THE METHOD OF AXES DISTANCE ERROR COMPENSATION IN CONVEXO-CONCAVE NOVIKOV GEARS

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S u m m a r y

Paper presents the method of axes distance error compensation in convexo-concave Novikov gears. The method is based on the selection of pressure angle, which enables that after gear assembly, contact pattern will take the desired position and shape. Moreover the example of application of this method was given.

Keywords: axes distance error, Novikov gears, contact pattern

Metoda kompensacji błędu rozstawienia osi kół w przekładniach zębatych typu Nowikowa

S t r e s z c z e n i e

W pracy przedstawiono metodę kompensacji błędu rozstawienia osi kół w przekładniach o zarysach wklęsło-wypukłych typu Nowikowa. Metoda polega na doborze wartości nominalnego kąta przyporu. Zapewni po montażu przekładni prognozowany kształt i położenie śladu styku. Podano przykład tej metody.

Słowa kluczowe: błąd rozstawienia osi, koła zębate Nowikowa, ślad styku

1. Introduction

Gears with convexo-concave teeth profiles are distinguished by higher surface load capacity than involute ones. They were the aim of theoretical analysis and experiments [1-14] carried out by many researchers around the world. There are examples of application of this king of gearing which led to an increase in load capacity [10] or to decrease in mass [1, 2, 4] of gearbox. Although the convexo-concave gears are reluctantly applied. It can be influenced by the fact that in contrast to involute gears there is no developed methodology of designing and analysing this king of gears. Moreover the machining of such kind of gears is more difficult. Usually there is a need to have two tools, which should be designed to machine the specific gear pair.

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The main disadvantage of convexo-concave teeth profile is its sensitivity to axes distance error. On the design stage of Novikov gear this error should be included by the constructor, who by applying the proper geometrical modifications can mitigate its negative impact. In this paper the method, which allows to compensate the negative effects connected with axes distance error in Novikov gears is presented.

2. Sensitivity to axes distance error

The above-mentioned sensitivity of gear to axes distance error reveals as a change of position of contact pattern, what was shown on the example of contact pattern of cylindrical external Novikov gear (Fig. 1).
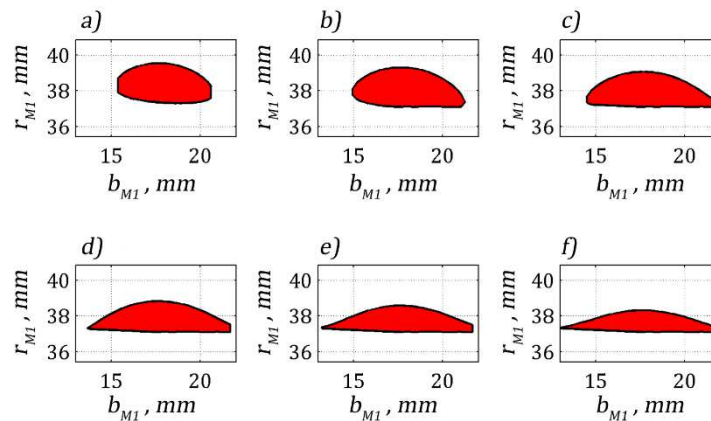

Fig. 1. The influence of axes distance error on contact pattern of cylindrical external Novikov gear: a) $\Delta a_x = 0$, b) $\Delta a_x = 0,01$ mm, c) $\Delta a_x = 0,02$ mm, d) $\Delta a_x = 0,03$ mm, e) $\Delta a_x = 0,04$ mm, f) $\Delta a_x = 0,05$ mm [11]

The increase of the axes distance error moves the contact pattern through the pinion tooth root (convex tooth). For some values of distance error (Fig. 1b) there is a maximum contact area. This phenomenon can be explained by the fact that contact point moves to the area where the ratio of average curvature radius of concave tooth to convex tooth is greater than in its previous position. This fact confirms the surface Hertz stress analysis in gear pair with axes distance error [12]. Further increase of axes distance error (Fig. 1d) leads to interference of convex tooth root with tip of concave tooth.

The effects caused by this error could be compensated by the proper selection of geometrical parameters, which moves contact point to the tip of convex tooth, so e.g. by the transverse pressure angle [11]. As a result the contact pattern should

take the desired by the constructor position and surface stresses should be decreased.

3. Selection of pressure angle

To determine the relation between axes distance error and operating transverse pressure angle, defining the location of contact point after spacing the axes, the geometrical construction shown on Fig. 2 can be used.
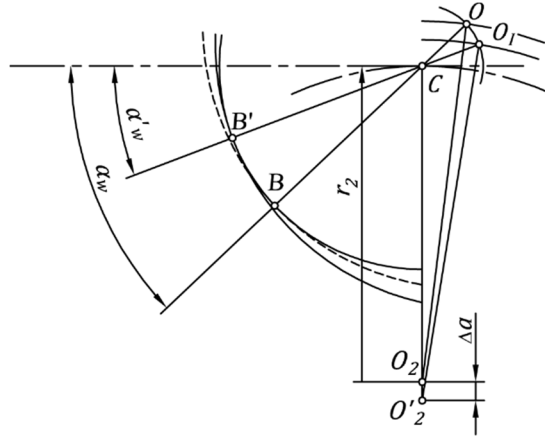

Fig. 2. Geometrical construction for obtaining the relation between axes distance error and operating transverse pressure angle, where: α_w – transverse pressure angle, α'_w – operating transverse pressure angle, Δa – axes distance error, r_2 – pitch radius of wheel, B – contact point for ideal gear pair, B' – contact point after axes distance error occurrence, C – central point of meshing, O – center of concave tooth profile, O_1 – center of concave tooth profile after axes distance error occurrence

The pinion with convex teeth is working with wheel which has the concave teeth. The center of convex tooth profile of ρ_1 radius covers the central point of meshing C. The center of concave tooth profile of ρ_2 radius in case of ideal gear pair lies in the O point. The location of contact point of profiles B is defined by transverse pressure angle α_w. After shifting the wheel by the axes distance error Δa the new location of contact point B' defined by the operating transverse pressure angle α'_w should be obtained. Therefore the center of concave tooth profile is in the O_1 point. From the law of cosines in O_2OC triangle, the square of $/O_2O/$ length can be obtained as (1):

$$\left|O_2O\right|^2 = r_2^2 + (\rho_2 - \rho_1)^2 + 2r_2(\rho_2 - \rho_1)\sin\alpha_w \qquad (1)$$

Next with the use of the same law in O'_2O_IC triangle the square of $/O'_2O_I/$ length is given as (2):

$$\left|O'_2O\right|^2 = (r_2 + \Delta a)^2 + (\rho_2 - \rho_1)^2 + 2(r_2 + \Delta a)(\rho_2 - \rho_1)\sin\alpha'_w \qquad (2)$$

Comparing the formulas (1) and (2) the required relation between operating pressure angle and axes distance error is obtained (3)

$$\sin\alpha'_w = \frac{r_2^2 + 2r_2(\rho_2 - \rho_1)\sin\alpha_w - (r_2 + \Delta a)^2}{2(r_2 + \Delta a)(\rho_2 - \rho_1)} \qquad (3)$$

The above formula with wheel pitch radius tending to infinite is convergent to simpler formula, what can be proved based on l'Hospital's rule and written as (4):

$$\lim_{r_2 \to \infty} \sin\alpha'_w = \sin\alpha_w - \frac{\Delta a}{(\rho_2 - \rho_1)} \qquad (4)$$

From that reason for most cases, in which r_2 radius is large enough (it is usually satisfied in case of gears used in machine building industry) formula (3) can be simplified to (5):

$$\sin\alpha'_w = \sin\alpha_w - \frac{\Delta a}{(\rho_2 - \rho_1)} \qquad (5)$$

The relation between operating pressure angle, axes distance error and difference between teeth profile radii for transverse pressure angle $\alpha_w = 20°$ is shown on Fig. 3.

If the teeth profile radii are less mismatched the sensitivity to axes distance error increases (Fig. 3). For instance for axes distance error $\Delta a = 0{,}01$ mm of gear pair, in which the difference between teeth profile radii is $\rho_2 - \rho_1 = 0{,}45$ mm the operating transverse pressure angle is $\alpha'_w = 18{,}65°$. However, in case of gear pair in which the difference between teeth profile radii is $\rho_2 - \rho_1 = 0{,}05$ mm this angle is about 2,3 times lower and is $\alpha'_w = 8{,}1°$. Less mismatched profiles of convex and concave teeth increases the surface load capacity of gear. Therefore, the designer of Novikov gear is faced with the need of achieving the compromise between the sensitivity to axes distance error and keeping the required surface strength.
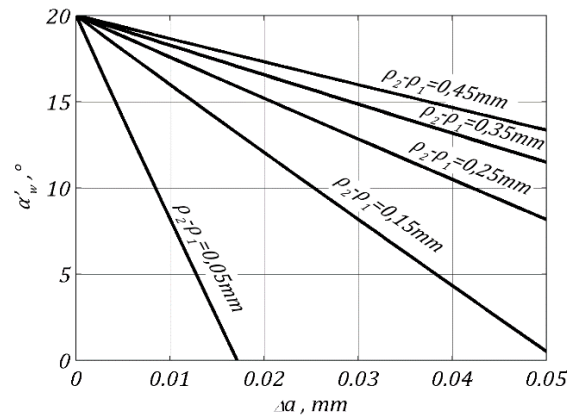
Fig. 3. Operating pressure angle in relation with axes distance error
and difference between teeth profile radii for transverse pressure angle $\alpha_w = 20°$

4. Example of axes distance error compensation

The axes distance error compensation will be demonstrated on the example of Novikov gear pair with data presented in Table 1.

Table 1. Data of analysed Novikov gear pair

Parametres	Pinion	Wheel
Normal module, mm	$m_n = 3$	$m_n = 3$
Number of teeth	$z_1 = 30$	$z_2 = 47$
Overlap ratio	$\varepsilon_\beta = 1,21$	$\varepsilon_\beta = 1,21$
Gear width, mm	$b = 30$	$b = 30$
Translation of convex tooth profile, mm	$d_{CO'} = 0$	$d_{CO'} = 0$
Profile	convex	concave
Profile radius, mm	$\rho_1 = 6,33$	$\rho_2 = 6,55$
Pitch diameter, mm	$d_1 = 97,4$	$d_2 = 152,6$
Tip diameter, mm	$d_{a1} = 104,3$	$d_{a2} = 152,6$
Root diameter, mm	$d_{f1} = 95,3$	$d_{f2} = 143,6$

It was assumed that due to strength, the location of contact point of teeth profiles should be defined by transverse pressure angle $\alpha_w = 16°$. Moreover, the expected axes distance error was $\Delta a = 0,02$ mm. The contact patterns obtained with the aid of method, in which the distance is measured through the surface unit normal [11] for different error values were shown on Fig. 4.

In ideal case (Fig. 4a) contact pattern is taking the desired position and shape, which is favorable due to strength. After axes distance error occurrence (Fig. 4b) there is an edge contact, what can lead to an increase of vibration amplitudes and stress concentration. To avoid this situation gear pair should be manufactured with

increased transverse pressure angle. Taking the operating pressure angle as $\alpha'_w = 16°$, axes distance error $\Delta a = 0,02$ mm, difference between teeth profile radii $\rho_2 - \rho_1 = 0,22$ mm (Table 1) and using the formula (5) the nominal value of transverse pressure angle can be obtained approximately as $\alpha_w = 21,5°$. Figure 4 presents contact patterns for gear pair with pressure angle of $\alpha_w = 21,5°$.
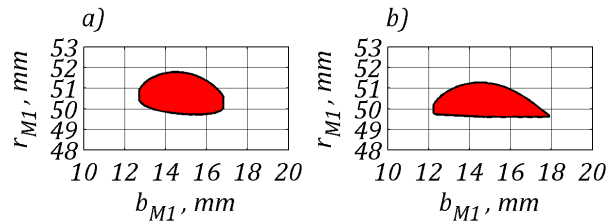

Fig. 4. Contact patterns of Novikov gear pair (Table 1) with transverse
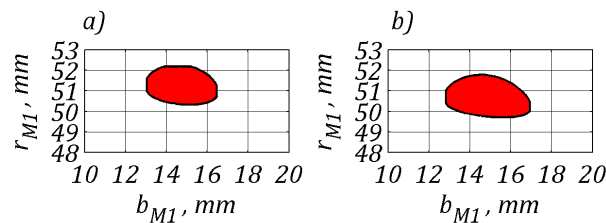pressure angle $\alpha_w = 16°$: a) $\Delta a = 0$, b) $\Delta a = 0,02$ mm

Fig. 5. Contact patterns of Novikov gear pair (Table 1) with transverse
pressure angle $\alpha_w = 21,5°$: a) $\Delta a = 0$, b) $\Delta a = 0,02$ mm

After increase of the nominal transverse pressure angle for gear pair without axes distance error, contact pattern is shifted through tip of the convex tooth, which cause the edge contact (Fig. 5a). After axes distance error occurrence, contact pattern takes the desired due to strength, position and shape (Fig. 5b), which meet the case presented on Fig. 5a.

5. Conclusions

Presented method of Novikov gears axes distance error compensation allows to avoid the negative effects connected with it. It allows the constructor to choose the proper pressure angle at the design stage, providing the proper position and shape of contact pattern. Presented method was used in designing the Novikov gear pair (Fig. 6) subjected to fatigue tests, which confirmed its effectiveness [13].

Fig. 6. Novikov gear pair, in which the presented method
of axes distance error compensation was used

References

[1] A. DYSON, H.P. EVANS, W. SNIDLE: Wildhaber-Novikov cicrular-arc gears: some properties of relevance to their design. *Proceedings of The Royal Society A Mathematical Physical and Engineering Sciences,* **425**(1989)1825, 341-363.

[2] A. DYSON, H.P. EVANS, W. SNIDLE: Wildhaber-Novikov circular arc gears: geometry and kinematics. *Proc. of The Royal Society a Mathematical Physical and Engineering Sciences*, **403**(1986)1825, 313-349.

[3] C-B. TSAY, Z.H. FONG: The mathematical model of Wildhaber-Novikov gears applicable to finite element stress analysis. *Mathematical and Computer Modelling*, **12**(1989)8, 939-946.

[4] D.V. ELLIS: The Westland Lynx. *The RUSI Journal*, **125**(1980)4, 70-73.

[5] F.L. LITVIN, et al.: New version of Novikov–Wildhaber helical gears: computerized design, simulation of meshing and stress analysis. *Computer methods in applied mechanics and engineering*, **191**(2002)49-50, 5707-5740.

[6] H. OPITZ, et al.: Der Einfluß der Fertigungsgenauigkeit und der Schmierfilmausbildung auf die Flankentragfähigkeit ungehärteter Stirnräder. Springer Fachmedien Wiesbaden 1965.

[7] J.R. COLBOURNE: The contact stress in Novikov gears. *Mechanism and Machine Theory*, **24**(1989)3, 223-229.

[8] K. LINGAIAH, K. RAMACHANDRA: Photoelastic Optimization of the Profiles of Wildhaber-Novikov Gears. *Experimental Mechanics*, **16**(1976)3, 116-120.

[9] K. LINGAIAH, K. RAMACHANDRA: Three-dimensional Photoelastic Study of the Load-carrying Capacity/Face Width Ratio of Wildhaber-Novikov Gears for Automotive Applications. *Experimental Mechanics*, **17**(1977)10, 392-397.

[10] LS PETROCHEM: LS Pumping Units Catalog. LS Petrochem 2008.

[11] M. BATSCH, T. MARKOWSKI: Tooth contact analysis of Novikov convexo-concave gears. *Advances in Manufacturing Science and Technology*, **39**(2015)1.

[12] M. BATSCH, W. HOMIK, T. MARKOWSKI: Cylindrical gears with increased contact area – proposal of application in watercrafts power transmission systems. *Solid State Phenomena*, **236**(2015), 26-30.

[13] M. BATSCH: Analysis of the contact region geometry of the cylindrical convexo-concave Novikov gear. PhD dissertation, Rzeszów University of Technology, Rzeszów 2015.

[14] S-C. YANG: Mathematical model of a stepped triple circular-arc gear. *Mechanism and Machine Theory*, **44**(2009)5, 1019-1031.

[15] V.I. KOROTKIN: Comparison of Novikov transmissions and evolvent transmissions. *Russian Engineering Research*, **29**(2009)1, 1-6.