.05$, $C_p/C_c = 0.2$, $C_c = 500$)

As the model 1, an inspection cost study is done. *Figure 12* shows that the optimal condition-based maintenance cost increases with the inspection cost. In this case, if the inspection cost is less than 46 it is more interesting to choose the condition-based maintenance instead of the age-replacement policy.

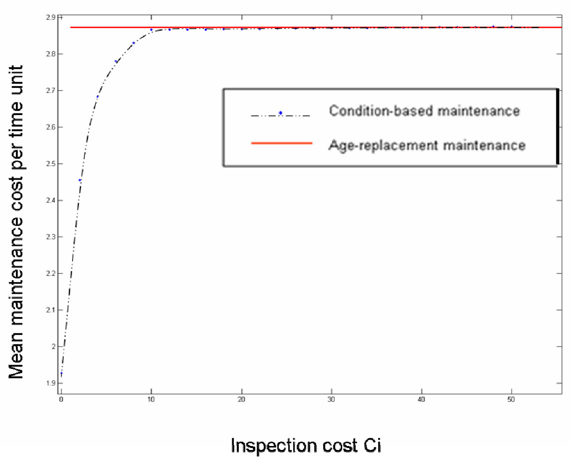

Figure 12. Model 2: Mean maintenance cost per unit time as a function of the inspection cost.

5.3. Perspectives

Further works can be developed including a variable inspection time interval for condition-based maintenance according to age and usage. In this study, we considered only a fixed inspection time interval.

At the beginning of the component life, the inspection time interval should be higher because of the low probability of failure. This inspection frequency should increase with time. Another parameter should also be considered: the usage. When the conditions of use are more severe, the inspection time should be shorter in order to detect earlier the preventive replacement and avoid the component failure. It would be interesting to define the inspection time interval according to each condition of use and time. Then, the influence of time and the inspection frequency will be more important on the condition-based maintenance cost.

The critical threshold should also be variable. Indeed this threshold is supposed to be constant but it could depend on the operating environment. Since the time-to-failure depends on the critical threshold, the variable of this setting will influence the occurrence of corrective replacement in condition-based maintenance. Thus, the cost per time unit will vary due to the corrective cost and the time of the operation cycle.

These perspectives could provide a better follow-up of the system degradation. It would be interesting to compute the condition-based maintenance cost per time unit with these assumptions. Then, it would be possible to evaluate the cost benefit of this policy with the optimal age-replacement maintenance cost.

6. Conclusion

This paper is only a preliminary study in order to compare two different maintenance policies with the cost criterion. Age-replacement policy is a maintenance policy which consists in replacing the system when it reaches a fixed operational age or when it fails. Condition-based maintenance is a monitoring maintenance which decides the maintenance operation (inspection, preventive or corrective replacement) according to the degradation level of the system.

These policies are compared by simulation on a same component in two different operating environments. If the component operates in a same environment (modelled by gamma process), there is a small margin to have a condition-based maintenance less expensive than an age-replacement policy.

Nevertheless more and more systems like truck components are subjected to different stresses which depend on the operating environment and on the conditions of use which usually change (modelled by Markov additive process). In this case, the limit of the inspection cost to have a cheaper condition-based maintenance is higher than the previous case.

This paper shows that it would be interesting to study further the parameters which influence the inspection cost limit which determines the cheaper maintenance policy for a deteriorating system in a dynamic environment.

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