# SCATTERING OF PLANE ACOUSTIC WAVES AT ELASTIC PARTICLES WITH ROUGH SURFACES

LEIF BJØRNØ

RESON A/S
Fabriksvangen 13, DK-3550 Slangerup, Denmark
prof.lb@get2net.dk

*A comprehensive theoretical and numerical study of the influence of surface roughness of elastic particles in water on the scattering of ultrasonic waves has been carried out. For near spherical shape of the particles and with small rms-roughness heights a perturbation method has been developed. In this method the first-order perturbation contribution predicts the contribution to the incoherently scattered acoustic field due to surface roughness, and the second-order perturbation contribution predicts the change in the coherent field and will satisfy the requirement of energy conservation. The second-order perturbation contribution is evaluated using the concept of form function, while the first-order perturbation to the total scattered acoustic field is evaluated using the scattering cross-section. As a function of the ka-value and for different rms-roughness heights a numerical study of the forward and backward scattering from rough, elastic particles has been carried out and a substantial roughness influence on the scattered field has been verified. Some experimental results from measurements of scattering from glass and cast iron spheres have given evidence to the numerical predictions.*

## INTRODUCTION

While the Kirchhoff approximation is frequently applied to the calculation of the scattering from a randomly rough surface [1], scattering of plane waves from certain geometries like for instance a sphere can only be calculated using the Kirchhoff approximation by small parts of the spherical surface due to nearly grazing angle of incidence on most of the surface of the sphere. A better candidate for the scattering calculation, also when the rms-roughness is small, is the use of a perturbation procedure. That an analytical solution [2] already exists to the scattering of plane waves from smooth spheres also points at an application of a perturbation solution.

Since Faran's work on scattering of plane acoustic waves by elastic smooth spheres several other approaches to the scattering problems by spheres have been worked out like the use of the concept of *form function* [3 - 4] and the use of the resonance scattering theory [5 – 8]. While several studies of scattering by rough, planar interfaces have been performed over recent years [9], only a few studies have been carried out of the scattering from rough, non-planar, elastic objects [19 -11]. Also a study [12] on scattering by rough spheres having Dirichlet boundary conditions using the null field method has been reported.

The problem to be studied in this paper originates from an investigation of the scattering of sound from sediment particles being transported by the flow near the seabed in an attempt to measure the sediment concentration and its variation from the seabed and up through the water column. This study formed a part of an EU-funded project [13] and it formed partly a basis for a Ph.D. study reported in [14]. Based on the resonance scattering theory for elastic spheres with smooth surfaces and the perturbation theory for the scattering from planar rough surfaces, the scattering of plane acoustic waves from single elastic spheres with rough surfaces are discussed in this paper.

## 1. SPHERES WITH SMOOTH SURFACES

A plane acoustic wave $p_{inc}$ incident on a sphere along the z-axis, see Figure 1, may be expressed by [2] as:

$$P_{inc} = P_o exp(iKz) = P_o exp(iKRcos\theta) = P_o \sum (2n+1) i^n j_n(KR) P_n(cos\theta) \quad (1)$$

where $P_o$ is the amplitude of the incident wave, $K = 2\pi f/c$ is the acoustic wavenumber and $c$ is the velocity of sound in the surrounding fluid, $f$ is the frequency of the incident wave, $j_n$ is the $n$th-order spherical Bessel function, and $P_n(cos\theta)$ is the $n$th-order Legendre polynomial. No dependence on the angle $\varphi$ should be considered due to symmetry. And the outgoing scattered wave $p_{sc}$ may be written as:

$$P_{sc} = P_o \sum c_n [j_n(KR) + iy_n(KR)] P_n(cos\theta) \quad (2)$$

where $y_n$ is the $n$th-order spherical Neumann function, and $c_n$ is an unknown coefficient for the $n$th-partial wave to be determined through the boundary conditions. These boundary conditions are:

1. The pressure in the surrounding fluid $(p_{inc} + p_{sc})$ must at the surface of the sphere be equal to the normal component of the stress $(\sigma_{RR})$ in the sphere.
2. The normal component of displacement $(u_{R,inc} + u_{R,sc})$ in the fluid must at the surface of the sphere be equal to the normal component of displacement in the sphere $(u_R)$.
3. The tangential components of the stress in the sphere $(\sigma_{R\theta}, \sigma_{R\varphi})$ must vanish.

Using these boundary conditions, the $c_n$ in (2) may be derived, and for a large distance $R$ from the scattering sphere the scattered wave field may be described by:

$$P_{sc} = \{(P_o exp(iKR)\} / (KR) \sum (2n+1) sin\eta_n e^{-i\eta} P_n(cos\theta) \quad (3)$$

which may be expressed by a *form function*, $f_\infty$, defined by [3]:

$$p_{sc} = \{P_o a f_\infty exp(iKR)\} / (2R) \quad (4)$$

where

$$f_\infty = -\{2/Ka\}\sum(2n+1)\sin\eta_n e^{-i\eta} P_n(\cos\theta) \quad (5)$$

where $a$ is the radius of the sphere, and where $\eta_n$ is the phase shift of the $n$th scattered partial wave [2, 3]. The form function (5) may be divided into contributions from a rigid mobile sphere and contributions from natural vibrations of an elastic sphere [4].

## 2. SPHERES WITH ROUGH SURFACES

When roughnesses are present on the surface of a sphere, and if the roughness height is small compared with the incident wavelength and the mean slope of the rough surface is small compared with unity, i.e.

$$K|\zeta(\theta,\varphi)| << 1 \quad (6)$$

and

$$|\nabla \zeta(\theta,\varphi)| << 1 \quad (7)$$

where $\zeta(\theta,\varphi)$ demotes the wave height of the surface roughness of the sphere, then the perturbation theory can be used for a study of scattering of acoustic waves from the surface roughness [15]. And the whole scattered field is then composed by contributions from the following sources:

1. A rigid mobile sphere ($p^r_{sc}$)
2. The natural vibrations of an elastic sphere ($p^e_{sc}$)
3. The surface roughness of the sphere ($p^\zeta_{sc}$)

As shown in [14] the first-order perturbation contribution can be used to predict the change in the incoherently scattered acoustic field due to the effects of the surface roughness, and the second-order perturbation contribution can be used to predict the change in the coherent field due to the effects of the surface roughness while also satisfying the requirement of energy conservation.

Two assumptions forming the basis for the use of the perturbation method for the scattering from a rough sphere may, when the reference surface is the surface of a smooth sphere $R = a$, be written as:

1. The value of each field quantity on the rough surface of the sphere can be expanded into Taylor series with respect to the reference surface as:

$$f(R_s,\theta,\varphi)=f(a,\theta,\varphi)+\zeta f'(a,\theta,\varphi)+(\zeta^2/2)f''(a,\theta,\varphi)+..., \quad R_s=a+\zeta(\theta,\varphi), \quad (8)$$

where $(R_s,\theta,\varphi)$ denotes a point on the rough sphere surface, $(a,\theta,\varphi)$ represents a point on the reference surface, and the prime denote derivatives with respect to $R$.

2. The scattered field is composed of several terms as:

$$p_{sc}(R,\theta,\varphi)=p_{sc,o}(R,\theta,\varphi)+p_{sc,1}(R,\theta,\varphi)+p_{sc,2}(R,\theta,\varphi)+... \quad (9)$$

All other field quantities, i.e. displacement and stress in the sphere, are assumed to be undisturbed by the presence of the surface roughness. The boundary conditions for the scattering by the smooth sphere may now be rewritten to include the effects of the roughness, thus leading to new and effective boundary conditions for the rough sphere scattering expressed by:

$$P_{sc,o}(R) = -\{\sigma_{RR}(R) + p_{inc}(R)\},\ at\ R = a \quad (10)$$

$$P_{sc,1}(R) = -\zeta\{p'_{inc}(R) + p'_{sc,o}(R)\},\ at\ R{=}a \quad (11)$$

$$P_{sc,2} = (\zeta^2/2)\{p''_{inc}(R) + p''_{sc,o}(R)\},\ at\ R{=}a \quad (12)$$

where it has been assumed, that the derivatives of $\sigma_{RR}$ with respect to $R$ are small enough to be ignored. The three effective boundary conditions (10) – (12) show:

1. When $p_{sc,o}$ is used for the initially scattered field, the results for the scattering by a smooth elastic sphere will be obtained.
2. When $p_{sc,1}$ is used for the initially scattered field, the first-order perturbation corrections (i.e. the change in the incoherent field) to the scattering from a rough elastic sphere will be obtained.
3. When $p_{sc,2}$ is used for the initially scattered field, the second-order perturbation corrections (i.e. the change in the coherent field) to the total field scattered from a rough elastic sphere will be obtained.

When using the boundary conditions (10) – (12) together with Helmholtz' integral equation, the first-order perturbation correction to the field scattered by a rough elastic sphere may be expressed by:

$$P_{sc,1}(\mathbf{R}) = \int_{So}\zeta(\theta_o,\varphi_o)[p''_{inc}(a,\theta_o) + p''_{sc.o}(a,\theta_o)]G^*(\mathbf{R},\mathbf{R}_o)dS_o \quad (13)$$

where the double primes denote the second derivative with respect to $R$, $\mathbf{R}$ and $\mathbf{R_o}$ are the vectors to the field point and the point on the surface of the sphere, respectively. The integration is carried out over the whole reference surface $S_o$. The half-space Greens function on the reference surface is given by:

$$G^*(\mathbf{R},\mathbf{R}_o) = exp(iK|\mathbf{R} - \mathbf{R}_o|)/2\pi|\mathbf{R} - \mathbf{R}_o| \quad (14)$$

By use of the effective boundary conditions and of Helmholts integral equation, the second-order perturbation corrections (the coherent part) to the total acoustic field scattered by a rough elastic sphere may be expressed by:

$$<p_{sc,2}(\mathbf{R})> = K^3h^2P_oa^2exp(iKR)(4\pi R)^{-1}\int d\varphi_o\int d\theta_o sin\theta_o F_3F_4 \quad (15)$$

where $F_3$ and $F_4$ are expressed by:

$$F_3 = F_3(\theta_o) = \sum(2n+1)i^n\{j_n''' + [y_n''' - ij_n''']sin\eta_n exp(i\eta_n)\}P_n(cos\theta_o) \quad (16)$$

$$F_4 = F_4(\theta,\theta_o,\varphi_o) = exp[-iKa(cos\theta cos\theta_o + son\theta sin\theta_o cos\varphi_o)] \quad (17)$$

where $h$ denotes the rms-roughness height, $\eta_n$ is a phase angle and $P_o$ is the amplitude of the incident field.

The second-order perturbation corrections to the total scattered acoustic field (15) may also be expressed in terms of the form function $f_\infty^{sc,2}$ given by:

$$<p_{sc,2}(\mathbf{R})> = P_o a f_\infty^{sc,2} exp(iKR)/(2R) \qquad (18)$$

with

$$f_\infty^{sc,2} = K^2h^2Ka(2\pi)^{-1}\int d\varphi_o \int d\theta_o sin\theta_o F_3F_4 \qquad (19)$$

For a backscattering geometry, i.e. for $\theta = \pi$, $\sin\theta = 0$ and $\cos\theta = -1$, the form function may be written as:

$$f_\infty = -2(Ka)^{-1}\sum(2n+1)(-1)^n sin\eta_n(exp(-i\eta_n))(exp(-D_n^2K^2h^2) \qquad (20)$$

where $D_n^2$ is given by:

$$D_n^2 = (-i)^n[Kaj_n'''(ka)\{sin\eta_n(exp(-i\eta_n))\}^{-1} + Kay_n'''(Ka) - iKaj_n'''(Ka)]\ x$$

$$\int KaP_n(x)e^{iKax}dx$$

The last term in (20), $exp(-D_n^2K^2h^2)$, represents the contribution of the surface roughness to the scattered field, while the other terms in (20) represents the backscattering form function for a smooth sphere.

The total scattered acoustic field by a rough elastic sphere, consisting of the first- and second-order perturbation contributions to be added to the scattered field from a smooth elastic sphere, may now be written as:

$$p_{sc} = p_{sc}^r + p_{sc}^e + p_{sc}^\zeta \qquad (21)$$

where the first two terms in (21) denote the reflection from a rigid, smooth sphere and the resonance vibration due to elasticity of the sphere, respectively. These two terms describe together the scattered field from a smooth, elastic sphere, and they constitute the $0^{th}$-order result for the scattering from a rough, elastic sphere. The last term in (21) is the contribution of the sphere's surface roughness to the total scattered field, and it consists of the sum of the first- and the second-order perturbation contributions.

## 3. NUMERICAL RESULTS

Numerical calculations of the backscatter form function for a smooth and a rough cast iron ball in water and for the scattering directivity patterns have been carried out [14] for the following data: Water density: 998 kg/m$^3$; Sound velocity in water: 1,496 m/s; Cast iron density: 7,200 kg/m$^3$; Cast iron shear velocity: 2,690 m/s; Cast iron longitudinal velocity: 4,990 m/s.

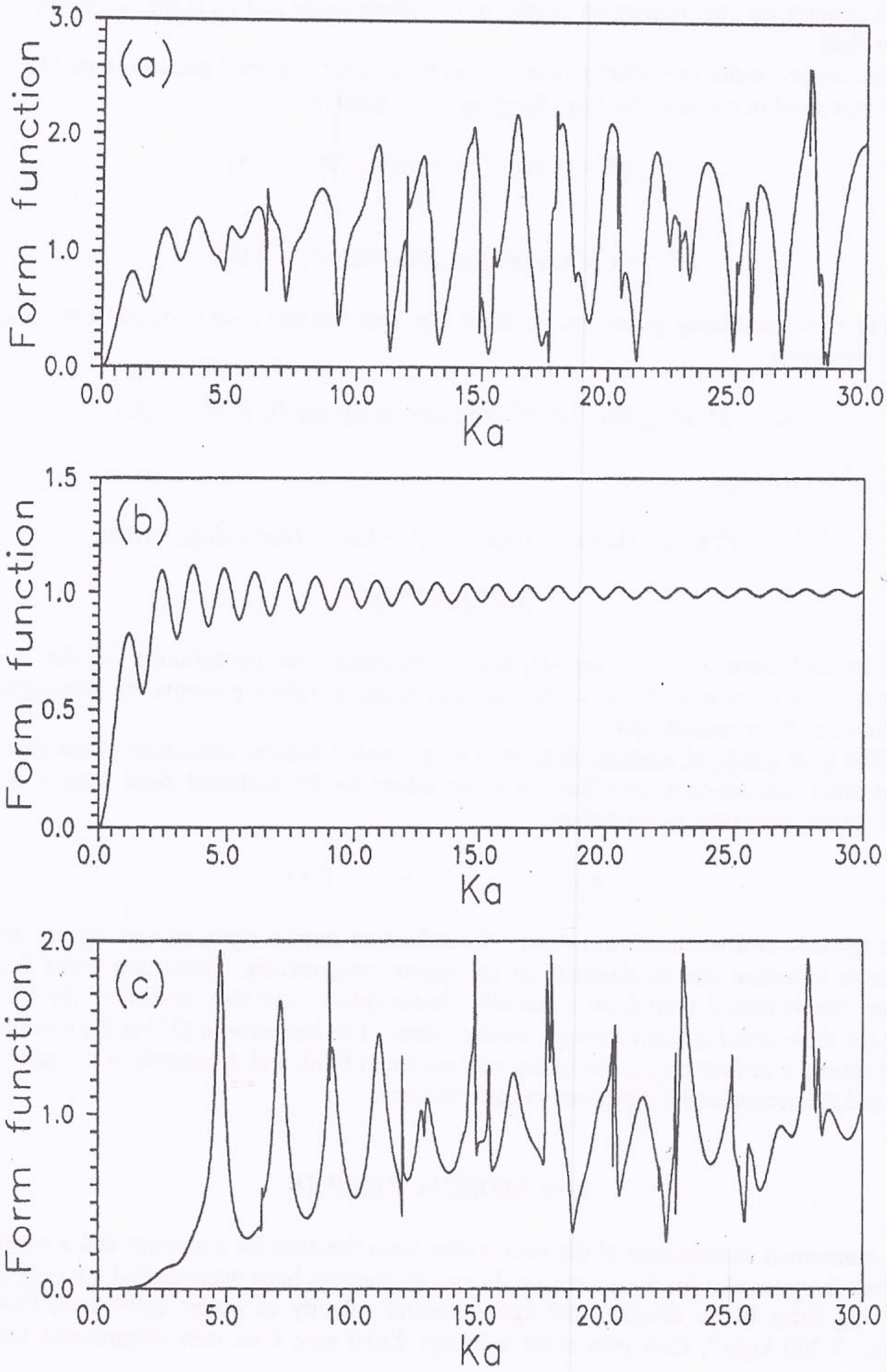


Figure 1

Figure 1 (a) shows the backscattering form function as a function of the Ka-values for a smooth, massive cast iron ball in water. This backscattering form function receives contributions from the rigid mobile ball, figure 1 (b), and from the elastic vibrations of the smooth, massive cast iron ball, figure 1 (c). The superposition of the solutions, figures 1 (b) and 1 (c), forms the solution given in figure 1 (a). While the backscatter form function approaches unity for high Ka-values for the rigid mobile ball, showing that the reflection of plane waves from a very large rigid ball will behave like the reflection from a rigid wall, the resonance nature of the backscattering from the massive, elastic cast iron ball may clearly be seen.

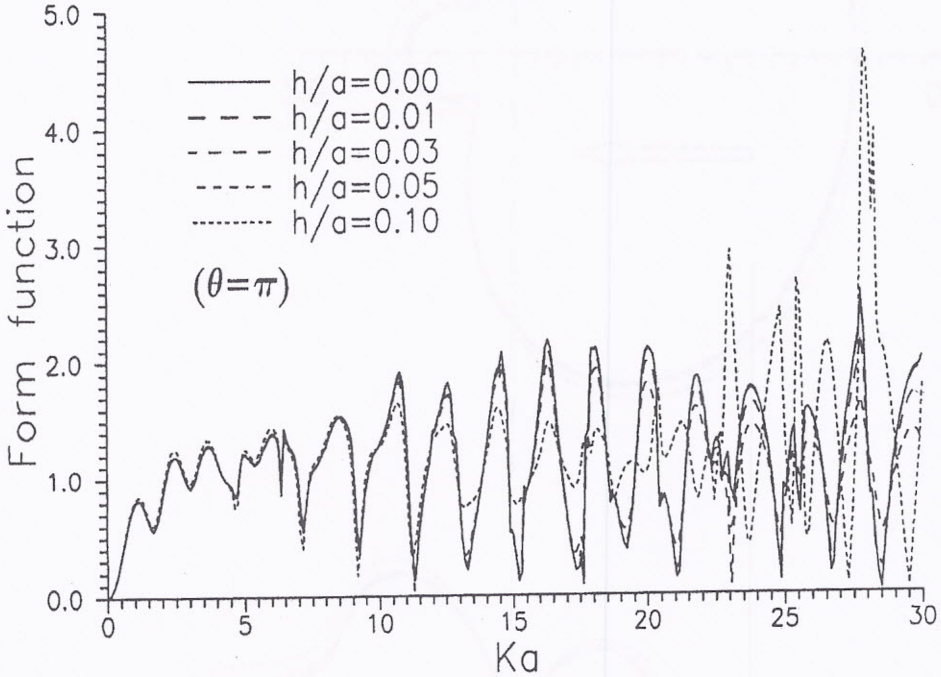


Figure 2

Figure 2 shows the backscattering form function as a function of the Ka-values for a rough, massive cast iron ball for various rms surface roughness heights $h$ measured relative to the radius $a$ of the smooth surface cast iron ball. The solid line describes the backscattering form function for a smooth surface. As the backscatter-ring form function in figure 2 is a function of the frequency – via dependence on the wave number $K$ – the $Kh$-value will also change with frequency. This means that for increasing $Ka$-values ($a$ being fixed), the effective roughness of the ball surface will show an apparent increase. This explains why for some $Ka$-values the form function for a rough ball is smaller than for a smooth ball, while for other $Ka$-values the form function has higher values than for a smooth ball.

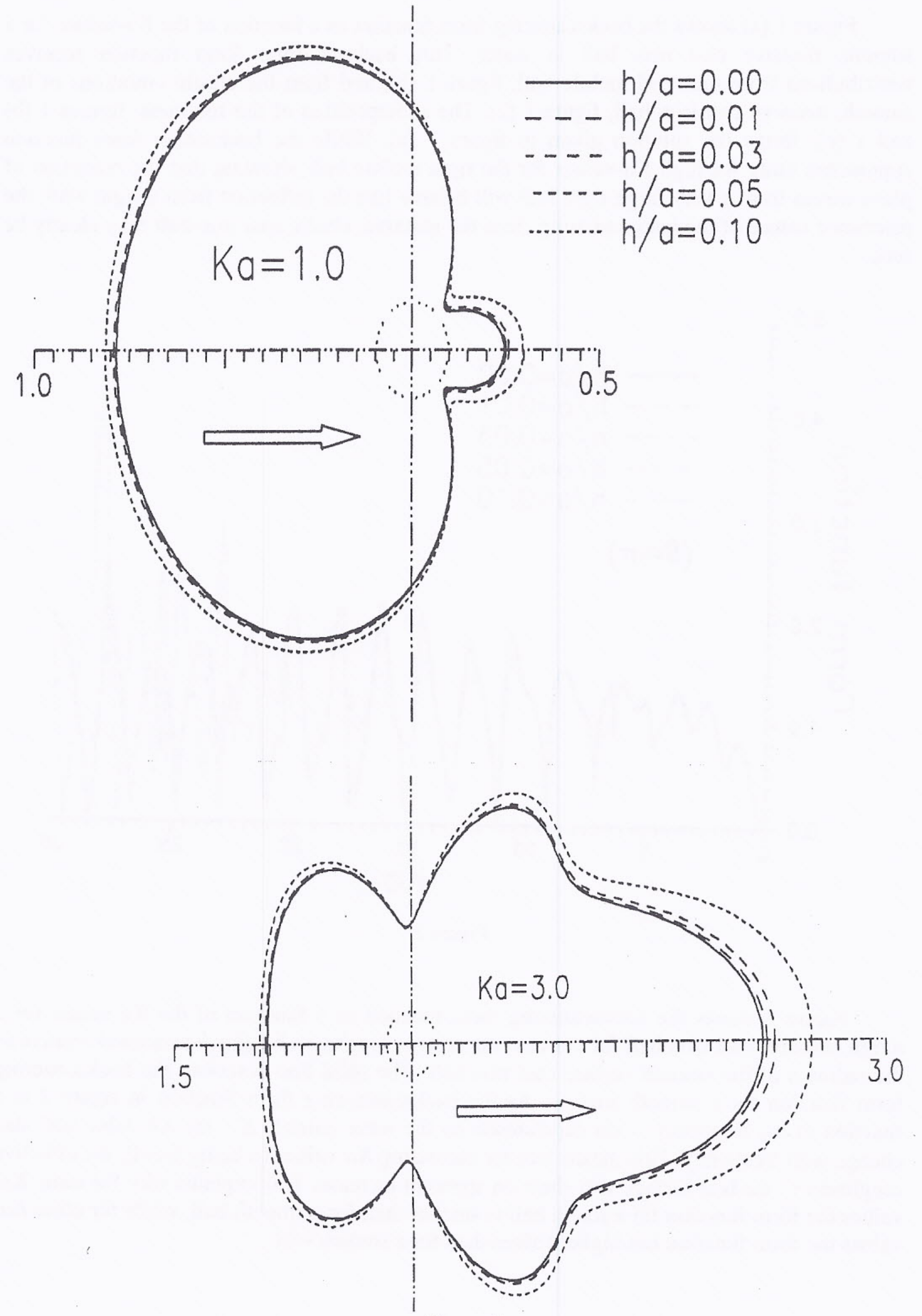


Figure 3

Figure 3 shows the directivity patterns for the scattering from a rough, massive elastic cast iron ball for $Ka = 1$ and $Ka = 3$, and for different rms-roughness heights $h$. The solid line describes the directivity patterns for the scattering from a smooth, massive elastic cast iron ball, and the other curves describes the directivity patterns for various $h/a$-values. For $Ka = 1$ the contributions to the directivity patterns are small, and vary with the scattering angle. In the forward and backward directions the influence of the roughness on the scattered field is higher than in certain other angles. In most directions the roughness increases the scattered field amplitude, showing that the requirement of energy preservation is not perfectly satisfied due to this adaptation of the perturbation procedure to the special shape of a ball. Figure 3 also shows for $Ka = 3$, that a stronger influence of the roughness shall be expected for higher $Ka$-values, in particular in the forward direction.

## 4. EXPERIMENTAL DATA AND COMPARISON WITH THEORY

The backscattering measurements from a smooth and a rough cast iron ball, both of a diameter on 25 mm, were carried out in the frequency range of 200 – 800 kHz, using single cycle pulses. A calibration of the experimental set-up was performed measuring the scattering directivity and the backscattering amplitude from a stainless steel ball with a smooth surface and comparing the experimental data with theoretical/numerical values already available.

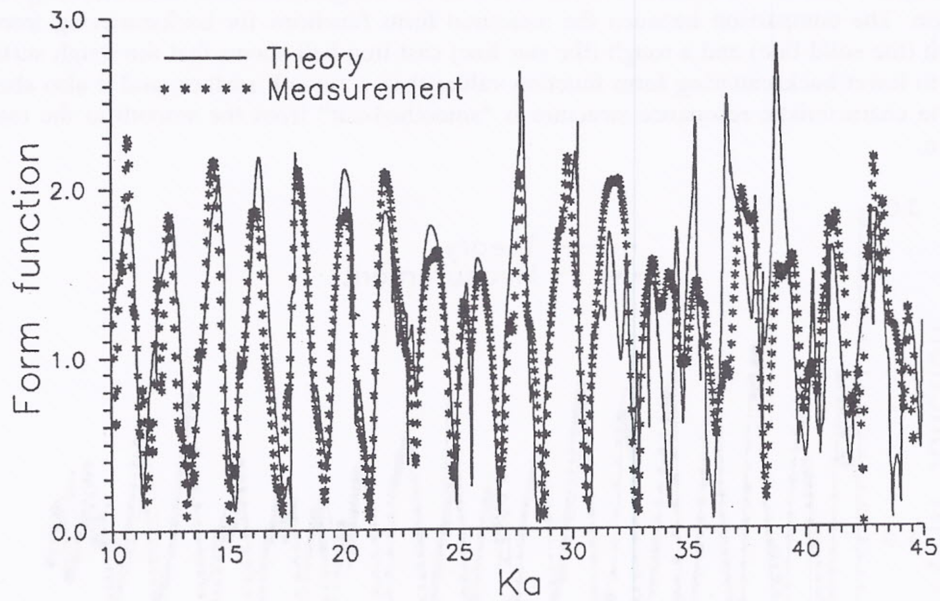


Figure 4

Figure 4 shows a comparison between the measured and calculated form functions for backscattering from a smooth, massive cast iron ball. A good agreement between the measured and calculated data is in general obtained, but the fact that the cast iron ball surface is not perfectly smooth has an influence on the agreement at certain $Ka$ = values.

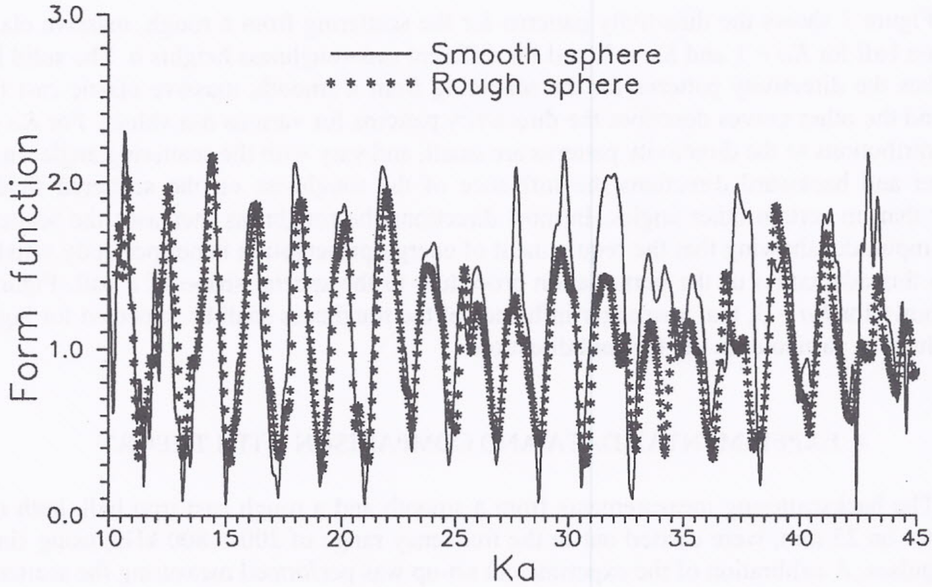


Figure 5

Figure 5 shows the influence of the cast iron ball roughness on the backscattering form function. The comparison between the measured form functions for backscattering from a smooth (the solid line) and a rough (the star line) cast iron ball shows that the rough surface leads to lower backscattering form function values than a smooth surface, and it also shows that the characteristic resonance structure is "smoothed-out" from the smooth to the rough surface.

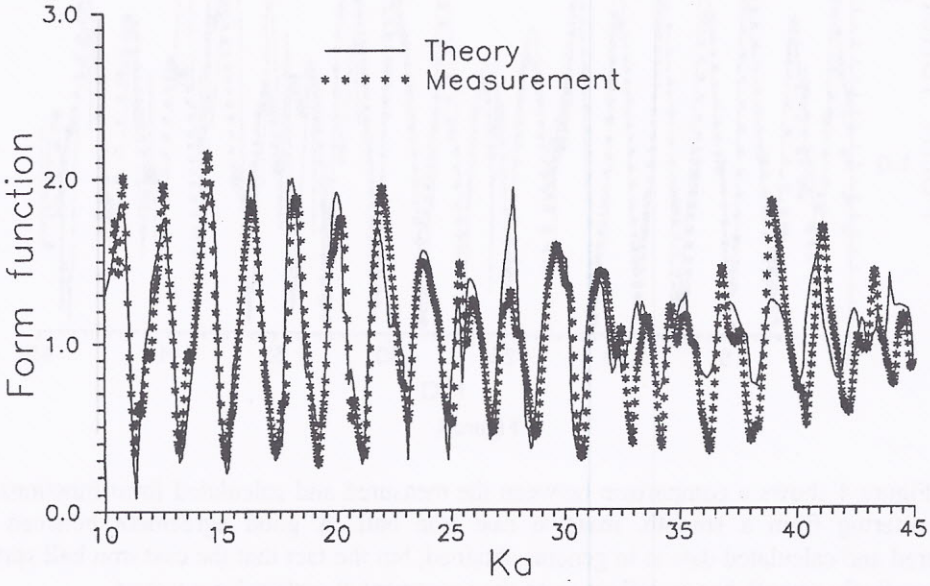


Figure 6

Figure 6 shows, finally, a comparison between the calculated and the measured form function for backscattering from a rough, massive elastic cast iron ball of 25 mm in diameter. The a very good agreement between the theoretical/numerical values (the solid line) and the measured data (the star values), in particular for lower *Ka*-values, gives evidence to the applicability of the perturbation method for calculation of the scattering from rough, massive elastic spheres.

## 5. CONCLUSIONS

A second-order perturbation solution for scattering by elastic spheres with rough surfaces has been established, numerical results have been presented and experimental verification of the applicability of the perturbation solution has been obtained. It has been shown that the influence of the surface roughness on the backscattered field is a decrease in the amplitude of the form function, mainly being caused by the coherently scattered acoustic energy, and a smoothing out of the resonance structure of the form function, mainly being caused by the incoherently scattered energy. The investigations also showed that as long as the *Kh*-values are small such that the perturbation theory is applicable, the higher the *Ka*-values, the better the prediction becomes of the scattered acoustic field from a rough, elastic sphere using the approach presented in this paper.

## REFERENCES

1. L. Bjørnø and S. Sun, "Use of the Kirchhoff Approximation in scattering from elastic, rough surfaces. Acoustical Physics, Vol. 41, (5), pp. 637 – 648, 1995.
2. J.J. Faran, "Sound scattering by solid cylinders and spheres", J. Acoust. Soc. Amer., Vol. 23, pp. 405 – 418, 1951.
3. A.E. Hay and D.G. Mercer, "On the theory of sound scattering and viscous absorption in aqueous suspensions at medium and short wavelengths", J. Acoust. Soc. Amer., Vol 78, pp. 1761 – 1771, 1985.
4. P.D. Thorne and S.C. Campbell, "Backscattering by a suspension of spheres", J. Acoust. Soc. Amer., Vol. 92, pp. 978 – 986, 1992.
5. L. Flax, G.C. Gaunaurd and Überall, H., "Theory of elastic resonance excitation by sound scattering", J. Acoust. Soc. Amer., Vol. 63, pp. 723 – 731, 1978.
6. G.C. Gaunaurd and Überall, H., "RST analysis of monostatic and bistatic acoustic echoes from an elastic sphere", J. Acoust. Soc. Amer., Vol. 73, pp. 1 – 12, 1983.
7. W.G. Neubauer, "Reflection and vibrational modes of elastic spheres", in *Acoustic Resonance Scattering*, H. Überall (Ed.), Gordon & Breach Science Publ., Philadelphia, USA, 1992.
8. A.D. Pierce and R.N. Thurston, *Underwater Scattering and Radiation,* Academic Press, 1993.
9. S.T. McDaniel, "A numerical study of rough-surface scatter", J. Comp. Acoust., Vol. 8, (3), pp. 443 – 458, 2000.
10. T.K. Stanton, "Sound scattering by rough elongated elastic objects. I. Means of scattered field", J. Acoust. Soc. Amer., Vol. 92, pp. 1641 – 1664, 1992.
11. J.V. Gurley, and T.K. Stanton, "Sound scattering by rough elongated elastic objects. III. Experiments." J. Acoust. Soc. Amer., Vol. 94, pp. 2746 – 2755, 1993.

12. P.-Å. Jansson, "Acoustic scattering from a rough sphere", J. Acoust. Soc. Amer., Vol. 93, pp. 3032 – 3042, 1993.
13. EU-funded project: SMART/ISUSAT, contract No. MAST-CT92-0082.
14. S. Sun, "Scattering of acoustic waves from rough interfaces between elastic media". Ph.D. thesis. Department of Industrial Acoustics, Technical Universi-ty of Denmark, November 1995.
15. J.L. Ogilvy, *"Theory of wave scattering from ransom rough surfaces",* Adam Hilger Publ., 1991.