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Elaboration of stochastic models to comprehensive evaluation of occupational risks in complex dynamic systems

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ABSTRACT

Purpose: Elaborate stochastic models to comprehensive evaluation of occupational risks in “man - machine - environment” systems taking into account the random and dynamic nature of the impact on the employee of negative factors over time.

Design/methodology/approach: Within study, the methods of probability theory and the theory of Markov processes - to find the limit distribution of the random process of dynamic impact on the employee of negative factors over time and obtain main rates against which the level of occupational risks within the "man - machine - environment" systems can be comprehensively evaluated; Erlang phases method, Laplace transform, difference equations theory, method of mathematical induction - to elaborate a method of analytical solution of the appropriate limit task for a system of differential equations in partial derivatives and appropriate limit conditions were used.

Findings: A system of differential equations in partial derivatives and relevant limit conditions is derived, which allowed to identify the following main rates for comprehensive evaluation of occupational risks in systems "man - machine - environment": probability of excess the limit of the employee's accumulation of negative impact of the harmful production factor; probability of the employee’s injury of varying severity in a random time. An method to the solution the limit task for a system of differential equations, which allows to provide a lower bounds of the probability of a certain occupational danger occurrence was elaborated.

Research limitations/implications: The elaborated approach to injury risk evaluation is designed to predict cases of non-severe injuries. At the same time, this approach allows to consider more severe cases too, but in this case the task will be more difficult.

Practical implications: The use of the elaborated models allows to apply a systematic approach to the evaluation of occupational risks in enterprises and to increase the objectivity of the evaluation results by taking into account the real characteristics of the impact of negative factors on the employee over time.

Originality/value: For the first time, a special subclass of Markov processes - Markov drift processes was proposed and substantiated for use to comprehensive evaluation of occupational risks in “man - machine - environment” systems.

Keywords: Safety and Health Management, Dynamic systems, Markov drift processes, Risk evaluation, Occupational risk

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INDUSTRIAL MANAGEMENT AND ORGANISATION

1. Introduction

This work is a continuation of the study [1], which is devoted to the elaboration of occupational risk evaluation models that take into account the dynamic and random characteristics of the impact on the employee of negative factors occurring within real systems "man – machine – environment". The analysis of literature sources and international standards concerning occupational risk evaluation in the study [1] showed that none of the known evaluation methods is designed to taking into account such characteristics, which significantly impact on the objectivity of the results [2-11].

Thus, in study [2], occupational risk evaluation is proposed using the Monte Carlo method, which makes it possible to take into account the random nature of the impact on the employee of harmful factors that occur during the work shift. At the same time, the proposed method cannot be used to evaluate such occupational risks as accidents of varying severity, occupational poisoning, etc. Study [3] is devoted to the development of a set of indicators of safety and occupational health of employees in the workplace, particularly for evaluating occupational risk. A questionnaire method is used to develop these indicators and evaluate occupational risk. Given that this method is expert, the main disadvantage of its using is the low objectivity of the results. First of all, this applies to the results of the evaluation of the risk of occupational diseases, since the characteristics of the intensities of the impact on the employee of most harmful factors can not be objectively taken into account in this case. The method of occupational risk evaluation presented in the study [4] allows to take into account the intensity of the impact of negative factors on the employee in time, but the characteristics of such impact are constant through time, that does not meet the real conditions of “man - machine - environment” systems within which its using is considered. In addition, this approach can be used to evaluate occupational risks only for a limited range of negative factors and only for a specific profession, which significantly restricts its use. The main disadvantage of the study [5] is the authors focus on the evaluation of psychosocial occupational risks exclusively, which is proposed to conduct on the results of the questionnaire. This approach is narrowly focused and can be used only in some cases. The method proposed in [6] is aimed at evaluating the risks of accidents caused by equipment failure only, which is its main disadvantage. However, the method does not allow to take into account the random nature of equipment failures, which can significantly reduce the objectivity of the results. Studies [7,8] are devoted to the analysis of trends in occupational risk management and analysis of known methods of their evaluation, in particular the analysis of the application for the relevant procedure of methods developed on the basis of the Markov processes theory. These methods requires to construct Markov chains with a countable set of states and a continuous time for experimental systems. This approach is elementary and does not allow to take into account the dynamic characteristics of the impact of negative factors on the employee over time. This significantly reduces the objectivity of the application of these methods to evaluate the risks of accidents (caused by equipment failure) and does not allow them to be used to evaluate the risks that can lead to occupational diseases or poisoning. A method for evaluating the effects of harmful factors on humans, taking into account the severity of the consequences of possible occupational dangers was developed in study [9]. At the same time, the characteristics of the intensity of accumulation of the negative impact of harmful factors in the human are considered as constant over time, that does not meet the real conditions of "man - machine - environment" systems. In addition, the method is narrowly focused, since it is not intended to evaluate occupational risks that can result in occupational injuries. In studies [10,11] to evaluate occupational risks an information-analytical system that processes existing arrays of statistical data using statistical analysis methods was proposed to use. The need to use large arrays of statistical data to evaluate occupational risks restricts the possibility of applying these system and approach due to a limited number of large enterprises, whose databases contain the necessary information. In addition, the inability to take into account the random and dynamic characteristics of the impact of negative factors on the employee over time can affect the objectivity of the evaluation results and restricts the application of this approach to evaluate the risks of occupational diseases and poisonings.

Thus, based on the results of the analysis, it is clear that each of the existing methods is intended solely to evaluate the certain occupational risks (e.g. or accident, or occupational disease) and does not take into account random and dynamic characteristics of negative factors on the employee that have taken place always within real "man – machine – environment" systems. The existence of these problems does not allow to apply a systematic approach to the evaluation of occupational risks in these systems and affects the objectivity of the evaluation results.

The stochastic models for occupational risk evaluation due to the negative impact on the employee only a certain range of negative factors, the so-called harmful factors (HF) were elaborated within study [1]. The consequence of the impact on the employee of these HF is occupational diseases or poisoning. These models allow to take into account the random and dynamic characteristics of the impact on the employee of the HF in time.

However, the need to apply a systematic approach during occupational risks evaluation in "man – machine – environment" systems also requires considering cases of impact on the employee of another negative factors, the so-called danger factors (DF) [12-14]. The consequences of such DF are industrial injuries of varying severity. Therefore, there is a need to elaborate the stochastic models that will allow a comprehensive evaluation of the risks of both occupational diseases (poisoning) and the employee's occupational injury in a random period (during the work shift).

It is also of interest to find an analytical solution of the limit task specified in the study [1] for a system of differential equations, which will simplify the use of elaborated stochastic models to occupational risk evaluation in practice.

2. Materials and methods

The methods of probability theory and the theory of Markov processes were used in the study:

- to find the limit distribution of the random process with parameters that describe the change in employment of the employee over time, the change in the efficiency of the employee and production equipment, as well as the level of accumulation of negative impact of harmful factor in the employee;
- to obtain the main rates (the probability of exceeding the accumulation of the negative impact of the harmful production factor of normalized values, the probability of industrial injury by an employee with varying severity in a random period, etc.) against which the level of occupational risks within the "man – machine – environment" systems can be quantified.

Erlang phases method was used to simplify the mathematical structure of the algorithm for solving the limit task. The application of this method allowed finding an appropriate solution for the system of ordinary differential equations instead of finding a solution of the limit task for the system of differential equations in partial derivatives. The Laplace transform was used to solve a limit task for a system of ordinary differential equations by obtaining the appropriate system of linear algebraic equations. The difference equations theory, method of mathematical induction were used to solve a system of linear algebraic equations and find the moments of probability distribution of the level of accumulation of negative impact of harmful factors in employee, which allows using Chebyshev's inequality to give a lower bounds for probability of the appropriate occupational risk.

3. Results and discussion

Consider the real production process within the system "man – machine – environment", when random negative factors impact on the employee. The fact that during the work shift the employee is negatively impacted by a certain HF, which occurs during the operation of a certain unit of production equipment, is as a prerequisite, under [1]. As a result, there is an accumulation of the negative impact of the this HF in the employee. It also be assumed that the duration of work shift and non-working time are mutually independent random variables with distribution functions $A_0(t)$ and $A_1(t)$, respectively. That is, changes in working and non-working hours form an alternating renewal process.

During working hours the production equipment gives into the work area certain HF with an intensity of W, which in turn accumulate in the employee. During non-working hours the negative impact of HF is removed from the employee with an intensity of $U < W$ [1]. Production equipment can fail at random times and be restored over a random period, both with different statistical characteristics in working and non-working conditions. It will be considered two types of equipment failures, namely: the first type – when equipment failure does not impact on the employee's health, and the second type – when any deviations from the normal mode of operation of production equipment lead to employee's injury. In this case, it should be considered the failures of production equipment of two types at the same time – which did not lead to and led to employee's injury. During the restoration process of production equipment, after the failure of the first type, the functional systems of the employee's body are restored from the consequences of the negative impact of HF that occurred during the operation of the equipment. This restoration occurs with a certain intensity U [1]. And during the treatment and rehabilitation of the employee, in addition to restoration from injury, there is also a restoration of the employee's functional systems from the negative impact of HF that occur during the performance of duties. This restoration occurs with an intensity of U_I. After treatment and rehabilitation procedures, the employee assumes its duties.

The time of failure-free operation of production equipment is adopted as a random variable distributed according to the exponential law, both the average operating time for failure of the k-th type in the i-th period is equal to λ_{ik}^{-1}. The restoration time of the equipment after the failure of the k-th type in the i-th period is also considered as random variable, distributed according to the exponential law with an average value μ_{ik}^{-1}, i = 0,1; k =1,2. For simplicity, the restoration time of production equipment after the failure of the second type is adopted as more than the treatment time of the employee from the injury during the failure. Such cases, according to accident investigation statistics, are the most common, as they characterize the non-severe injuries [15-19]. However, if necessary, the using a similar approach within the Markov drift processes allows to consider other cases where the treatment and rehabilitation of the employee requires considerable time, but in this case the task solution will be more difficult. For a formalized description of the research process, the following notation will be introduced:

- $\alpha(t)$ – variable that describes the employment state of the employee during the work shift ($\alpha(t)=1$) and during non-working hours ($\alpha(t)=0$) at time t;
- $\gamma(t)$ – a variable that describes the efficiency state of production equipment and the employee. $\gamma(t)=0$, if the production equipment at time t is operable and the employee is not injured; $\gamma(t)=1$, if at time t the production equipment is inoperable and the employee is not injured; $\gamma(t)=2$, if equipment fail, that resulted in employee's injury;
- $\xi(t)$ – the level of accumulation of the negative impact of HF in the employee at time t.

The task consists in finding the distribution of a random vector ($\xi(t)$, $\alpha(t)$, $\gamma(t)$). This process is not Markov for arbitrary distribution functions $A_i(t)$, i = 0,1. However, adding an additional continuous component $\eta(t)$ – the time remaining from time t to change the state of the alternating process (the process that describes the change of working and non-working time of the employee), the process becomes Markov [1]. Therefore, within the task, it is necessary to study the random process:

$$\Xi(t)=(\xi(t),\alpha(t),\gamma(t),\eta(t)). \tag{1}$$

For greater clarity, the possible states of the first two components of this random process, ie ($\alpha(t)$, $\gamma(t)$): should be described in detail:

- (0,0) – the time of the work shift is over, the intensity of removing of the negative impact of the HF is equal to U;
- (0,1) – non-working hours, production equipment fails (failure of the first type) and is restored, the employee is not injured, the intensity of removing of the negative impact of the HF is equal to U;
- (1,0) – working hours, the equipment is operable, intensity of removing of negative impact of the HF is equal to U;
- (1,1) – working hours, production equipment fails (failure of the first type), the employee is not injured, the intensity of removing of the negative impact of the HF is equal to U;
- (1,2) – working hours, production equipment fails (failure of the second type), the employee is injured, the intensity of removing of the negative impact of the HF is equal to $U_I < U$.

To find the limit distribution of a random process $\Xi(t)=(\xi(t),\alpha(t),\gamma(t),\eta(t))$, the following probability densities for the this random process will be introduced:

$$P\{\alpha(t)=i,\gamma(t)=k,\tau<\eta(t)<\tau+d\tau,x<\xi(t)<x+dx\}=$$
$$=q_{ik}(x,\tau,t)(1-A_i(\tau))d\tau dx, i=0,1; k=0,1,2; x>0,\tau>0,$$
$$P\{\alpha(t)=0,\gamma(t)=k,\tau<\eta(t)<\tau+d\tau,\xi(t)=0\}=$$
$$=\bar{q}_{0k}(\tau,t)(1-A_0(\tau))d\tau,\tau>0,k=0,1,2;$$
$$P\{\alpha(t)=1,\gamma(t)=k,\tau<\eta(t)<\tau+d\tau,\xi(t)=0\}=$$
$$=\bar{q}_{1k}(\tau,t)(1-A_i(\tau))d\tau,k=1,2;\tau>0,$$

where τ – the time elapsed from the beginning of the work shift (from the beginning of the non-working hours) to the moment t; x – the amount of harmful substances in the employee at time t; q – the density of the probability of joint distribution of the amount of harmful substances and the time remaining before the change of the alternating process; i and k – discrete variables describing the values of the variables α and γ, respectively, at time t.

The corresponding limit distribution is given by the following functions:

$$\begin{aligned} &q_{ik}(x,\tau)=\lim_{t\to\infty}q_{ik}(x,\tau,t), i=0,1; k=0,1,2,\\ &\bar{q}_{00}(\tau)=\lim_{t\to\infty}\bar{q}_{00}(\tau,t),\\ &\bar{q}_{0k}(\tau)=\lim_{t\to\infty}\bar{q}_{0k}(\tau,t), k=0,1,2;\\ &\bar{q}_{1k}(\tau)=\lim_{t\to\infty}\bar{q}_{1k}(\tau,t), k=1,2. \end{aligned} \tag{2}$$

For functions (2) the following system of differential equations in partial derivatives can be derived by the standard method [1]:

$$
\begin{aligned}
&(-U\frac{\partial}{\partial x}+\frac{\partial}{\partial \tau})q_{00}(x,\tau)=-\lambda_0 q_{00}(x,\tau)+\mu_{01}q_{01}(x,\tau)+\mu_{02}q_{02}(x,\tau),\\
&(-U\frac{\partial}{\partial x}+\frac{\partial}{\partial \tau})q_{01}(x,\tau)=-\mu_{01}q_{01}(x,\tau)+\lambda_{01}q_{00}(x,\tau),\\
&(-U_1\frac{\partial}{\partial x}+\frac{\partial}{\partial \tau})q_{02}(x,\tau)=-\mu_{02}q_{02}(x,\tau)+\lambda_{02}q_{02}(x,\tau), x>0, \tau>0,
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
&(V\frac{\partial}{\partial x}+\frac{\partial}{\partial \tau})q_{10}(x,\tau)=-\lambda_1 q_{10}(x,\tau)+\mu_{11}q_{11}(x,\tau)+\mu_{12}q_{12}(x,\tau),\\
&(-U\frac{\partial}{\partial x}+\frac{\partial}{\partial \tau})q_{11}(x,\tau)=-\mu_{11}q_{11}(x,\tau)+\lambda_{11}q_{10}(x,\tau),\\
&(-U_1\frac{\partial}{\partial x}+\frac{\partial}{\partial \tau})q_{12}(x,\tau)=-\mu_{12}q_{12}(x,\tau)+\lambda_{12}q_{10}(x,\tau),\\
&\qquad x>0, \tau>0,
\end{aligned} \tag{4}
$$

where, U and U_1 – the intensity of removing of the consequences of the negative impact of the HF from the uninjured and injured employee, respectively; $V = W - U$ – the intensity of the accumulation of negative impact in the employee of HF; W – the intensity of the accumulation of the negative impact in the employee of HF; λ_{ik} – the intensity of the flow of equipment failures of the k-th type in the i-th period; μ_{ik} – the intensity of the flow of restoration of the employee after the failure of the equipment of the k-th type in the i-th period, $i = 0,1$; $k = 1,2$;
$\lambda_i = \lambda_{i1} + \lambda_{i2}$, $i = 0,1$.

The limit conditions for the system of differential equations (3) and (4) are following.

For the non-working hours:

$$
\begin{aligned}
&\frac{d}{d\tau}\bar{q}_{00}(\tau)-Uq_{00}(0,\tau)=-\lambda_0\bar{q}_{00}(\tau)+\mu_{01}\bar{q}_{01}(\tau),\\
&\frac{d}{d\tau}\bar{q}_{01}(\tau)-Uq_{01}(0,\tau)=\lambda_0\bar{q}_{00}(\tau)-\mu_0\bar{q}_{01}(\tau),\\
&\frac{d}{d\tau}\bar{q}_{02}(\tau)-U_1q_{02}(0,\tau)=\lambda_{02}\bar{q}_{00}(\tau)-\mu_{02}\bar{q}_{02}(\tau),
\end{aligned} \tag{5}
$$

For the working hours:

$$
\begin{aligned}
&\frac{d}{d\tau}\bar{q}_{11}(\tau)-Uq_{11}(0,\tau)=-\mu_{11}\bar{q}_{11}(\tau), \tau>0;\\
&\frac{d}{d\tau}\bar{q}_{12}(\tau)-U_1q_{12}(0,\tau)=-\mu_{12}\bar{q}_{12}(\tau), \tau>0;
\end{aligned} \tag{6}
$$

Conditions describing the change of working and non-working hours:

$$
\begin{aligned}
&q_{00}(x,0)=\int_0^\infty q_{10}(x,\tau)dA_1(\tau),\\
&q_{01}(x,0)=\int_0^\infty q_{11}(x,\tau)dA_1(\tau),\\
&q_{02}(x,0)=\int_0^\infty q_{12}(x,\tau)dA_1(\tau),\\
&q_{10}(x,0)=\int_0^\infty q_{00}(x,\tau)dA_0(\tau),\\
&q_{11}(x,0)=\int_0^\infty q_{01}(x,\tau)dA_0(\tau),\\
&q_{12}(x,0)=\int_0^\infty q_{02}(x,\tau)dA_0(\tau);
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
&\bar{q}_{01}(0)=\int_0^\infty \bar{q}_{11}(\tau)dA_1(\tau),\\
&\bar{q}_{11}(0)=\int_0^\infty \bar{q}_{01}(\tau)dA_0(\tau),\\
&\bar{q}_{12}(0)=\int_0^\infty \bar{q}_{02}(\tau)dA_1(\tau),\\
&\bar{q}_{02}(0)=\int_0^\infty \bar{q}_{12}(\tau)dA_1(\tau),\\
&\bar{q}_{00}(0)=0.
\end{aligned} \tag{8}
$$

Condition describing the transition from a state when the level of negative impact of HF is zero, to a state when this level begins to increase:

$$
\begin{aligned}
&V\int_0^\infty q_{10}(0,\tau)(1-A_1(\tau))d\tau=\mu_{11}\int_0^\infty \bar{q}_{11}(\tau)(1-A_1(\tau))d\tau+\\
&+\mu_{12}\int_0^\infty \bar{q}_{12}(\tau)(1-A_1(\tau))d\tau+\int_0^\infty \bar{q}_{00}(\tau)dA_0(\tau);
\end{aligned} \tag{9}
$$

Rationing condition:

$$
\begin{aligned}
&\int_0^\infty[(\bar{q}_{00}(\tau)+\bar{q}_{01}(\tau))(1-A_0(\tau))+(\bar{q}_{11}(\tau)+\bar{q}_{12}(\tau))(1-A_1(\tau))]d\tau+\\
&+\int_0^\infty\int_0^\infty[(q_{00}(x,\tau)+q_{01}(x,\tau)+q_{02}(x,\tau))(1-A_0(\tau))+\\
&+(q_{10}(x,\tau)+q_{11}(x,\tau)+q_{12}(x,\tau))(1-A_1(\tau))]dxd\tau=1.
\end{aligned} \tag{10}
$$

As a result of solving the limit task (3)-(10), the number of main indicators that characterize the level of occupational risk can be obtained. Namely:

- The probability of exceeding the accumulation of the negative impact of HF of normalized values σ (hygienic standard):

$$\int_0^\infty\int_\sigma^\infty [(q_{00}(x,\tau)+q_{01}(x,\tau)+q_{02}(x,\tau))(1-A_0(\tau))+ +(q_{10}(x,\tau)+q_{11}(x,\tau)+q_{12}(x,\tau))(1-A_1(\tau))]dxd\tau. \quad (11)$$

- The probability that in a random time the consequences of the negative impact of HF in the employee will be completely absent:

$$\int_0^\infty (\bar{q}_{00}(\tau)+\bar{q}_{01}(\tau))(1-A_0(\tau))d\tau+\int_0^\infty (\bar{q}_{11}(\tau)+\bar{q}_{12}(\tau))(1-A_1(\tau))d\tau. \quad (12)$$

- The probability of injury by an employee in a random time:

$$\int_0^\infty \bar{q}_{12}(\tau))(1-A_1(\tau))d\tau+ +\int_0^\infty\int_0^\infty [q_{02}(x,\tau)(1-A_0(\tau))+q_{12}(x,\tau)(1-A_1(\tau))]dxd\tau. \quad (13)$$

- The average level of accumulation of negative impact in the employee of HF:

$$M\xi=\int_0^\infty x\int_0^\infty [(q_{00}(x,\tau)+q_{01}(x,\tau)+q_{02}(x,\tau))(1-A_0(\tau))+ +(q_{10}(x,\tau)+q_{11}(x,\tau)+q_{12}(x,\tau))(1-A_1(\tau))]d\tau dx. \quad (14)$$

As an example, a method to solution the simpler limit task for the case of an impact on an employee of the HF will be elaborated. The basic system of differential equations and limit conditions for this task were derived in a previous study (see (6)-(12) [1]). The solution this limit task is a complex mathematical problem, so in the elaborating an algorithm for its solution, the following assumptions, which are correct for the real conditions of functioning of systems "man – machine – environment", are made. Namely, it will be assumed that:

- random failures of production equipment that produces the HF and its restoration after failures occur only during the working hours (during the work shift). In other words, it will be assumed that the parameters $\mu_0 = 0$, $\lambda_0 = 0$.
- the alternating duration of working and non-working hours distributed according to Erlang's law of the same order $r \geq 1$.

These assumptions will simplify the mathematical structure of task because it will be possible to find a suitable solution for a system of ordinary differential equations instead of finding a solution to the limit task for a system of differential equations in partial derivatives. Formally, such simplification is achieved by the use of the Erlang phases method.

Thus, it is accepted that:

$$A_k(t)=1-e^{-a_k t}\sum_{j=0}^{r-1}\frac{(a_k t)^j}{j!}, k=0,1. \quad (15)$$

In addition, the Erlang distribution has a very important property for the research model. Because, as is known, under $r\to\infty$, $1/a_k\to 0$, it converges to a degenerate distribution, i.e. the duration of the working the non-working hours tend to constant values T_k, which corresponds to the real conditions of functioning of systems "man – machine – environment", i.e. [20]:

$$r / a_k \to T_k, k=0,1.$$

Also, the Erlang distribution (15) is a distribution of the sum r of mutually independent random variables, each of which is distributed exponentially with the same parameter a_k This makes it possible to change the non-Markov process to the Markov process, due to the introduction of fictitious phases.

To be able to use the properties of the Erlang distribution, a new random variable $\nu(t)$, that will indicate the number of the current phase of the Erlang distribution will be introduced. Thus, instead of $\Xi(t)=(\xi(t),\alpha(t),\gamma(t),\eta(t))$ [1] a new Markov process will be introduced to consideration:

$$\Xi_1(t)=(\xi(t),\alpha(t),\gamma(t),\nu(t)), \quad (16)$$

where all other variables have an initial meaning.

Accordingly, the following probabilistic characteristics of the process $\Xi_1(t)$: will be introduced:

$$P\{\alpha(t)=i,\nu(t)=j,\gamma(t)=k,x<\xi(t)<x+dx\}= =q_{ijk}(x,t)dx, i=0,1; k=0,1; j=0,1,2,...,r-1; x>0,$$

$$P\{\alpha(t)=0,\nu(t)=j,\gamma(t)=k,\xi(t)=0\}=\bar{p}_{0jk}(t),$$

$$P\{\alpha(t)=1,\nu(t)=j,\gamma(t)=1,\xi(t)=0\}= =\bar{p}_{ij1}(t), i=0,1; j=0,1,...,r-1.$$

Below the following limit distribution of the introduced random process will be considered, namely:

$$q_{ijk}(x)=\lim_{t\to\infty}q_{ijk}(x,t), i=0,1; k=0,1; j=0,1,2,...,r-1;$$
$$p_{0jk}^{-}=\lim_{t\to\infty}p_{0jk}^{-}(t), \tag{17}$$
$$p_{ij1}^{-}=\lim_{t\to\infty}p_{ij1}^{-}(t), i=0,1; j=0,1,...,r-1.$$

It should be noted that the state ($\alpha(t)=0$, $\nu(t)=j$, $\gamma(t)=1$, $\xi(t)=0$) is possible with a positive probability p_{0j1}^{-}, because during the non-working hours the production equipment that failed in one of the previous work shift, may be waiting to restoration in subsequent working hours.

The introduced Markov process belongs to the class of Markov drift processes. Therefore, by standard probabilistic considerations based on the consideration of transitions of the Markov process from one state to another in a short time, as well as the application of the total probability formula to find the introduced probability density limits and probabilities (17), the following system of ordinary differential equations can be derived [21, 22]:

$$-Uq'_{000}(x)=-a_0q_{000}(x)+a_1q_{1,r-1,0}(x),$$
$$-Uq'_{0j0}(x)=-a_0q_{0j0}(x)+a_0q_{0,j-1,0}(x), \tag{18}$$
$$j=1,2,...,r-1,$$

$$-Uq'_{001}(x)=-a_0q_{001}(x)+a_1q_{1,r-1,1}(x),$$
$$-Uq'_{0j1}(x)=-a_0q_{0j1}(x)+a_0q_{0,j-1,1}(x), \tag{19}$$
$$j=1,2,...,r-1,$$

$$-Uq'_{101}(x)=-(a_1+\mu)q_{101}(x)+\lambda q_{100}(x)+a_0q_{0,r-1,1}(x),$$
$$-Uq'_{1j1}(x)=-(a_1+\mu)q_{0j1}(x)+\lambda q_{1j0}(x)+a_1q_{1,j-1,1}(x), \tag{20}$$
$$j=1,2,...,r-1,$$

$$Vq'_{100}(x)=-(a_1+\lambda)q_{100}(x)+\mu q_{101}(x)+a_0q_{0,r-1,0}(x),$$
$$Vq'_{1j0}(x)=-(a_1+\lambda)q_{0j1}(x)+\mu q_{1j1}(x)+a_1q_{1,j-1,0}(x), \tag{21}$$
$$j=1,2,...,r-1; x>0.$$

The corresponding limit conditions for the system of differential equations (18)-(21) describe the behaviour $\Xi_1(t)=(\xi(t),\alpha(t),\gamma(t),\nu(t))$ near the zero value of the process of accumulation of negative impact of HF in the employee and have the following form:

$$-Uq_{001}(0)=-a_0p_{001}^{-}+a_1p_{1,r-1,1}^{-},$$
$$-Uq_{101}(0)=-(a_1+\mu)p_{101}^{-}+a_0p_{0,r-1,1}^{-},$$
$$-Uq_{1j1}(0)=-(a_1+\mu)p_{1j1}^{-}+a_1p_{1,j-1,1}^{-}, \tag{22}$$
$$-Uq_{0j1}(0)=-a_0p_{0j1}^{-}+a_0p_{0,j-1,1}^{-},$$
$$-Uq_{0j0}(0)=-a_0p_{0j0}^{-}+a_0p_{0j-1,0}^{-},$$

$$Vq_{100}(0)=a_0p_{0,r-1,0}^{-}+\mu p_{101}^{-},$$
$$q_{1j0}(0)=0, j=1,2,...,r-1. \tag{23}$$

Conditions (22) describe the entry of the Markov process into a state with zero level of accumulation of negative impact of HF in the employee, and conditions (23) – the exit from such a state. Finally, the rationing condition is as follows:

$$\sum_{j=0}^{r-1}(p_{0j0}^{-}+p_{0j1}^{-}+p_{1j1}^{-})+$$
$$+\sum_{j=0}^{r-1}\int_0^{\infty}(q_{0j0}(x)+q_{0j1}(x)+q_{1j1}(x)+q_{1j0}(x))dx=1. \tag{24}$$

The standard method for solving the limit task (18)-(24) is based on the use of the Laplace transform. To solve it, the following notations will be introduced:

$$\int_0^{\infty} e^{-sx}q_{ijk}(x)dx=q^{*}_{ijk}(s), \mathrm{Re}\,s\geq 0;$$
$$l,i=0,1; k=0,1; j=0,1,2,...,r-1. \tag{25}$$

Applying the Laplace transform to a system of differential equations (18)-(21), using its known properties, the following system of linear algebraic equations, relative to (25) was obtained:

$$(a_0-Us)q^{*}_{000}(s)-a_1q^{*}_{1,r-1,0}(s)=-Uq_{000}(0)$$
$$(a_0-Us)q^{*}_{0j0}(s)-a_0q^{*}_{0,j-1,0}(s)=-Uq_{0j0}(0), j=1,2,...,r-1, \tag{26}$$

$$(a_0-Us)q^*_{001}(s)-a_1q^*_{1,r-1,1}(s)=-Uq_{001}(0)$$
$$(a_0-Us)q^*_{0j1}(s)-a_0q^*_{0,j-1,1}(s)=-Uq_{0j1}(0), j=1,2,...,r-1, \tag{27}$$

$$(a_1+\mu-Us)q^*_{101}(s)-a_0q^*_{0,r-1,1}(s)-\lambda q^*_{100}(s)=-Uq_{101}(0)$$
$$(a_1+\mu-Us)q^*_{1j1}(s)-a_1q^*_{1,j-1,1}(s)-\lambda q^*_{1j0}(s)=-Uq_{1j1}(0), \tag{28}$$
$$j=1,2,...,r-1,$$

$$(a_1+\lambda+Vs)q^*_{100}(s)-a_0q^*_{0,r-1,0}(s)-\mu q^*_{101}(s)=Vq_{100}(0),$$
$$(a_1+\lambda+Vs)q^*_{1j0}(s)-a_1q^*_{1,j-1,0}(s)-\mu q^*_{1j1}(s)=Vq_{1j0}(0), \tag{29}$$
$$j=1,2,...,r-1.$$

The rationing condition (24) under the new notation will take the following form:

$$\sum_{j=0}^{r-1}(\bar{p}_{0j0}+\bar{p}_{0j1}+\bar{p}_{1j1}+q^*_{0j0}(0)+q^*_{0j1}(0)+q^*_{1j1}(0)+q^*_{1j0}(0))=1. \tag{30}$$

So from equations (26) it is got:

$$q^*_{000}(s)=\frac{a_1}{a_0-Us}q^*_{1,r-1,0}(s)-\frac{U}{a_0-Us}q_{000}(0), \tag{31}$$

$$q^*_{0j0}(s)=(\frac{a_0}{a_0-Us})^j[q^*_{000}(s)-\frac{U}{a_0}\sum_{l=1}^{j}(\frac{a_0-Us}{a_0})^{l-1}q_{0l0}(0)], \tag{32}$$
$$j=1,2,...,r-1,$$

but from equations (27) it is got, similarly:

$$q^*_{001}(s)=\frac{a_1}{a_0-Us}q^*_{1,r-1,1}(s)-\frac{U}{a_0-Us}q_{001}(0), \tag{33}$$

$$q^*_{0j1}(s)=(\frac{a_0}{a_0-Us})^j[q^*_{001}(s)-\frac{U}{a_0}\sum_{l=1}^{j}(\frac{a_0-Us}{a_0})^{l-1}q_{0l1}(0)], \tag{34}$$
$$j=1,2,...,r-1.$$

Furthermore, from equations (28) and (29), the following equation in finite differences of the 2nd order concerning the function $q^*_{1j1}(s)$, j=1,2,…,r-1 will be got:

$$q^*_{1j1}(s)[(a_0+\lambda+Vs)(a_1+\mu-Us)-\lambda\mu]-$$
$$-a_1q^*_{1j-1,1}(s)[2a_0+\lambda+\mu+(V-U)s]+a_1^2q^*_{1,j-2,1}(s)= \tag{35}$$
$$=\lambda Vq_{1j0}(0)-U(a_0+\lambda+Vs)q_{1j1}(0)+a_1Uq_{1,j-1,1}(0), j=2,...,r-1.$$

Based on the theory of difference equations, the general solution of the inhomogeneous difference equation (35) has the following form [23]:

$$q^*_{1j1}(s)=\frac{1}{x_2(s)-x_1(s)}\{[x_1^j(s)x_2(s)-x_1(s)x_2^j(s)]q^*_{101}(s)+$$
$$+[x_2^j(s)-x_1^j(s)]q^*_{111}(s)\}+Q_j(s), \quad j=0,1,...,r-1, \tag{36}$$

where $x_{1,2}(s)$ – the roots of the characteristic quadratic equation for the corresponding homogeneous difference equation, $Q_j(s)$ – partial solution of an inhomogeneous equation (35)

$$\delta(s)x^2-a_1\beta(s)x+a_1^2=0,$$

while:

$$x_{1,2}(s)=\frac{a_1}{2\delta(s)}\{\beta(s)\pm\sqrt{(\beta(s))^2-4\delta(s)}\},$$
$$\beta(s)=2a_0+\lambda+\mu+(V-U)s;$$
$$\delta(s)=(a_0+\lambda+Vs)(a_1+\mu-Us)-\lambda\mu;$$

Using the method of mathematical induction, it can show that:

$$Q_j(s)=\frac{1}{(r-1)!(j-2)!}\frac{\partial^{j-2}\partial^{r-1}}{\partial z^{j-2}\partial y^{r-1}}\left.\frac{\sum_{l=2}^{r+1}d_l(s)z^{l-2}y^{l-2}}{(1-y)[\delta(s)-zya_1\beta(s)+a_1^2z^2y^2]}\right|_{z=y=0}, \tag{37}$$
$$j=2,3,...,r-1;$$

where

$$d_j(s)=Vq_{1j0}(0)-(a_0+\lambda+Vs)Uq_{1j1}(0)/\lambda+a_1Uq_{1,j-1,1}(0)/\lambda,$$
$$j=1,2,..,r-1.$$

Functions $q^*_{1j0}(s)$, $j=1,2,\ldots,r$-1 are expressed through functions $q^*_{1j1}(s)$, using the correlation (28):

$$\lambda q^*_{1j0}(s)=-(a_1+\mu-Us)q^*_{1j1}(s)-a_1q^*_{1,j-1,1}(s)+Uq_{1j1}(0), \quad (38)$$
$$j=1,2,...,r-1.$$

Thus, to find unknown functions

$$q^*_{000}(s),\ q^*_{001}(s), q^*_{100}(s),\ q^*_{101}(s), q^*_{111}(s) \quad (39)$$

all the necessary equations (31), (33), (36)-(38) are found. As a result, functions (39) will be expressed through as yet unknown constants

$$q_{1j0}(0), q_{1j1}(0), q_{1,j-1,1}(0), j=1,2,..,r-1,$$

which are expressed through probabilities

$$\bar{p}_{001}, \bar{p}_{1,r-1,1}, \bar{p}_{101}, \bar{p}_{0,r-1,1}, \ \bar{p}_{0,r-1,0},$$

using limit conditions (22), (23). The remaining probabilities can be found using the standard technique, which is often used in applied probabilistic tasks, namely using the property of analyticity of functions (25) in the right half-plane of the complex plane. In our case this condition is reduced to the simultaneous conversion to zero of the numerator and denominator of the fraction (37) for several values of the variable s (denominator zeros). It will give the additional relations to determine the search constants (together with the rationing condition (30)).

After finding the search probability densities and probabilities of states of the experimental system, the necessary indicators that characterize the negative impact of HF on the employee can be found.

The greatest interest is finding the probability that in the random period of work shift (production equipment is operational) the accumulation of negative impact of HF in the employee will not exceed the values σ. In the accepted notation this probability will be equal to

$$\int_0^{\sigma}\sum_{j=0}^{r-1} q_{1j0}(x)dx$$

Since clear expressions for densities $q_{1j0}(x)$, $j=0,1,\ldots,r$-1 are very difficult to find, as can be seen from the above calculations, the evaluation of the given probability using, for example, one of the modifications of the Chebyshev inequality can be used. To do this, only the first two moments of the necessary probability distribution should be found

$$F(x)=\int_0^{x}\sum_{j=0}^{r-1} q_{1j0}(u)du \Big/ \int_0^{\infty}\sum_{j=0}^{r-1} q_{1j0}(u)du$$

or using of Laplace transform

$$sF^*(s)=\sum_{j=0}^{r-1} q^*_{1j0}(s) \Big/ \sum_{j=0}^{r-1} q^*_{1j0}(0)$$

These moments of the given distribution are calculated by the formulas:

$$M_1=-\frac{d}{ds}sF^*(s)\Big|_{s=0}=-\sum_{j=0}^{r-1}\frac{d}{ds}q^*_{1j0}(s)\Big|_{s=0}\Big/\sum_{j=0}^{r-1}q^*_{1j0}(0),$$

$$M_2=\frac{d^2}{ds^2}sF^*(s)\Big|_{s=0}=\sum_{j=0}^{r-1}\frac{d^2}{ds^2}q^*_{1j0}(s)\Big|_{s=0}\Big/\sum_{j=0}^{r-1}q^*_{1j0}(0).$$

Chebyshev inequality will be used in the following form [24]:

$$F(\sigma)\geq\frac{(M_1-\sigma)^2}{(M_1-\sigma)^2+M_2-M_1^2}.$$

The expression F(σ) is the probability that the level of accumulation of the negative impact of HF in the employee does not exceed the maximum allowable (normalized) values. The last inequality gives a lower bounds for the search probability.

Similarly, an algorithm to solve a more complex limit task (3)-(10) presented in this study (the impact on the employee HF and DF) can be elaborated. In the general case, the practical implementation of occupational risk evaluation to obtain the necessary quantitative results can be done by numerically solving the system of differential equations and limit conditions using special packages of computer programs, such as Matlab.

4. Conclusions

1. The task of comprehensive evaluation of occupational risks within "man – machine – environment" systems, taking into account the random and dynamic nature of

the impact on the employee of negative factors over time is presented as finding the probability distribution of the Markov drift process. The parameters of this process describe the change in the state of employment of the employee over time, the change in the efficiency of the employee and production equipment, as well as the level of accumulation of negative impact of harmful factors in the employee.
2. In order to find the limit distribution of the appropriate random Markov process (solution of the limit task), the system of basic differential equations in partial derivatives and appropriate limit conditions is derived. Stochastic models that allow to evaluate comprehensively the level of occupational risk within the "man – machine – environment" systems: the probability of exceeding the accumulation of the negative impact of the HF of normalized values; the probability of an industrial injury by an employee in a random period with a certain severity, and others were elaborated.
3. A method of analytical solution of the limit task for the system of differential equations in Laplace transform for the case of Erlang distribution of working and non-working periods is elaborated. The application of this method allows to simplify the use of elaborated models in practice and to quantify the level of occupational risk within the "man – machine – environment" systems.

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