Sławomir ŻABA, Maciej SIWCZYŃSKI, Andrzej DRWAL
DEPARTMENT OF ELETRICAL AND COMPUTER ENGINEERING, CRACOW UNIVERSITY OF TECHNOLOGY
24 Warszawska St., 31-155 Cracow, Poland

Energy-optimal current distribution in a complex linear electrical network with pulse or periodic voltage and current signals. Suboptimal control

Abstract

In the circuits of electrical signals belonging to the L^1-impulses space or periodic signals space, occurring there real distribution of electrical currents does not meet the principle of minimum energy losses [1, 2]. The solution to this problem is to introduce the control system as current-dependent voltage sources vector, entered into a meshes set of a complex RLC network. It has been shown that the control is energy-neutral (optimal control) [2]. For energy-optimal controlling, to obtain the control operator, the inversion of **R**(*s*) operator is required. It is the matrix operator and the dispersive operator (it depends on frequency). Inversion of such operators is inconvenient because it is algorithmically complicated. To avoid this, the operator **R**(*s*) is replaced by the **R'** operator which is a matrix, but nondispersive one (does not depend on *s*). Such control is called the suboptimal control.

Keywords: L^1-impulses, linear circuits, principle of minimum energy losses, operators, suboptimal control.

1. Introduction

The issues relating to the quality of electrical energy distribution and minimization of energy losses usually refer to minimization of some energy indicators, such as reactive power, or to obtain an optimal, on account of energy, currents distribution.

In the DC circuits there is the minimum energy principle, according to which the currents distribution in a complex network is such that the total energy loss is minimal [3, 4]. However, this rule usually no longer works in the sinusoidal current circuits [5].

On the other hand, in nonsinusoidal signals domain, the term "reactive power" makes no sense, which means that this term should not be used during testing the quality of electrical energy distribution in the network [6, 7]. However, the compensation problems aimed at resetting the indicator of reactive power can be solved as optimization tasks consisting in minimizing energy losses in the network or as related tasks of minimizing the RMS value of currents [8, 9].

In the articles [1, 2] was shown that in the circuits with the signals belonging to the linear L^1-impulses space, there actually occurring current distribution does not satisfy the principle of minimum energy losses. To make it so, it is necessary to use the control system. It was considered a complex network powered multicurrent and with a multidimensional current-voltage control. It has been shown that the system of controlled sources is energy neutral. Thus, the process of minimal energy controlling is energy neutral.

In Fig. 1 the RLC network with power given as a vector of current signals $\boldsymbol{i_0}$ is shown. Distribution of mesh currents within the network is determined by the vector of current signals $\boldsymbol{i}$. The network is characterized by the so-called "internal operators matrix" $\mathbf{Z}(s)$ (s=d/dt), and the matrix so-called "contact operators" $\mathbf{Z}_0(s)$. The equations of network operator assume the form:

$$\begin{aligned} \mathbf{Zi} - \mathbf{Z}_0\mathbf{i}_0 &= \mathbf{0} \\ -\mathbf{Z}_0^{\mathrm{T}}\mathbf{i} + \mathbf{Z}_{00}\mathbf{i}_0 &= \mathbf{u}_0 \end{aligned} \tag{1}$$

(**0** – zero vector (or zero operator), T - a sign of transposition)

All impedance matrices have a distribution of: Hermitian **R** and skew-Hermitian **X** parts:

$$\mathbf{Z}(s) = \mathbf{R}(s) + \mathbf{X}(s) \tag{2}$$

i.e. such that

$$\mathbf{R}(-s) = \mathbf{R}(s); \qquad \mathbf{X}(-s) = -\mathbf{X}(s) \tag{3}$$

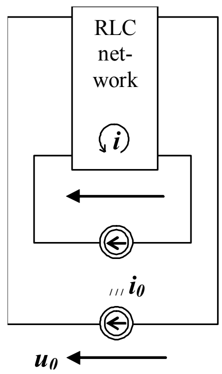


Fig. 1. The complex network with multicurrent power; $\boldsymbol{i}$ – internal mesh currents vector; $\boldsymbol{i_0}$ external current vector

In Fig. 2 the structure of system of equations (1) is illustrated. In this figure the sizes of the matrix and vectors are shown.

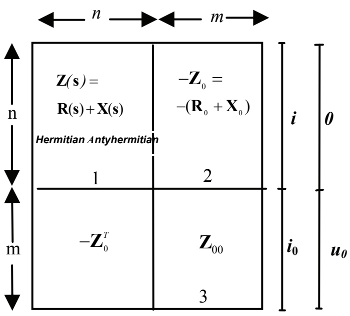


Fig. 2. Scheme the system of equations (1); 1 - internal operators matrix, 2- contact operators matrix, 3 - external operators matrix, **0** - vector (or operator) zero

The signals $i(t)$ - the coordinates of the current vector, belong to the $\mathbf{L}^1$ signal space, so-called the $\mathbf{L}^1$-impulses space:

$$\mathbf{L}^1 = \{x(t): \int_{-\infty}^{\infty} |x(t)| dt < \infty\} \tag{4}$$

or to generated by it the T-periodic signals space $\mathbf{P_T}$ [10]:

$$\mathbf{P}_{\mathrm{T}} = \{\tilde{x}(t): \quad \tilde{x}(t) = \sum_{p=-\infty}^{\infty} x(t+pT); \; x(t) \in \mathbf{L}^1\} \tag{5}$$

In these spaces the inner product is defined, in $\mathbf{L}^1$:

$$(u,i) = \int_{-\infty}^{\infty} u(t)i(t)dt \tag{6}$$

and in $\mathbf{P}_T$

$$(u,i)=\int_o^T u(t)i(t)dt \quad (7)$$

All operators are convolutional type, i.e. in a time representation they have the form:

$$Zi(t)=\int_{-\infty}^{+\infty} z(t-t')i(t')dt' \quad \text{in} \quad \mathbf{L}^1 \quad (8)$$

or

$$Zi(t)=\int_O^T z(t\Theta t')i(t)dt' \quad \text{in} \quad \mathbf{P}_T \quad (9)$$

Θ - operation of subtraction modulo T.

In the notation using the Fourier transform:

$$ZI(s)=Z(s)I(s) \quad (10)$$

The impedance operator Z is decomposed into two components R and X [11]:

$$Z=\frac{1}{2}(Z+Z^*)+\frac{1}{2}(Z-Z^*)=R+X \quad (11)$$

i.e., operator

$$R=\frac{1}{2}(Z+Z^*) \quad (12)$$

which is the Hermitian operator (self-adjoined), ie: $R^*=R$, and the operator

$$X=\frac{1}{2}(Z-Z^*) \quad (13)$$

which is skew-Hermitian, i.e.: $X^*=-X$.

Operator Z^* is an adjoined operator relative to Z, i.e. such that for any signals x,y occurs $(Zx,y)=(x,Z^*y)$.

The R operator represents the active component of the impedance operator Z and the operator X is the passive component. This means that the following conditions for quadratic forms are fulfilled for any signal i:

$$(Zi,i)=(Ri,i); \qquad (Xi,i)=0 \quad (14)$$

It can be shown that the functions $Z(t)$ and $Z(s)$ determined by the convolution operators meet the conditions [12]:

$$Z^*(t)=Z(-t); \qquad Z^*(s)=Z(-s) \quad (15)$$

2. The principle of minimum energy losses in the electrical network at $\mathbf{L}^1$ and $\mathbf{P}_T$ spaces. Optimal control

Current functional

$$f(\mathbf{i})=[\mathbf{i}^T,\mathbf{i}_0^T]\begin{bmatrix}\mathbf{R} & -\mathbf{R}_0\\ -\mathbf{R}^T & \mathbf{R}_{00}\end{bmatrix}\begin{bmatrix}\mathbf{i}\\ \mathbf{i}_0\end{bmatrix} \quad (16)$$

has a value that is equal to the energy losses in the electrical network. Equation (16) creates an operator-matrix inner product, i.e. if

$$\mathbf{A}=[A_{pq}];\ \mathbf{X}=[x_q],\ \mathbf{Y}=[y_q] \quad (17)$$

the bilinear form takes a form

$$\begin{aligned}&\mathbf{x}^T\mathbf{A}\mathbf{y}=\sum_{p,q}\left(x_p,A_{pq}y_q\right)=\\&\sum_{p,q}\sum\int x_p(t)\left[\int A_{pq}(t-t')y_q(t')dt'\right]dt=\\&\sum_{p,q}\sum\iint A_{pq}(t-t')x_p(t)y_q(t')dtdt'\end{aligned} \quad (18)$$

Integrals in the formula (18) are taken in the interval (-∞,+∞) or in [0,T] depending on which space $\mathbf{L}^1$ or $\mathbf{P}_T$ is used. And depending on this, the operators are linear or cyclic convolutions.

The principle of minimum requires the functional (16), which value is equal to the total energy losses in the network, to reach a minimum. Thus, the condition of minimum energy functional (16), i.e.

$$\bigwedge_{d\mathbf{i}} df(\mathbf{i})>0 \quad (19)$$

takes the form [2]:

$$\mathbf{R}\mathbf{i}-\mathbf{R}_0\mathbf{i}_0=\mathbf{0} \quad (20)$$

or the system of operator equations form

$$\mathbf{R}\mathbf{i}=\mathbf{R}_0\mathbf{i}_0 \quad (21)$$

The system of equations (21) must be reconciled with the system of equations (1):

$$\mathbf{Z}\mathbf{i}=\mathbf{Z}_0\mathbf{i}_0 \quad (22)$$

the solution of which is the real distribution of mesh currents inside the network. The systems of equations (21) and (22) have the same structure, but the system (21) is Hermitian type (part of) the system of equations (22).

The solution of the system of equations (21) is energy optimal distribution of mesh currents minimizing energy losses within the network:

$$\mathbf{i}^{opt}=\mathbf{R}^{-1}\mathbf{R}_0\mathbf{i}_0 \quad (23)$$

called the optimal distribution. Whereas the solution system of equations (22):

$$\mathbf{i}=\mathbf{Z}^{-1}\mathbf{Z}_0\mathbf{i}_0 \quad (24)$$

gives the actual distribution of mesh currents in the network called "current divider" distribution.

For DC currents the optimal distribution matches the distribution of the current divider, but it is not only in this case. However, these distributions do not match in general.

This means that the optimal distribution is achieved by using a current-voltage control system:

$$\begin{aligned}&\mathbf{e}^{st}=\mathbf{Z}\mathbf{i}^{opt}-\mathbf{Z}_0\mathbf{i}_0=\\&\mathbf{R}\mathbf{i}^{opt}-\mathbf{R}_0\mathbf{i}_0+\mathbf{X}\mathbf{i}^{opt}-\mathbf{X}_0\mathbf{i}_0=\\&(\mathbf{X}\mathbf{R}^{-1}\mathbf{R}_0-\mathbf{X}_0)\mathbf{i}_0\end{aligned} \quad (25)$$

Equation (25) gives the voltage signal sources which must be plugged into internal network meshes to induce energy-optimal current distribution. That formula is written in the form

$$\mathbf{e}^{st}=\mathbf{X}^{st}\mathbf{i}_0 \quad (26)$$

where:

$$\mathbf{X}^{st}=\mathbf{X}\mathbf{R}^{-1}\mathbf{R}_0-\mathbf{X}_0 \quad (27)$$

is skew-Hermitian, matrix control operator.

3. Suboptimal control

According to the formulas (25), (26), (27), for energy-optimal controlling to obtain $\mathbf{X}^{st}(s)$ operator, the inversion of $\mathbf{R}(s)$ operator is required. Apart from the fact that it is the matrix operator, it is also the dispersive operator, i.e. dependent on $s = \frac{\mathrm{d}}{\mathrm{d}t}$ (depends on frequency). Dependence on s is most often a rational function. Inversion of such operators is inconvenient because it is algorithmically complicated. To avoid this, the operator $\mathbf{R}(s)$ is replaced by the $\mathbf{R}'$ operator which is a matrix, but nondispersive one (does not depend on s). Such control is called the *suboptimal control*. It is determined by the formula:

$$\mathbf{i}^{sub} = (\mathbf{R}')^{-1}\mathbf{R}'_0\mathbf{i}_0 = \mathbf{\Gamma}\mathbf{i}_0 \tag{28}$$

where

$$\mathbf{\Gamma} = (\mathbf{R}')^{-1}\mathbf{R}_0\,, \tag{29}$$

is so-called suboptimal current distribution operator, and

$$\begin{aligned} \mathbf{e}^{sub} &= \mathbf{Z}\mathbf{i}^{sub} - \mathbf{Z}_0\mathbf{i}_0 = \\ &= (\mathbf{R}(\mathbf{R}')^{-1}\mathbf{R}'_0 - \mathbf{R}_0)\mathbf{i}_0 + \\ &(\mathbf{X}(\mathbf{R}')^{-1}\mathbf{R}'_0 - \mathbf{X}_0)\mathbf{i}_0 = (\Delta\mathbf{R} + \mathbf{X}^{sub})\mathbf{i}_0 \end{aligned} \tag{30}$$

where: $\Delta\mathbf{R} = \mathbf{R}(\mathbf{R}')^{-1}\mathbf{R}'_0 - \mathbf{R}_0$ - *differential operator of mismatch resistance* (matrix - dispersive); $\mathbf{X}^{sub} = \mathbf{X}(\mathbf{R}')^{-1}\mathbf{R}'_0 - \mathbf{X}_0$ - *suboptimal control operator*.

The resulting formula (30) is a generalization of the formula of optimal control (25) to the suboptimal control, because for $\mathbf{R}' = \mathbf{R}$ and $\mathbf{R}'_0 = \mathbf{R}_0$ the operator $\Delta\mathbf{R}$ disappears. Therefore it is important to make appropriate selection of the $\mathbf{R}'$ operator and hence the suboptimal control. One of the many possibilities of the matrix $\mathbf{R}'$ selection is the method of a power series. Decomposing the impedance matrix $\mathbf{Z}(s)$ in the Taylor series, it is obtained: („*RL*” decomposition)

$$\begin{aligned} &\mathbf{Z}(s) = \mathbf{R}(s) + \mathbf{X}(s) = \\ &\mathbf{r} + \sum_{n=1}^{\infty}\mathbf{r}_n s^{2n} + s(\mathbf{L} + \sum_{n=1}^{\infty}\mathbf{L}_n s^{2n}) \end{aligned} \tag{31}$$

It can be assumed that:

$$\mathbf{R}' = \mathbf{r} = \mathbf{Z}(0)$$

The suboptimal control operator then has the form:

$$\begin{aligned} &\mathbf{X}^{sub}(s) = \mathbf{X}(s)\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{X}^0(s) = \\ &= s[(\mathbf{L}\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{L}^0) + \sum_{n=1}^{\infty}s^{2n}(\mathbf{L}_n\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{L}_n^0)] \end{aligned} \tag{32}$$

It can be also used the „*RC*” decomposition:

$$\begin{aligned} &\mathbf{Z}(s) = \mathbf{R}(s) + \mathbf{X}(s) = \\ &\mathbf{r} + \sum_{n=1}^{\infty}\mathbf{r}_n s^{-2n} + s^{-1}(\mathbf{\Sigma} + \sum_{n=1}^{\infty}\mathbf{\Sigma}_n s^{-2n}) \end{aligned} \tag{33}$$

and then assuming $\mathbf{R}' = \mathbf{Z}(\infty)$ it is obtained:

$$\begin{aligned} &\mathbf{X}^{sub}(s) = s^{-1}[(\mathbf{\Sigma}\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{\Sigma}^0) \\ &+ \sum_{n=1}^{\infty}s^{-2n}(\mathbf{\Sigma}_n\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{\Sigma}_n^0)] \end{aligned} \tag{34}$$

where: $\mathbf{r}, \mathbf{r}^0, \mathbf{r}_n$ - resistance matrices; $\mathbf{L}, \mathbf{L}_n, \mathbf{L}^0, \mathbf{L}_n^0$ - inductance matrices; $\mathbf{\Sigma}, \mathbf{\Sigma}_n, \mathbf{\Sigma}^0, \mathbf{\Sigma}_n^0$ - elastance matrices (the inverse of the capacity).

Differential operators of mismatch resistance then have the form:

$$\begin{aligned} &\Delta\mathbf{R}(s) = \mathbf{R}(s)\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{R}_0(s) = \\ &\left(\mathbf{r} + \sum_{n=1}^{\infty}\mathbf{r}_n s^{2n}\right)\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{r}^0 - \sum_{n=1}^{\infty}\mathbf{r}_n^0 s^{2n} = \\ &\sum_{n=1}^{\infty}s^{2n}(\mathbf{r}_n\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{r}_n^0) \end{aligned} \tag{35}$$

for „*RL*” decomposition, and

$$\Delta\mathbf{R}(s) = \sum_{n=1}^{\infty}s^{-2n}(\mathbf{r}_n\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{r}_n^0) \tag{36}$$

for „*RC*” decomposition.

Presented above operators are easily achievable using the multiple differentiators or the integrator systems. In addition, comfortable algorithms of decomposition of rational functions in power series of variables s and s^{-1} can be formulated.

Example

For the distribution of current signal 01 into two parallel branches 1, 2 (Fig. 3), determine the *current-voltage suboptimal control operator* $X^{sub}i_{01}$.

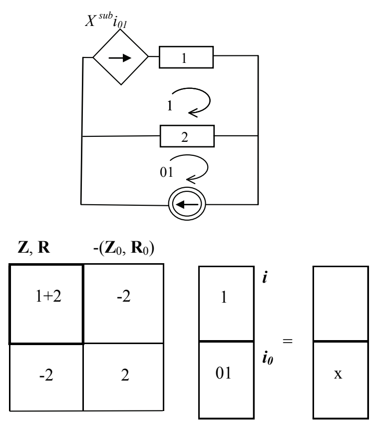
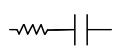


Fig. 3. Energy-suboptimal distribution of the current signal 01 into two parallel branches 1, 2; below there is the structure of the equations (8) and (7)

It is obtained for branches 1, 2 having the structure *RL* type,

$$\begin{aligned} &\mathbf{X}^{sub}(s) = s(\mathbf{L}\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{L}^0) = \\ &s(\frac{L_1 + L_2}{r_1 + r_2}r_2 - L_2) = s\frac{L_1 r_2 - L_2 r_1}{r_1 + r_2} \end{aligned} \tag{37}$$

or for branches 1, 2 *RC* type:



$$\begin{aligned} &\mathbf{X}^{sub}(s) = s^{-1}(\mathbf{\Sigma}\mathbf{r}^{-1}\mathbf{r}^0 - \mathbf{\Sigma}^0) = \\ &s^{-1}(\frac{\Sigma_1 + \Sigma_2}{r_1 + r_2}r_2 - \Sigma_2) = s^{-1}\frac{\Sigma_1 r_2 - \Sigma_2 r_1}{r_1 + r_2} = \\ &s^{-1}\frac{r_2 C_2 - r_1 C_1}{C_1 C_2(r_1 + r_2)} \end{aligned} \tag{38}$$

These are appropriately: the differential operators (for *RL* type of branches) and the integral operators (for *RC* type of branches). For these simple circuits, the obtained results are the same as in the case of optimal control operators.

4. Summary

The study [2] showed that in the complex *RLC* network, besides the currents flows arising from the normal laws of Kirchhoff - called *current divider*, through appropriate controls there may also be received other distributions of currents, resulting from certain optimization criteria. The distribution that satisfies the condition of minimum energy losses within the network, calling it the *energy-optimal distribution* was examined there.

Optimal distribution itself is not reachable in the way the distribution of current divider, but in order to make it, the necessary is *optimal control* carried out by the control operator $\mathbf{X}^{st}(s)$ (27), generating appropriately distributed signal of the voltage source $\mathbf{e}^{st}$ (26):

$$\mathbf{e}^{st} = \mathbf{X}^{st}(s)\mathbf{i}_0$$
$$\mathbf{X}^{st}(s) = \mathbf{X}(s)[\mathbf{R}(s)]^{-1}\mathbf{R}_0(s) - \mathbf{X}_0(s)$$

The optimal control operator $\mathbf{X}^{st}(s)$ is skew-Hermitian, i.e. $\mathbf{X}^{st}(-s) = -\mathbf{X}^{st}(s)$, which makes that skew-Hermitian is also operator:

$$[\mathbf{R}_0(s)]^T[\mathbf{R}(s)]^{-1}\mathbf{X}^{st}(s)$$

and thus disappears quadratic form

$$(\mathbf{i}^{opt})^T\mathbf{e}^{st} = \mathbf{i}_0^T\mathbf{R}_0^T\mathbf{R}^{-1}\mathbf{X}^{st}\mathbf{i}_0$$

Thus, the controlled sources $\mathbf{e}^{st}$ do not produce energy – the optimal control is *energy-neutral.*

To obtain $\mathbf{X}^{st}(s)$ operator, the inversion of $\mathbf{R}(s)$ operator is required. It is the matrix and the dispersive (frequency dependent) operator. Inversion of such operator is inconvenient because it is algorithmically complicated. To avoid this, the operator $\mathbf{R}(s)$ is replaced by the $\mathbf{R}'$ operator which is a matrix, but nondispersive one (does not depend on *s*). Such control is called the *suboptimal control*.

The suboptimal control is not *energy-neutral.* This is due to *differential operator of mismatch resistance*:

$$\Delta\mathbf{R}(s) = \mathbf{R}(s)(\mathbf{R}')^{-1}\mathbf{R}_0' - \mathbf{R}_0(s)$$

Therefore, it is important to make appropriate selection of the $\mathbf{R}'$ operator and hence the suboptimal control. One of the many possibilities of the matrix $\mathbf{R}'$ selection is the method of a power series, which in the case of „*RL*" or „*RC*" decomposition gives the possibility of using the multiple differentiator or the integrator systems.

5. References

[1] Siwczyński M., Drwal A., Żaba S.: Minimally - energy distribution of electrical current signals in the linear L1 - impulses space. Przegląd Elektrotechniczny, R90 (2014), nr 12, 47-51 (in Polish).

[2] Siwczyński M., Drwal A., Żaba S.: Energy-optimal current distribution in a complex linear electrical network with pulse or periodic voltage and current signals. Optimal control. Bull. of the Polish Acad. of Science, Techn. Scie. (2015) (accepted for print).

[3] Desoer C. A..: The maximum power transfer theorem for n-ports. IEEE Trans., Vol. CT-20 (1979), 228-230.

[4] Rohrer R. A.: Optimal matching: A new approach to the matching problem for real invariant one port networks. IEEE Trans., Vol. CT-15 (1968), 118-124.

[5] Siwczyński M., Drwal A., Żaba S.: Minimum-energetic sinusoidal signals distribution in electrical circuits. Wiadomości Elektrotechniczne, 9 (2014), pp. 22-25 (in Polish).

[6] Czarnecki L.S.: Discussion on "a uniform concept of reactive power of nonsinusoidal currents in a time-domain". Przegląd Elektrotechniczny, R85 (2009), nr 6, 164-166 (in Polish).

[7] Rens A. P.: Validation of popular nonsinusoidal power theories for the analysis and management of modern power systems. North-West University, Potchefstroom Campus, 2006.

[8] Walczak J., Pasko M.: The Minimization of losses of active power and the symmetrization of power flow in the nonsinusoidal systems. Electrical Power Quality and Utilisation, 5 (1999), nr. 1, 55-59 (in Polish).

[9] Czarnecki L. S.: Currents' Physical Components (CPC) concept: a fundamental for power theory. Przegląd Elektrotechniczny, R84 (2008), nr 6, 28-37.

[10] Siwczyński M., Jaraczewski M.: The L1 –impulse method as an alternative to the Fourier series in the power theory of continuous time systems. Bull. of the Polish Acad. of Science, Techn. Scie., 57 (2009), n. 1, 79-85.

[11] Siwczyński M.: The distribution: active current, scattered current, reactive current, the time domain approach - the mathematical theory. Przegląd Elektrotechniczny, R87 (2011), nr 3, 254-257 (in Polish).

[12] Siwczyński M.: Energy theory of electrical circuits. Wyd. Instytutu Gospodarki Surowcami Mineralnymi i Energią PAN, Kraków 2003 (in Polish).

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Sławomir ŻABA, PhD, eng.

Graduated (1993) from Faculty of Electrical Engineering at Cracow University of Technology. PhD in 2001 at AGH University of Science and Technology. Currently works at Faculty of Electrical and Computer Engineering (Cracow University of Technology). The research area includes many aspects focused on the processing and transmission of signals and theory of circuits and signals.

e-mail: szaba@pk.edu.pl

Prof. Maciej SIWCZYŃSKI, DSc, eng.

MSc. (1969), PhD (1974) and DSc (1983) from Electrical Engineering of Silesian University of Technology. Since 1998 he was full professor at Cracow University of Technology. Author and co-author about 200 scientific papers (8 monographs) mainly related to: the theory of circuits and signals in relation to the optimal control, the application of functional and harmonic analysis, the power theory of electrical circuits using the operator methods in the time domain.

e-mail: e-3@pk.edu.pl

Andrzej DRWAL, PhD, eng.

In 1993 graduated from Faculty of Electrical Engineering (Cracow University of Technology). PhD in 2002 at AGH University of Science and Technology. Currently works in Institute of Electrical Engineering and Computer Science at Cracow University of Technology. His scientific interests mainly center around theory of circuits and signals and the transmission and processing of signals.

e-mail: adrwal@pk.edu.pl