# 17$^{th}$ SYMPOSIUM ON HYDROACOUSTICS

Jurata May 23-26, 2000


## THEORETICAL INVESTIGATIONS OF THE BEAM PATTERN OF THE PARAMETRIC ACOUSTIC ARRAYS


A. Baranowska

Naval Academy, ul. Smidowicza 71, 81-919 Gdynia, Poland

e-mail: abar@amw.gdynia.pl



*The aim of this paper is theoretical analysis of the beam pattern of the parametric acoustic arrays. The procedure of the beam pattern calculation and an example of numerical calculations were presented. Mathematical model was built on the basis of the KZK equation. A circular transducer was the source of finite amplitude waves.*


### INTRODUCTION

Parametric acoustic wave generation is one of the practical aims of nonlinear acoustics [1, 3 - 5]. Finite amplitude waves during their propagation in water interact. The formation of the different frequency waves, another that primary one (especially sum and difference frequency waves), is the effect of it. Existence of the difference frequency wave has the most important practical application. The generation of it has application in construction of the parametric acoustic arrays. Narrow beam pattern with low frequency for small size of the transducer and high frequencies of primary waves are the most important characteristics of them.

The paper presents an example of the results of theoretical investigations of the beam patterns of the difference frequency waves obtained during finite amplitude waves interaction in water.

### 1. SOLUTION OF THE PROBLEM

Mathematical model of the finite amplitude interaction problem was built assuming that the circular transducer which is the primary waves source is placed in the y-z plane and waves are propagated in the x direction. It means that the x axis corresponds to the sound beam axis (Fig. 1).

Assuming the axial symmetry of the source of the primary waves it is comfortably to solve the problem in cylindrical coordinates (x,r), where $r=\sqrt{y^2+z^2}$.

The wave interaction problem in water is built on the basis of the KZK equation:

$$\frac{\partial}{\partial \tau}\left(\frac{\partial p'}{\partial x}-\frac{\varepsilon}{\rho_o c_o^3}p'\frac{\partial p'}{\partial \tau}-\frac{b}{2\rho_o c_o^3}\frac{\partial^2 p'}{\partial \tau^2}\right)=\frac{c_o}{2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p'}{\partial r}\right) \qquad (1)$$

where p' denotes an acoustic pressure, $\rho_o$ - medium density at rest, $c_o$ – speed of sound, ε - nonlinearity parameter, b – dissipation coefficient of the medium and variable $\tau = t - x / c_o$ is time in the coordinate system fixed in the zero phase of the propagating waves.

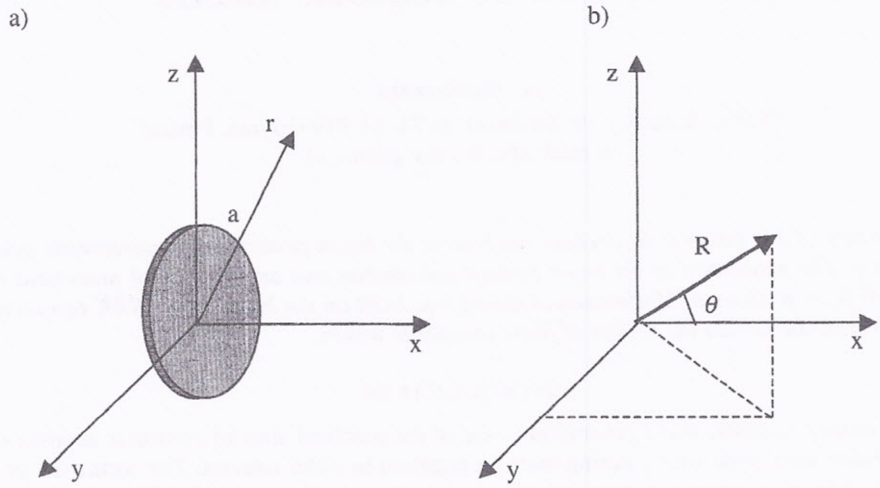


Fig.1. Coordinate systems for the problem: a – pressure changes along the sound beam computation, b – beam pattern computation

Pressure distribution on the transducer is defined as a sum of two harmonic functions with fixed amplitudes $A_1$(r), $A_2$(r) and angular frequencies $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$ respectively:

$$p'(x=0,r,\tau) = A_1(r)\sin\omega_1\tau + A_2(r)\sin\omega_2\tau \qquad (2)$$

for $r \le a$ (a – transducer radius) and $p'(x=0,r,\tau)=0$ for $r > a$.

To solve the KZK equation the finite difference method is used. The pressure changes along the sound beam are obtained after computer calculations. Spectrum changes are calculated using the Fast Fourier Transform (FFT) algorithm. Next, the beam pattern for fixed frequency wave is computed as a ratio:

$$D(\theta) = \frac{p_f(R,\theta)}{p_f(R,\theta=0)} \tag{3}$$

where $p_f(R,\theta)$ is a value of pressure of the fixed spectrum component at the distance R from the source and an angle θ is measured from the x axis (Fig. 1).

## 2. NUMERICAL INVESTIGATIONS

The numerical investigations were made assuming that the finite amplitude waves are produced by circular transducer with radius a=25 mm. So the waves with frequency $f_1$=1.2 MHz, $f_2$=1 MHz respectively and equal amplitudes are propagated in water.
Figure 2 presents normalized difference frequency pressure as a function of distance from the source. The curve number 1 shows pressure changes on the beam axis and the curve number 2 the pressure changes at distance r=a from the beam axis.
Figure 3 shows the beam pattern of the difference frequency wave as a function of angle. Calculations were made using formula (3) for R=0.5 m. Exact analysis of the beam patterns which were obtained for different distances R shows that the beam pattern aperture of difference frequency wave is equal about $1^o$.

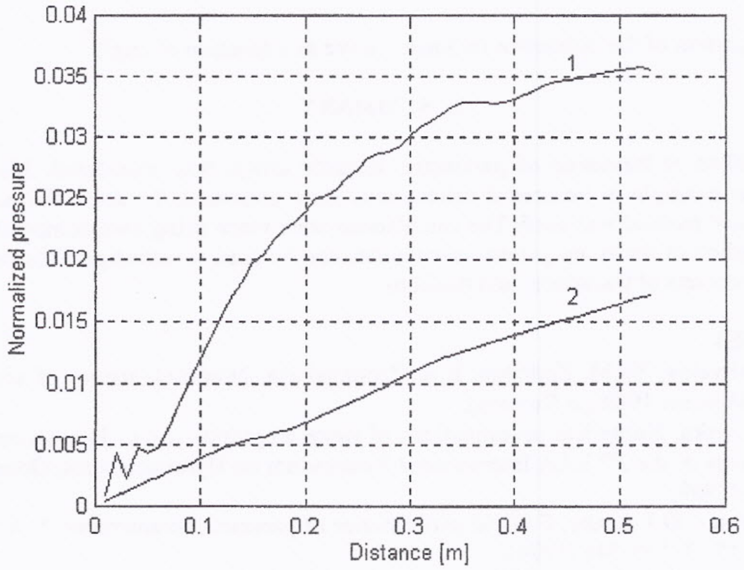


Fig.2. Normalized difference frequency pressure as a function of distance from the source: 1 – r=0, 2 – r=a

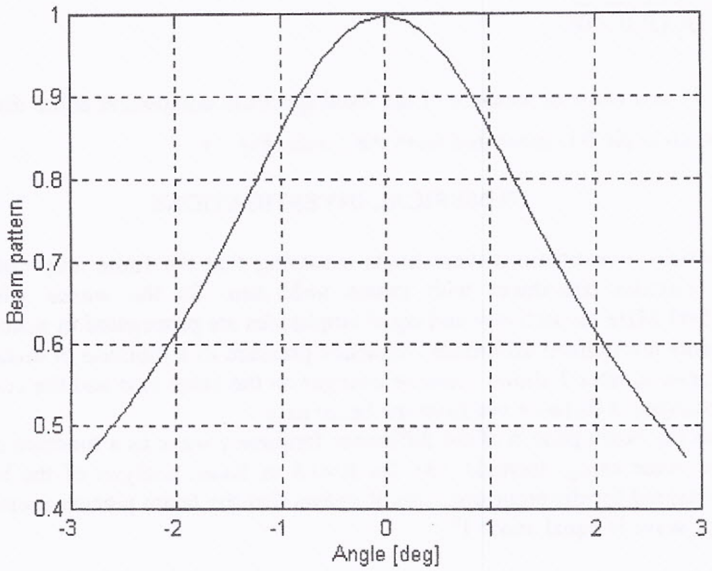


Fig.3 Beam pattern of the difference frequency wave as a function of angle

## 3. SUMMARY

The problem of formation of parametric acoustic arrays was considered. Mathematical model and an example of numerical calculations were presented. To solve the problem the finite difference method was used. The calculations were made using own computer program. Proposed method of the beam pattern calculations can be used to investigate the problem for different parameters of transducer and medium.

REFERENCES

1. N.S. Bakhvalov, Ya.M. Zhileikin, E.A. Zabolotskaya, Nonlinear theory of sound beam, Nauka, Moscow 1982 (in Russian).
2. A.Baranowska, Numerical investigations of finite amplitude waves interaction in water, Proceedings of the 2nd EAA International Symposium on Hydroacoustics, Gdansk-Jurata, 247-250 (1999).
3. H.O. Berktay, D.J. Leahy, Farfield performance of parametric transmitter, J. Acoust. Soc. Am., V. 55, 3, 539-546 (1974).
4. V.A. Voronin, S.P. Tarasov, V.I. Timoshenko, Application of parametric arrays in Marine research, Sov. Phys. Acoust. V. 41, 3, 405-409 (1995) (in Russian).
5. L.K. Zarembo, V.I. Timoshenko, Nonlinear acoustics, Moscow University Press, Moscow 1984 (in Russian).