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COMPUTER MODELLING OF THE DYNAMIC SYSTEM «POROUS MEDIUM – MOISTURE – CHEMICAL SUBSTANCE» IN THE CASE OF SOIL DESALINIZATION BY RAINFALLS (A THREE-DIMENSIONAL CASE)

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Summary. Mathematical model of moisture transport taking into account variable porosity has been investigated numerically. Changes in porosity are caused by dissolution of chemical substances associated with soil skeleton. The finite element solution of the problem in the case of regular rainfall has been found. Program realization of the corresponding algorithms has been implemented in FreeFem++ computational environment. Numerical experiments have been carried out and the impact of rainfall on desalinization of soil with high concentration of salts in the solid component has been determined.

Keywords: mathematical model, porous medium, moisture transport, solute transport, finite element method

1. INTRODUCTION

Computer and mathematical modelling of soil moisture transport processes is an essential part of various applied problems. For example, it is used in the construction of hydraulic structures, evaluating strength of potential slide soils, crop growth etc.

The difficulty of soil water flow modelling lies in the fact it is influenced by a considerable number of another factors and processes. Diffusion and transport of solutes, solute transfer between the phases of the soil, heat transport, net radiation, precipitation dynamics, plant growth, root water uptake, soil erosion by surface and underground waters are some of them [9]. They cause changes in water content rate and soil properties, which, in their own turn, influence mentioned processes. For instance, porosity depends greatly on soil salt content and solute transport dynamics; hydraulic conductivity is influenced by temperature, chemical content and soil moisture [6]. These dependencies are complex and often ambiguous, e.g. hysteresis [13].

Mathematical modelling of interconnected processes in heterogeneous porous media is rapidly developing nowadays. For example, mathematical models of mentioned processes for the purpose of crop yield forecasting have been built in [13]. In [7, 10] the models describing transport of water, vapour and air in the soil under non-isothermal conditions have been proposed. The processes of soil heat and water transport during the wildfire have been described in [8]. Modelling of solute transport during the wetting of saturated-unsaturated soil has been done in [16]. Also, the impact of anthropogenic factors on the water flow is being studied. For instance, in [5] the filtration processes in earth dams with regard to solute transport and suffusion have been modelled. The way the acid rains can cause landslides has been discussed in [17].

The purpose of this paper is a computer modelling of moisture transport in the soil with variable porosity. Porosity variation affects water content of porous medium, and so impacts moisture indirectly. The model built in this paper is based on the concepts presented in the papers [3, 4] and the model proposed in [7], which is also taking into account non-isothermal conditions and movement of chemical solutes in the soil. The model is specified for the purpose of numerical experiments with a three-dimensional problem of soil desalinization by rainfalls.

Numerical solving of nonlinear boundary value problem for a system of partial differential equations in a three-dimensional case requires complex computational algorithms and computer resources. Their program realization is often simplified by using premade software packages. The problem described in this paper is solved with the use of FreeFem++ integrated development environment. It implements numerical solving of boundary value problem with the finite element method (FEM) [2].

2. MATHEMATICAL MODEL

2.1. EQUATIONS GOVERNING MOISURE, HEAT AND SOLUTE FLOW

Assume we have a three-dimensional soil layer $\left(\Omega \supset \mathbf{X} \in \left(x_1, x_2, x_3\right)\right)$ with variable porosity, consisting of the solid, liquid and gaseous components. In addition, the soil is salinized and is under non-isothermal conditions.

Mathematical model for the given problem is built based on the model proposed in [7]. We neglect the flow of vapour and air for the purpose of this problem. Referred to equation for the water flow is:

$$\theta\left(\frac{\partial \rho_l}{\partial c}\frac{\partial c}{\partial t}+\frac{\partial \rho_l}{\partial T}\frac{\partial T}{\partial t}\right)+\rho_l\left(\left(\sigma-\theta_{\min}\right)\frac{\partial s}{\partial t}+s\frac{d\sigma}{dt}\right)+\nabla\left(\rho_l \mathbf{q_l}\right)=0, \tag{1}$$

where:

- θ – volumetric water content;
- c – solute concentration in pore water;
- T – temperature;
- ρ_l – pore water density;
- σ – porosity;
- $\theta_{\min}$ – residual (minimum) water content;
- $\mathbf{q_l}$ – soil water flux;
- t – time.

To determine the pore water flux we use a modified Darcy-Klute's law with consideration of osmosis, written as follows:

$$\mathbf{q_l} = -\mathbf{D}\nabla\theta - \mathbf{K_l}\nabla x_3 + \mathbf{K_c}\nabla c + \mathbf{K_T}\nabla T,$$

where:

$\mathbf{D}$ – soil moisture diffusion coefficient;

$\mathbf{K_l}$ – unsaturated hydraulic conductivity;

$\mathbf{K_c}$ – coefficient of chemical osmosis;

$\mathbf{K_T}$ – coefficient of thermal osmosis.

Providing we assume porosity $\sigma(N,T)$ to be dependent on the concentration of salts in the solid component N and temperature T, we can rewrite total differential of porosity as:

$$\frac{d\sigma}{dt} = \frac{\partial\sigma}{\partial t} + \frac{\partial\sigma}{\partial N}\frac{\partial N}{\partial t} + \frac{\partial\sigma}{\partial T}\frac{\partial T}{\partial t}.$$

Besides, to take into account variable porosity, it is suggested to modify equation (1) by changing volumetric water content θ to the degree of saturation s using:

$$\theta = (\sigma - \theta_{\min})s + \theta_{\min},$$

$$\frac{d\theta}{dt} = (\sigma - \theta_{\min})\frac{\partial s}{\partial t} + s\frac{d\sigma}{dt}, \quad \nabla\theta = (\sigma - \theta_{\min})\nabla s + s\nabla\sigma.$$

Then (1) becomes:

$$\begin{aligned} &\left(\frac{\partial\rho_l}{\partial c}\frac{\partial c}{\partial t} + \frac{\partial\rho_l}{\partial T}\frac{\partial T}{\partial t}\right)\left(s(\sigma - \theta_{\min}) + \theta_{\min}\right) + \rho_l(\sigma - \theta_{\min})\frac{\partial s}{\partial t} + \\ &+\rho_l s\left(\frac{\partial\sigma}{\partial t} + \frac{\partial\sigma}{\partial N}\frac{\partial N}{\partial t} + \frac{\partial\sigma}{\partial T}\frac{\partial T}{\partial t}\right) = \nabla\cdot\left(\rho_l\mathbf{D}\left((\sigma - \theta_{\min})\nabla s + s\nabla\sigma\right)\right) + \\ &+\nabla\cdot\left(\rho_l\left(\mathbf{K_l}\nabla x_3 - \mathbf{K_c}\nabla c - \mathbf{K_T}\nabla T\right)\right), \mathbf{X}\in\Omega, t>0. \end{aligned} \tag{2}$$

Solute transport equation in [7] is written as:

$$\begin{aligned} &\theta\left(1 - \frac{c}{\rho_l}\frac{\partial\rho_l}{\partial c}\right)\frac{\partial c}{\partial t} = \nabla\cdot(\theta\mathbf{D_c}\nabla c) - \mathbf{q_l}\left(1 - \frac{c}{\rho_l}\frac{\partial\rho_l}{\partial c}\right)\nabla c + \\ &+\frac{c}{\rho_l}\frac{\partial\rho_l}{\partial T}\left(\theta\frac{\partial T}{\partial t} + \mathbf{q_l}\nabla T\right) - \frac{\partial N}{\partial t}, \mathbf{X}\in\Omega, t>0, \end{aligned} \tag{3}$$

where:

$\mathbf{D_c}$ – dispersion coefficient.

As an equation of solute transport kinetics between the phases of the soil we choose a simple formulation, describing adsorption and desorption of dissolved salts under Henry isotherm [14, p. 175]:

$$\frac{\partial N}{\partial t} = \gamma_1 \left(c\sigma - \alpha N \right), \mathbf{X} \in \overline{\Omega}, \mathrm{t} > 0, \tag{4}$$

where:

γ_1 – adsorption and desorption velocity constant;

$\alpha = 1/\Gamma$ – distribution ratio;

Γ – Henry coefficient.

Heat transfer equation without considering gaseous component is represented as follows:

$$c_T \frac{\partial T}{\partial t} + T \frac{d\left(\rho_s c_s (1-\sigma)\right)}{dt} = \nabla \cdot \left(\lambda \nabla T\right) - \rho_l c_l \mathbf{q_l} \nabla T, \tag{5}$$

where:

c_T – volumetric heat capacity of porous medium;

ρ_s – density of solid soil particles;

λ – thermal conductivity;

c_s, c_l – heat capacities of solid and liquid soil phases, respectively.

Let us assume that solid soil phase consist of indissoluble solid particles constituting soil skeleton and soluble salts. Then:

$$\frac{d\left(\rho_s c_s (1-\sigma)\right)}{dt} = \frac{d}{dt}\left(c_N N + \rho_{s0} c_{s0} \left(1-\sigma_0\right)\right) =$$
$$= c_N \frac{\partial N}{\partial t} - \rho_{s0} c_{s0} \frac{\partial \sigma_0}{\partial t} + c_{s0} \left(1-\sigma_0\right) \frac{\partial \rho_{s0}}{\partial T} \frac{\partial T}{\partial t},$$

and equation (5) becomes:

$$c_T \frac{\partial T}{\partial t} + T\left(c_N \frac{\partial N}{\partial t} - \rho_{s0} c_{s0} \frac{\partial \sigma_0}{\partial t} + c_{s0} \left(1-\sigma_0\right) \frac{\partial \rho_{s0}}{\partial T} \frac{\partial T}{\partial t} \right) =$$
$$= \nabla \cdot \left(\lambda \nabla T\right) - \rho_l c_l \mathbf{q_l} \nabla T, \mathbf{X} \in \Omega, \mathrm{t} > 0, \tag{6}$$

where:

σ_0 – porosity of indissoluble solid soil skeleton;

c_N – density of salt in the solid soil component;

ρ_{s0}, c_{s0} – density and heat capacity of solid soil skeleton.

2.2. VARIABLE PROCESS PARAMETERS

2.2.1. Pore water density

For the purpose of modelling variable pore water density we use the properties of NaCl solution, for it is well studied and discussed in the literature. For instance, an empirical dependence of solution density on the salt concentration and temperature has been proposed in [11] in the form of:

$$\rho_l(c,T)=\sum_{i=1}^{3}\sum_{j=1}^{3}a_{ij}c^{j-1}T^{i-1}.$$

To use this in the model, it is required to find partial derivatives of the density, used in equation (2):

$$\frac{\partial\rho_l}{\partial c}=\sum_{i=1}^{3}\sum_{j=2}^{3}(j-1)a_{ij}c^{j-2}T^{i-1},\ \frac{\partial\rho_l}{\partial T}=\sum_{i=2}^{3}\sum_{j=1}^{3}(i-1)a_{ij}c^{j-1}T^{i-2}.$$

2.2.2. Porosity

According to [7], soil porosity can be determined as follows:

$$\sigma(N,T)=\sigma_0-\frac{N}{\rho_N}.$$

We shall assume soil skeleton porosity σ_0 and salt density ρ_N to be constant. Doing so, we get the following partial derivatives of porosity with respect to N and T: $\partial\sigma/\partial N = 1/\rho_N$, $\partial\sigma/\partial T = 0$.

2.2.3. Unsaturated hydraulic conductivity and diffusion coefficients

Numerous methods may be found to determine unsaturated soil conductivity and diffusion coefficient. In this paper we use a widespread approach called BC (Brooks and Corey) model [1]:

$$\begin{gathered}K_l(c,T,s,\sigma)=K_0(c,T,\sigma)s^{\frac{2+3\lambda}{\lambda}},\\ D(c,T,s,\sigma)=D_0(c,T,\sigma)s^{2+\frac{1}{\lambda}},D_0=\frac{K_0(c,T,\sigma)\psi_b}{\lambda(\sigma-\theta_{\min})},\end{gathered}\tag{7}$$

where:

$K_0(c,T,\sigma)$ – coefficient of permeability;

λ – pore-size distribution index;

ψ_b – pore air pressure.

2.2.4. Coefficient of permeability

Permeability is included as an input parameter in equations (7) for unsaturated soil conductivity and diffusion coefficient. It is affected by a great number of physical and chemical factors, including temperature and chemical composition of the pore water [3–5, 16]. It also depends greatly on the void ratio e, which is related to porosity σ as:

$$e = \frac{\sigma}{1-\sigma}.$$

One of the simplest empirical relations between permeability void ratio is Kozeny--Carman equation:

$$K_0 = k_0 \frac{1-e_0}{1-e}\left(\frac{e}{e_0}\right)^3,$$

where:

k_0, e_0 – initial values of permeability and void ratio.

The remaining coefficients (osmosis coefficients, dispersion coefficient, heat capacity etc.) are considered constant values.

2.3. BOUNDARY CONDITIONS

Assume we have a three-dimensional soil layer as showed on Fig. 1. Upper boundary of the layer contacts with the atmocphere. Lower boundaty is impermeable for some reason: either lies on a stone foundation or on the clayey soil of low permeability.

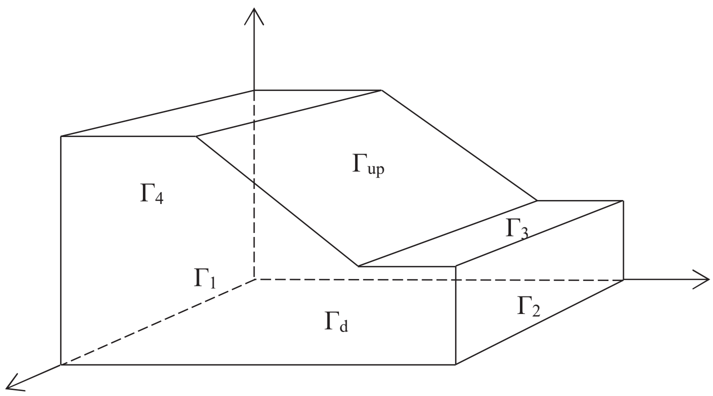


Fig. 1. Domain of the problem

Initial conditions for the given problem are as follows:

$$s(\mathbf{X},t) = S_0(\mathbf{X}), c(\mathbf{X},t) = C_0(\mathbf{X}), T(\mathbf{X},t) = T_0(\mathbf{X}), N(\mathbf{X},t) = N_0(\mathbf{X}), \mathbf{X} \in \overline{\Omega}. \quad (8)$$

On the lower boundary Γ_d we set boundary conditions that correspond to impermeability conditions (for moisture and solute flow):

$$\left.\frac{\partial s}{\partial n}\right|_{\Gamma_d} = \left.\frac{\partial c}{\partial n}\right|_{\Gamma_d} = 0, \quad \mathbf{X} \in \Gamma_d, t \ge 0. \quad (9)$$

On the side boundaries $\Gamma_s = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4$ we set symmetry conditions that are mathematically similar to the impermeability conditions and have the form of:

$$\left.\frac{\partial s}{\partial n}\right|_{\Gamma_s} = \left.\frac{\partial c}{\partial n}\right|_{\Gamma_s} = \left.\frac{\partial T}{\partial n}\right|_{\Gamma_s} = 0, \quad \mathbf{X} \in \Gamma_S, t \geq 0. \tag{10}$$

The top boundary condition on Γ_{up} ought to reflect weather conditions. We use a simple second type boundary condition that allows for water influx due to precipitation and transpiration. It can be written as:

$$\left. D\frac{\partial s}{\partial n}(\sigma - \theta_{\min})\right|_{\Gamma_{up}} = q_p(t) - q_t(t), \quad \mathbf{X} \in \Gamma_{up}, t > 0, \tag{11}$$

where:

$q_p(t)$ – moisture net influx due to precipitation for a unit of time;

$q_t(t)$ – transpiration rate for a unit of time.

Both $q_p(t)$ and $q_t(t)$ are considered non-negative here. Values $q_p(t_i) = 0$ stand for days without any rainfall. Actual transpiration rate is determined using a dependence proposed in [12]:

$$q_t(t) = q_{t0}\left(\sqrt{D_{slr}} - \sqrt{D_{slr} - 1}\right),$$

where:

D_{slr} – number of days since last rain;

q_{t0} – effective transpiration rate.

For the heat transport equation on the top boundary Γ_{up} Danckwerts boundary condition is set. It describes an inflow of clear surface water into the soil, resulting in its desalinization [15, p. 52]:

$$\left.\left[\mathbf{D_c}\frac{\partial c}{\partial n} - (\mathbf{q_l}, n)(c - C_1)\right]\right|_{\Gamma_{up}} = 0, \quad \mathbf{X} \in \Gamma_{up}, t > 0, \tag{12}$$

where:

C_1 – concentration of the salts in rainwater.

For the heat transport equation on both upper and lower boundaries we have the following first type boundary conditions:

$$\left.\mathrm{T}(\mathbf{X}, \mathrm{t})\right|_{\Gamma_d} = T_1, \left.\mathrm{T}(\mathbf{X}, \mathrm{t})\right|_{\Gamma_{up}} = T_{atm}, \quad \mathbf{X} \in \Gamma_{up}, t > 0, \tag{13}$$

where:

T_1 – temperature of the underlying soil layer;

T_{atm} – mean daily atmospheric temperature (in general is a function of time and can describe daily temperature variation as well).

Therefore, mathematical model of the problem includes equations (2)–(4), (6), initial conditions (8) and boundary conditions (9)–(13).

Besides, in this paper two principal cases of the problem are distinguished and their solutions compared:

a) classical moisture transport problem with only the Richard's water flow equation and variable soil conductivity and diffusion coefficient according to (7) taken into consideration;
b) moisture transport problem with regard to heat and solute transport and all parameters dependencies mentioned above.

3. NUMERICAL SOLUTION

3.1. WEAK FORMULATION

Let us multiply equation (2) by any function $v \in H_0$, where $H_0 = \left\{ v(x) : v(x) \in W_2^1, v(x)\big|_{\Gamma_1} = 0 \right\}$ and integrate it over Ω. We get:

$$\iint_{\Omega} \left(\frac{\partial \rho_l}{\partial c} \frac{\partial c}{\partial t} + \frac{\partial \rho_l}{\partial T} \frac{\partial T}{\partial t} \right) \left(s(\sigma - \theta_{\min}) + \theta_{\min} \right) v d\Omega +$$
$$+ \iint_{\Omega} \left(\rho_l (\sigma - \theta_{\min}) \frac{\partial s}{\partial t} + \rho_l s \left(\frac{\partial \sigma}{\partial t} + \frac{\partial \sigma}{\partial N} \frac{\partial N}{\partial t} \right) \right) v d\Omega =$$
$$= \iint_{\Omega} \left(\nabla \cdot \left(\rho_l D \left((\sigma - \theta_{\min}) \nabla s + s \nabla \sigma \right) \right) + \nabla \cdot \left(\rho_l \left(K_l \nabla x_3 - K_c \nabla c - K_T \nabla T \right) \right) \right) v d\Omega.$$

Then let us apply Ostrogradsky's theorem to the terms containing divergence operator and use corresponding boundary conditions. Time discretization is done in the form of implicit difference scheme. Doing so we get a weak formulation of equation (1):

$$\iint_{\Omega} \left(\frac{\partial \rho_l}{\partial c} \frac{c^{k+1} - c^k}{\Delta t} + \frac{\partial \rho_l}{\partial T} \frac{T^{k+1} - T^k}{\Delta t} \right) \left(s^{k+1} (\sigma - \theta_{\min}) + \theta_{\min} \right) v \, d\Omega +$$
$$+ \iint_{\Omega} \rho_l s^{k+1} \left(\frac{\sigma^{k+1} - \sigma^k}{\Delta t} + \frac{1}{\rho_N} \frac{N^{k+1} - N^k}{\Delta t} \right) v \, d\Omega +$$
$$+ \iint_{\Omega} \left(\rho_l (\sigma - \theta_{\min}) \frac{s^{k+1} - s^k}{\Delta t} v + \rho_l \left(K_l \nabla x_3 - K_c \nabla c^{k+1} - K_T \nabla T^{k+1} \right) \cdot \nabla v \right) d\Omega -$$
$$- \int_{\Gamma_{up}} \rho v \left(q_p - q_t + K_l \frac{\partial x_3}{\partial n} - \frac{K_c}{D} (c - C_1)(\mathbf{q_l}, n) \right) d\Gamma_{up} +$$
$$+ \iint_{\Omega} \left(\rho_l D \left((\sigma - \theta_{\min}) \nabla s^{k+1} + s^{k+1} \nabla \sigma \right) \right) \cdot \nabla v \, d\Omega = 0, k = 0,1,2...$$

Performing same procedures over equations (3) and (6) gives:

$$\iint_{\Omega} \theta^{k+1}\left(1-\frac{c^k}{\rho_l}\frac{\partial \rho_l}{\partial c}\right)\frac{c^{k+1}-c^k}{\Delta t} v\, d\Omega + \iint_{\Omega}\left(\theta^{k+1} D_c \nabla c^{k+1}\right)\cdot \nabla v\, d\Omega +$$

$$+\iint_{\Omega}\left(1-\frac{c^k}{\rho_l}\frac{\partial \rho_l}{\partial c}\right)\nabla c^{k+1}\cdot \mathbf{q_l} v\, d\Omega - \int_{\Gamma_{up}} \theta\left(c-C_1\right)\left(\mathbf{q_l}, n\right) v d\Gamma_{up} -$$

$$-\iint_{\Omega}\frac{c}{\rho_l}\frac{\partial \rho_l}{\partial T}\left(\theta\frac{T^{k+1}-T^k}{\Delta t}+\mathbf{q_l}\nabla T\right) v\, d\Omega + \iint_{\Omega}\frac{N^{k+1}-N^k}{\Delta t} v\, d\Omega = 0,$$

$$\iint_{\Omega}\left(c_T\frac{T^{k+1}-T^k}{\Delta t} v + T^{k+1}\left(c_N\frac{N^{k+1}-N^k}{\Delta t} - \rho_{s0} c_{s0}\frac{\sigma^{k+1}-\sigma^k}{\Delta t}\right) v\right) d\Omega +$$

$$+\iint_{\Omega}\lambda \nabla T^{k+1}\nabla v\, d\Omega + \iint_{\Omega}\rho_l c_l \mathbf{q_l}\cdot \nabla T^{k+1} v\, d\Omega,\ k=0,1,2...$$

Initial conditions in weak formulation are written in the form of:

$$\iiint_{\Omega} s(\mathbf{X},t) v(\mathbf{X})\, d\Omega = \iiint_{\Omega} S_0(\mathbf{X}) v(\mathbf{X})\, d\Omega,\ \iiint_{\Omega} c(\mathbf{X},t) v(\mathbf{X})\, d\Omega = \iiint_{\Omega} C_0(\mathbf{X}) v(\mathbf{X})\, d\Omega,$$

$$\iiint_{\Omega} T(\mathbf{X},t) v(\mathbf{X})\, d\Omega = \iiint_{\Omega} T_0(\mathbf{X}) v(\mathbf{X})\, d\Omega.$$

Equation (4) ought only to be discretized with respect to time:

$$\frac{N^{k+1}-N^k}{\Delta t} = \gamma_1\left(c^{k+1}\sigma - \alpha N^{k+1}\right), k=0,1,2...$$

Its solution can be calculated iteratively in the form of:

$$N^{k+1} = \frac{N^k + \gamma_1 c^{k+1}\sigma \Delta t}{1+\gamma_1 \alpha \Delta t}, k=0,1,2...$$

3.2. INPUT DATA

For the purpose of numerical experiments following parameter values have been used:

$$K_c = 3\cdot 10^{-6}\, m^5 / kg\cdot day,\ K_T = 10^{-6}\, m^2 / K\cdot day,\ D_c = 0.01 m^2 / day,$$

$$D_T = 10^{-4}\, kg / day\cdot m\cdot K,\ c_T = 2143 kJ / m^5\cdot K,$$

$$\lambda = 108\, {kJ}\big/{day \cdot m \cdot K},\ \rho_N = 2165\, {kg}\big/{m^3},\ c_N = 0.87\, {kJ}\big/{kg \cdot K},$$

$$c_l = 4.2\, {kJ}\big/{kg \cdot K},\ c_{m0} = 1\, {kJ}\big/{kg \cdot K},\ \rho_{m0} = 2167\, {kg}\big/{m^3},$$

$$\gamma_1 = 0.1\ day^{-1}, \alpha = 0.33,\ \lambda_1 = 3,\ \psi_b = 100\ kPa,\ \theta_{\min} = 0.065, \Delta t = 10\ days.$$

In the case when some variable parameter dependencies were not taken into consideration they have been assumed to be equal to the following values:

$$\rho_l = 1000\, {kg}\big/{m^3},\ k_0 = 0.013\, {m}\big/{day},\ K_l = 10^{-4}\, {m}\big/{day},$$

$$D = 0.02\, {m^2}\big/{day}.$$

Functions defining initial and boundary conditions of the problem have been set as follows:

$$S_0 = 0.5,\ C_0 = 70\, kg/m^3,\ T_0 = (10 + 2y\ )°C,\ N_0 = 100\, kg/m^3,$$

$$\sigma_0 = 0.4,\ C_1 = 2\, kg/m^3,\ T_{atm} = 24°C,\ T_1 = 10°C.$$

3.3. RESULTS OF NUMERICAL EXPERIMENTS

Numerical solution of the described problem has been found using FreeFem++ computational environment. It is a free software for solving boundary value problems with finite element method (FEM). It contains facilities for analytical description of the problem domain and boundary conditions, automated finite element mesh generation, solving boundary value problems in weak formulation and visualization of two and three-dimensional plots. FreeFem++ has its own C-based programing language that enables user to program computational algorithms, including discretization and linearization methods [2].

The given problem is solved for a period of half a year with 10-day time step. As said above, solutions of the classical moisture transport problem s and of the problem considering heat and solute flow $s*$ are compared in this paper. The resulting moisture distributions at the last time step for both cases, as well as their relative difference are shown on Fig. 2. The difference has been calculated as follows (providing $s \neq 0$):

$$\Delta_s = \frac{|s* - s|}{s} \cdot 100\%.$$

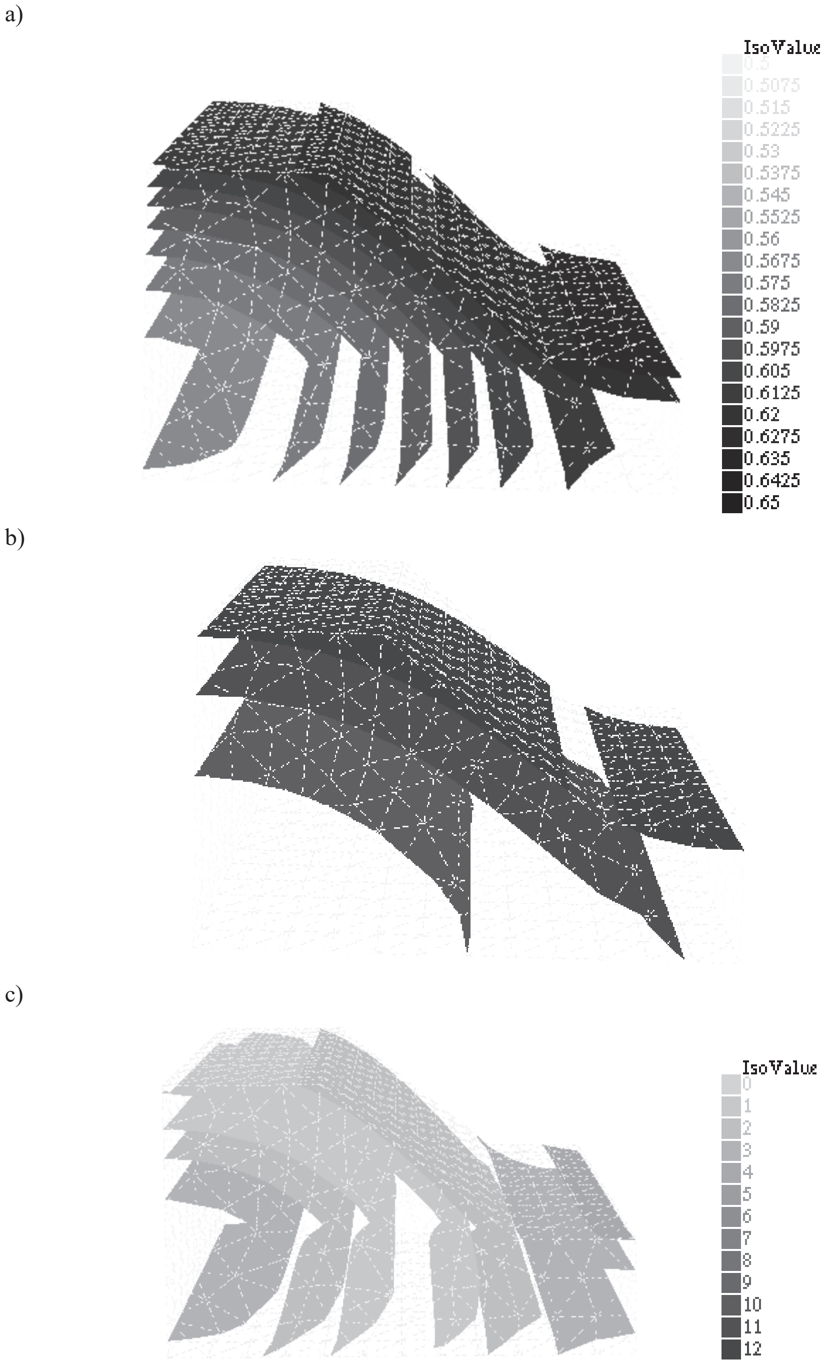
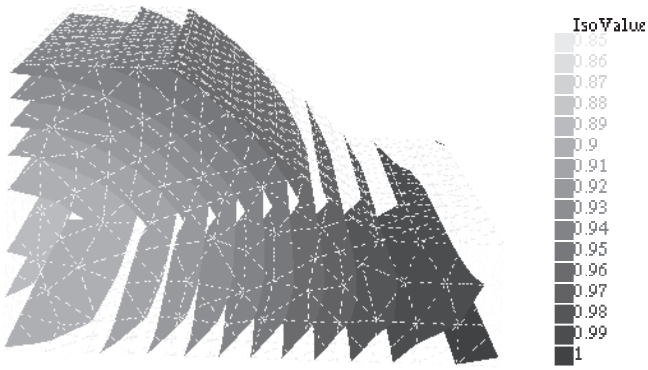


Fig. 2. Solution s of the classical moisture transport problem (a), solution $s*$ of the problem described in the paper (b) and their relative difference Δ_S , % (c)

From Fig. 2 we have that the degree of saturation ranges from 0.56 to 0.64. These values are not much different from the initial value because the chosen precipitation rate was not high. Relative difference between the found solutions is under 7%. The distribution in the classical problem is more regular than in the case considering solute transport and variable porosity. It can be explained by the high salinization rate of the soil that prevents moisture downward percolation. Also the slowness of pore water flow may have been caused by a high concentration of salts, which results in its high density. As for the concentration of salts in the solid phase, it has decreased from the initial value of 100 kg/m^3 to 94.4 kg/m^3 on the lower and 91.8 kg/m^3 on the upper boundary. Average rate of salt content reduction in the soil equals 6.1%.

Solution of the same problem in the case of more intense rainfall is shown on Fig. 3.

a)



b)

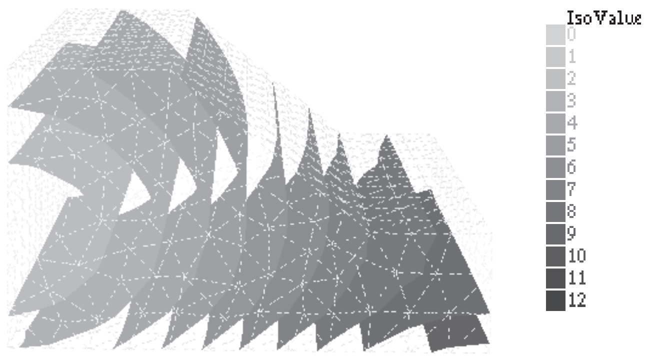


Fig. 3. Solution $s*$ of the problem described in the paper (a) and the relative difference Δ_s between it and the solution of classical problem, % (b)

In this case the relative difference between the solutions is no more than 12%. Concentration of salts in the solid phase has decreased to 94.6 kg/m^3 on the lower and 81.3 kg/m^3 on the upper boundary with average reduction by 8.5% over the whole domain.

4. CONCLUSIONS

A mathematical model of soil moisture transport with variable porosity has been studied in this paper. Some variable process parameters associated with solute transport have been considered, as well as some methods of their determination described in literature. A setting of the problem has been formulated for the case of soil desalinization caused by rainfalls. Program realization of the corresponding algorithms has been conducted using FreeFem++. In the course of numerical experiments solutions of the classical problem and the one considering solute and heat transport have been compared. As a result, it has been established that in the case of the latter moisture transport transpired more slowly. Relative difference between the solutions of corresponding problems amounts to 12% in the case of intense rainfall. During a period of six month for which a modelling has been conducted the concentration of salts in the solid component of the soil has decreased by 6.1% in the case of moderate and by 8.5% in the case of intense rainfall.

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MODELOWANIE KOMPUTEROWE SYSTEMU DYNAMICZNEGO «OŚRODEK POROWATY – WILGOĆ – SUBSTANCJA CHEMICZNA» NA PRZYKŁADZIE WPŁYWU OPADÓW NA ODSALANIE GRUNTU (PRZYPADEK PRZESTRZENNY)

Streszczenie

Numerycznie badano model matematyczny transportu wilgoci o zmiennej porowatości. Zmiany porowatości są spowodowane rozpuszczaniem substancji chemicznych związanych ze szkieletem gruntu. Znaleziono numeryczne rozwiązanie problemu w przypadku regularnych opadów. Programowa realizacja odpowiednich algorytmów została zaimplementowana w środowisku obliczeniowym FreeFem++. Przeprowadzono eksperymenty numeryczne i określono wpływ opadów na odsalanie gruntu przy wysokim stężeniu soli w twardym komponencie.

Słowa kluczowe: model matematyczny, ośrodek porowaty, przenoszenie wilgoci, przenoszenie masy, metoda elementów skończonych