

THE IMPACT OF ROTATIONAL SPEED SYNCHRONISATION OF EXCITERS ON THE DYNAMICS OF A VIBRATING SCREEN SIEVE

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Summary

In the paper the results of examinations and numerical analysis of the vibrating screen sieve dynamics are presented. The working movement of the sieve is forced by one or two inertial exciters. They should display a self-synchronization feature. Since such self-synchronisation effect is not always achieved, therefore the sieve vibration directions do not coincide with directions of loads transferred by the vibration isolators. In such case the frequencies of forced vibrations may be close to free vibration frequency for a given modal form and the system may work in resonance conditions.

Keywords: free vibration, resonance frequencies, self-synchronisation of excitation

WPLYW SYNCHRONIZACJI PRĘDKOŚCI OBROTOWEJ WZBUDNIKÓW NA DYNAMIKĘ SITA PRZESIEWACZA WIBRACYJNEGO

Streszczenie

W pracy przedstawione zostały wyniki badań oraz analiza numeryczna dynamiki sita przesiewacza wibracyjnego. Tego rodzaju maszyna modelowana jest zazwyczaj jako układ dyskretny o jednym stopniu swobody. Jego drgania mają charakter nadrezonansowy. Praktyka pokazuje jednak, że szereg efektów występujących podczas pracy przesiewacza można wytłumaczyć jedynie po przyjęciu modelu wielorezonansowego. Najlepsze odwzorowanie obiektu rzeczywistego na model uzyskuje się wówczas, gdy sito jest traktowane jako układ ciągły. Rozważany przypadek jest tego przykładem. Pokazuje przy tym, że nieprawidłowo dobrane cechy konstrukcyjne znacząco pogorszą własności eksploatacyjne urządzenia. Ruch roboczy sita wymuszają jeden lub dwa wzbudniki bezwładnościowe. Powinny one posiadać własność samosynchronizacji. Efekt ten nie zawsze jest osiągnięty, co powoduje, że kierunki drgań sita nie są zgodne z kierunkami obciążeń przenoszonych przez wibroizolatory. Częstotliwości drgań wymuszonych mogą być wówczas zbliżone do częstotliwości drgań własnych dla określonej postaci modalnej i układ pracuje w warunkach rezonansu.

Słowa kluczowe: drgania własne, częstotliwości rezonansowe, samosynchronizacja wzbudzenia

1. WSTĘP

Vibrating screens are machines used in many industries, particularly in the mining industry, agricultural and food processing industry as well as the construction materials industry. The working motion of a vibrating screen is vibration of its main structural element, i.e. the sieve, imposes many strength-related requirements on the entire machine. The design of the sieve should be compact and resistant to high loads varying with time. Optimisation of functional properties of the vibrating screen comes down to achieving correct proportions between its overall dimensions and inertia, which limits the power of the machine driving mechanisms. The sieve is driven by exciters consisting of unbalanced discs mounted on the motor shaft and being the source of centrifugal force which enforces oscillations of the sieve. The vibrating isolators which limit the magnitude of force transmitted to the supporting structure and simultaneously ensure correct conditions if screening process, constitute important elements of

the sieve foundation. To achieve suitable output of the machine, the velocity of sieve vibration must be properly adjusted to the type of the screened material allowing smooth flow of it through the sieve.

The most commonly used vibration isolators are spring systems. Cylindrical springs are used in machines where the sieve vibrates mainly in vertical direction. Suspension of the sieve on flat springs allows for its axial movements. The vibration isolators with elastic inserts and three axes of relative movement of the moving elements allows the sieve to move in two directions and therefore to obtain better screening characteristics. The vibrating screen supporting structure should be stiff and stable in order to obtain conditions in which the dynamics of the machine is determined only by the excitation-vibrating isolation-sieve inertia system, since it should be remembered that vibrating screens are large machines and their sieves are sometimes mounted at significantly high level. Free vibration of the sieve founded on vibrating isolators with low stiffness along the

working direction are characterised by low frequencies and the forcing frequencies reach the values of a dozen Hz; in this way the design engineers are striving to eliminate resonance in the system at the working speed of the machine. However, the sieve is not a stiff body and also has its own free vibrating frequencies, usually quite high. In turn, the vibration isolators feature high stiffness along the direction other than the working direction of the sieve. As a result, beyond the free vibration frequencies falling within the low-frequency range, the vibrating screen may also perform free vibrations with higher frequencies, close to the forcing frequency. Then the resonance phenomenon occurs along the direction other than the direction in which the vibrating screen works; and this resonance has an adverse effect on both the strength of machine elements and the dynamic characteristics of the screening process.

In the past, many scientists were interested in conducting research concerning vibrating screens. In Poland, these issues were widely studied at the AGH University of Science and Technology [1-3]. The studies were conducted with relation to the nature of vibration of industrial machines and creation of their numerical models. In modern over-resonance screens the phenomenon of self-synchronisation of inertial vibrators is employed. However, it was found that in case of improper self-synchronisation the phase shift between discs imbalance vectors occurs, leading to differentiation of vibration parameters in various points of the screen structure. The results of spring-supported vibrating screens operation analysis prove that in case of short-term interactions appearing during transition of the system through a resonance zone the magnitude of dynamic interactions is several times higher than during stabilized movement. For stability of sieve movement the random-type interactions occurring in the vibrators' drives and suspension systems are significant [4].

2. NUMERICAL ANALYSIS OF THE DYNAMICS OF A VIBRATING SCREEN SIEVE

In the agricultural products processing industry a vibrating screen manufactured by a Spanish company, Urtasun, is employed. Its design is shown on Fig.1. The main component of the screen is the sieve with the mass of about 3500 kg. Two inertial vibrators are fixed to a transverse beam located in the upper part of the sieve at an angle of 45° with respect to the horizontal plane. Since the original solution of sieve foundation was not adapted to the value of the oscillation exciting force necessary for achieving suitable efficiency of the screening process and therefore breakages and fractures in both the sieve and supporting elements have occurred, it was necessary to redesign the

foundation in order to meet the requirements relating to its strength and stability (Fig.1). The columns which were originally made of closed stiffened profiles were replaced with posts made of thick-wall tubes and instead of resting directly on the floor the screen was positioned on specially built foundation made of steel-reinforced concrete. As a result, the structure vibration velocity was reduced from several dozen down to just a few mm s^{-1} [5].



Fig.1. The vibrating screen after modernisation.

1. Sieve; 2. Exciters; 3. Supporting structure;
4. Belt conveyors; 5. Foundation

The sieve of the vibrating screen rests on vibration isolators produced by a Spanish company, ROSTA (Fig.2). These isolators have plastic inserts which play a role of elastic and damping elements and are selected depending on the weight of the oscillating mass. Low stiffness of the insert ensures low frequency of sieve free vibrations in $0y$ and $0x$ directions. This vibration isolator features low flexibility along the $0z$ direction, i.e. the direction perpendicular to its plane of deflection ($x0y$ plane). Thus, the frequency of free vibrations in this direction is high.

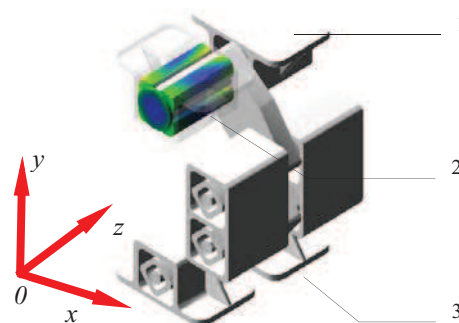


Fig.2. Vibration isolator with elastic insert.

1. Upper arm, 2. Elastic inserts, 3. Lower arm

Usually, the manufacturer declares the stiffness and damping characteristics of the vibration isolator. However, it applies only to these directions along which the isolator's deflections are expected. Other parameters should be determined either

experimentally or based on the analysis of the device model vibrations by comparing the measured values of free vibration frequencies with those obtained from the response analysis of a numerically simulated system.

Although simulation of a vibrating isolator with elastic inserts made of plastic material with high deformability turned out to be a quite complicated issue, the results obtained were very coincident with the results of examinations. The complexity of the problem was that the interactions between the inserts and surfaces of sockets are of contact character. To create a correct representation of the contact it is necessary to know the parameters of physical phenomena occurring within the zone of mutual contacts of the elements. The assumed stiffness of inserts corresponds to the actual values because the calculated static characteristic curve for the model of the vibrating isolator is close to such curve determined during examinations. The difference is that the calculated stiffness/deflection curve is almost linear for deflections within the range of up to 10 mm, while in this range the actual curve shows the change in vibrating isolator stiffness of more than 100 N mm^{-1} . Of course, the flatter the vibrating isolator stiffness characteristic curve is, the easier the selection of the vibrating isolator is.

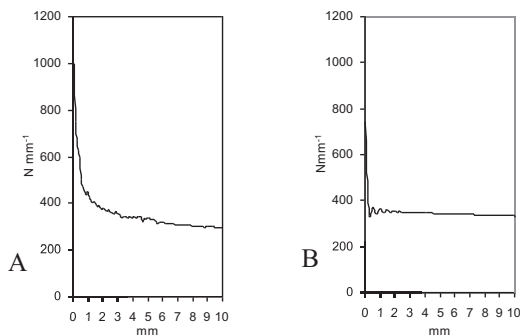


Fig.3. The stiffness / deformation characteristic curve of a vibrating isolator determined: (A) experimentally, (B) based on a numerical model

The model of inertial exciter produced by the ROSTA Company is shown in Fig.4A. On each end of the motor shaft an asymmetrical disc is fixed. By changing the angular displacement between these two discs it is possible to adjust the value of centrifugal force within the range of up to 64 kN. In the original design developed by the vibrating screen manufacturer the exciters are placed at an angle of 45° relative to the material feed plane. At first glance such arrangement of exciters may be surprising, since if the rotation of unbalance vectors for both discs is not synchronised, the centrifugal force can excite oscillations of the sieve in the direction perpendicular to its axis. As it was indicated above, vibrations in this direction are not suppressed by the vibration isolator. The movement

trajectory of the sieve gravity centre should have the form of an ellipse and in special cases – a circle lying in the vertical plane rather than the form of a three-dimensional curve. After approximately three seconds the exciter reaches the maximum rotational speed of almost 1000 rpm. The rise of force caused by unbalance of discs with time is illustrated in Fig.4B.

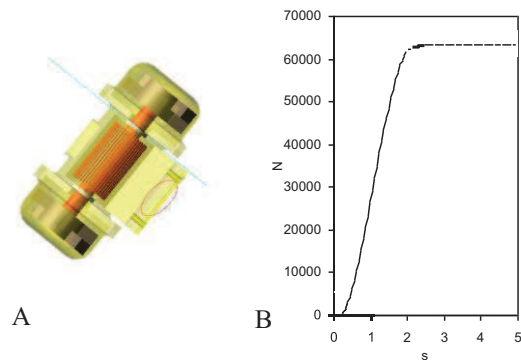


Fig.4. Numerical model (A) and the run-up time/force characteristic curve (B) of inertial exciter

In the arrangement of exciters as shown in Fig. 5 the projections of forces caused by disc unbalance on the straight line orthogonal to the vertical plane going through the sieve axis will not compensate in case of different values of resistance to the motion of their rotors. This is disadvantageous in the aspect of the vibrating isolators' design, particularly in cases where resonance oscillations occur in this direction. Such problem appeared in the vibrating screen being examined. The velocity of vibration in forbidden direction at the forcing frequency of 16.5 Hz reached the value of almost 100 mm s^{-1} .

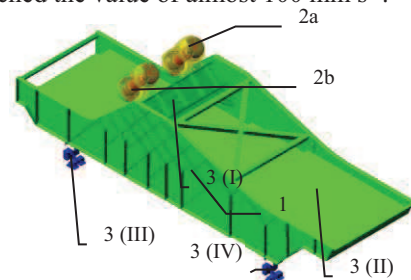


Fig.5. The model of vibrating screen simplified to the system of vibrating mass – elastic and suppressing elements – excitation.

1. Sieve, 2. Exciters, 3. Load bearing columns, 4. Foundation

To identify the reasons for rough running of the sieve it was necessary to perform detailed examinations. It was assumed that in order to take special countermeasures aimed at limiting the vibration amplitudes in a reasonable way, the numerical analysis of the system model should be performed first. The numerical model of the

vibrating sieve, including its main components, is shown in Fig. 5.

The presented method of vibrating screen modelling based on the dynamics of multi-body systems is a development of the method employed in the paper [6], where the solution of sieve motion equations derived on the ground of the calculus of variations was presented.

3. EXAMINATIONS OF THE SIEVE VIBRATION CHARACTER

The sieve dynamics was examined on the basis of modal analysis. Since the system vibrations were originally supposed to be over-resonant, the system response characteristics can be determined using a short-term Fourier transform of velocity vs. time waveforms of vibrations excited by the force generated by unbalance of vibrators' discs during run-up and run-down phases. Examinations were carried out during simultaneous operation of both exciters and during individual operation of each exciter independently.

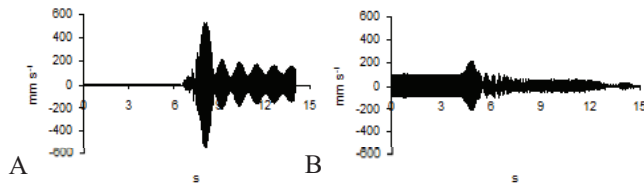


Fig.6. Velocity vs. time waveforms for sieve vibration in $0z$ direction during run-up (A) and run-down (B) phases with both exciters active

During operation of two exciters the velocity vs. time waveforms for sieve oscillations along the $0z$ direction during run-up and run-down phases are presented in Fig. 6A and Fig. 6B. In each case the dynamics of the system is different. During the run-up phase the transition through resonance zone at higher angular frequencies occurs with higher vibration velocity amplitudes than during the run-down phase. It occurs near the frequency of 16 Hz (see Fig. 6B). The resonance zone at low frequencies is better visible during the sieve run-down period. The sieve run-up is accompanied by a beat phenomenon caused by the fact that during the run-up phase the rotational speeds of exciters are slightly different.

In the image of short-time Fourier transform of vibration velocity (Fig.7) the area of resonance vibrations occurrence within the range of 15.5 Hz-16 Hz (Fig.7) is clearly visible. Also, the effect connected with differing resistance to motion of exciters can be noted. The vibrator marked with the symbol 2a in Fig.5 features higher resistance to motion; therefore the time to stop for this exciter is shorter than for the 2b exciter.

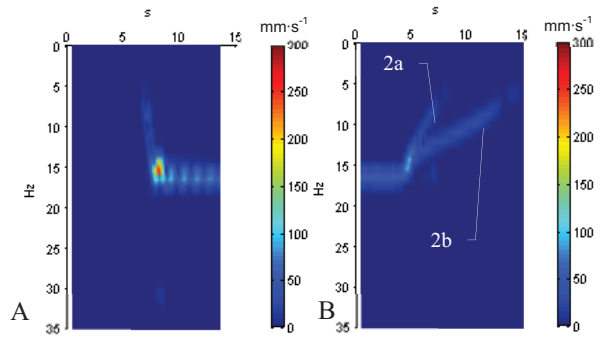


Fig.7. Image of short-time Fourier transform of time waveforms of sieve vibration velocity during the run-up (A) and run-down (B) phases of the system with two exciters in operation

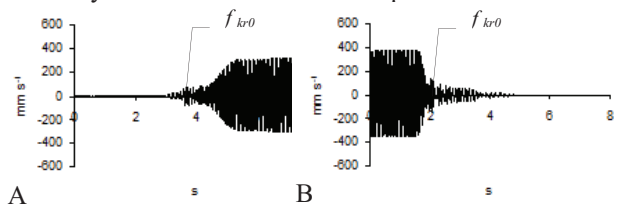


Fig.8. Velocity vs. time waveforms for sieve vibration in $0z$ direction during the run-up (A) and run-down (B) phases with exciter 2a in operation

If only exciter 2a is in operation the beat effect does not occur (Fig.8). The amplitudes of sieve vibration velocity are much higher than during simultaneous operation of both exciters. This is understandable, since the angular velocities of exciters have opposite senses and $0z$ components of forces generated by disc unbalance should mutually compensate. When only one exciter is in operation, such condition cannot occur.

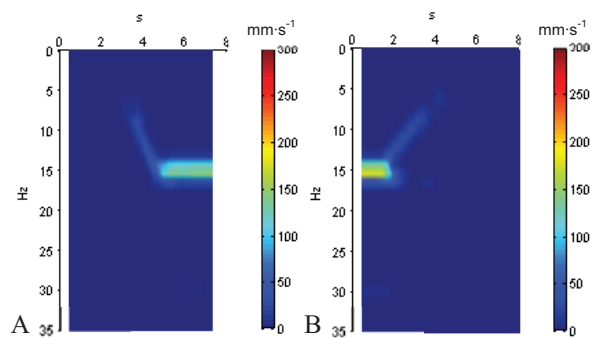


Fig.9. Image of short-time Fourier transform of time waveforms of sieve vibration velocity during the run-up (A) and run-down (B) phases of the system with exciter 2a in operation

In such circumstances the amplitude of sieve vibration is very high but as a result of intense damping in the system the resonance does not occur because the rotational frequency of the exciter is lower than free vibration frequency of the sieve (Fig.9).

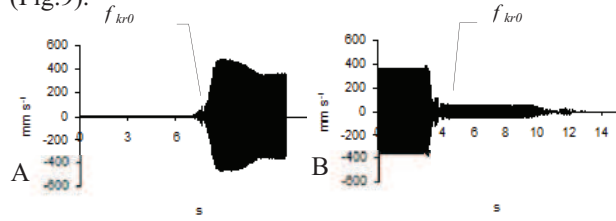


Fig.10. Velocity vs. time waveforms for sieve vibration in $0z$ direction during the run-up (A) and run-down (B) phases with exciter 2b in operation

Driving the sieve using only exciter 2b leads to very high amplitude values of its vibration velocity (Fig.10). The occurrence of resonance vibration is clearly visible, particularly during the sieve run-up phase.

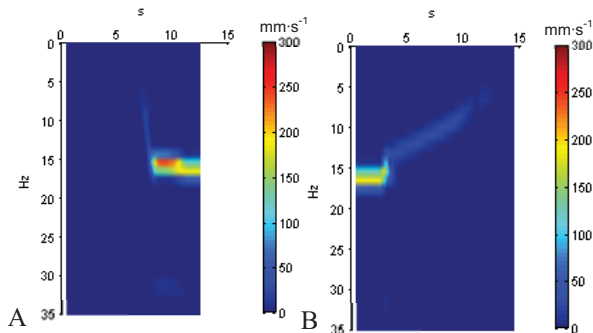


Fig.11. Image of short-time Fourier transform of time waveforms of sieve vibration velocity during the run-up (A) and run-down (B) phases of the system with exciter 2b in operation

The smooth increase of vibration velocity amplitude during the sieve run-up phase confirms the presence of high damping in the system along the $0z$ direction (Fig.11).

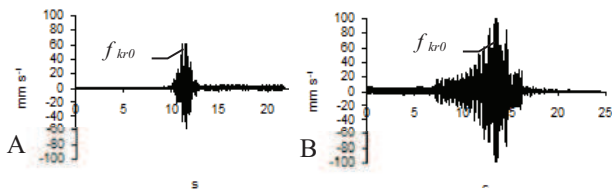


Fig.12. Velocity vs. time waveforms for sieve vibration in $0z$ direction during the sieve run-up (A) and run-down (B) phases with both exciters running, for the excitation frequency of 13.5 Hz

The amplitude of sieve vibration velocity in forbidden direction can be effectively reduced by pulling the system out of resonance. To this end,

using the frequency converters, the angular velocities of exciters were reduced down to the value of about 85 s^{-1} , with simultaneous increase of the exciters' disc unbalance to the maximum value at which the centrifugal force has the absolute value of 64 kN.

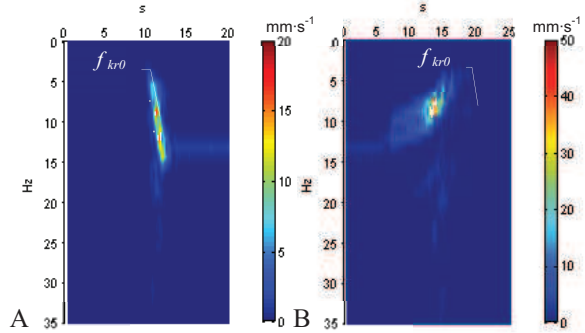


Fig.13. Image of short-time Fourier transform of time waveforms of sieve vibration velocity during the run-up (A) and run-down (B) phases with both exciters in operation, for the excitation frequency of 13.5 Hz

As a result of forcing frequency reduction, after passing through the critical frequency f_{kr0} the amplitude of vibration velocity was reduced to a few $\text{mm}\cdot\text{s}^{-1}$ (Fig.12-13). Since the amplitudes of vibration velocity in the directions $0y$ and $0x$ reach the values of almost $160 \text{ mm}\cdot\text{s}^{-1}$, the screen operates smoothly with assumed output (Fig.14).

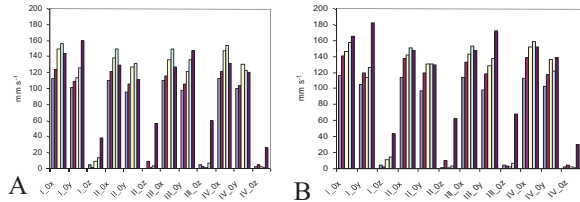


Fig.14. Values of amplitudes (A) and RMS (B) of sieve vibration velocity in directions $0x$, $0y$, $0z$ at excitation frequencies of 10 Hz, 11.5 Hz, 12.5 Hz, 13.25 Hz and 16.5 Hz, respectively

At excitation frequency of 16.5 Hz the level of sieve vibration in the $0z$ direction increases abruptly. The Roman numerals I, II, III, IV on Fig. 14 indicate the numbers of vibrating isolators in the system presented on Fig.5.

4. MODELLING SIEVE VIBRATIONS AND SCREENING PROCESS

The analysis of sieve vibration using the dynamics of multi-body systems allows considering the screen as a stiff body supported by elastic-damping elements. The mass of 3500 kg was assigned to the body simulating the sieve. The values of the stiffness of vibrating isolators determined by numerical modeling were verified experimentally. The vibrators attached to the sieve model were considered to be a set of stiff bodies linked by appropriate constraints. It was assumed that the

vibrators operate in self-synchronisation conditions and the vectors of angular velocities of rotors have opposite senses. The force generated by the disc unbalance was established at the maximum level possible for ROSTA vibrators, i.e. at the level of 2x64 kN at rotational frequency of 16.6 s⁻¹.

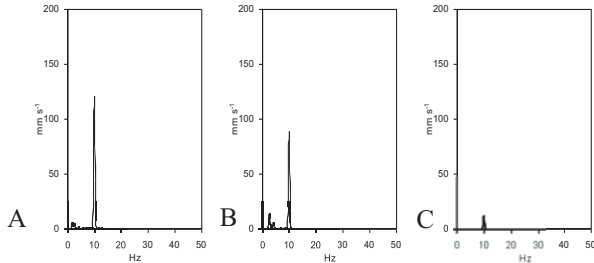


Fig.15. Amplitude-frequency characteristic curves obtained on the basis of vibration velocity vs. time waveforms for the frequency of 10 Hz in the directions: (A) - $0x$, (B) - $0y$, (C) - $0z$

Fig.15 shows the amplitude-frequency characteristic curves of sieve vibration velocity, determined on the basis of numerical calculations of time waveform for excitation frequency of 10 Hz. The vibration velocity amplitudes at this frequency are close to the measured values presented in Fig.14A as the first left stripe in each bar. The amplitudes of vibration velocity in $0z$ direction for the frequency of 16 Hz are lower than the actual values. This means that the assumed value of damping is lower than the value of damping which actually exists in the system.

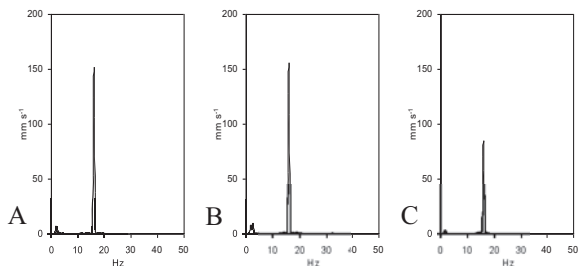


Fig.16. Amplitude-frequency characteristic curves obtained on the basis of vibration velocity vs. time waveforms for the frequency of 16 Hz in the directions: (A) - $0x$, (B) - $0y$, (C) - $0z$

By comparing the sieve vibration velocity determined for a given forcing frequency with values of vibration velocity amplitudes obtained from numerical calculations it is possible to evaluate the conformity of physical properties of the model with the real object. As these values do not differ significantly, it can be stated that such compliance is satisfactory.

When considering the potential for reduction of sieve excitation frequency it is important to take into account the possible change in screening process efficiency, because this efficiency should not be less than the assumed one. Otherwise, the machine would

be a weak link in the technological process. The best way to learn about the extent of the possible change in the efficiency is to simulate the screening process for various sieve excitation frequencies; however, it is an issue to simulate the loose feed material. In the majority of studies two approaches to this issue are employed – the first is to simulate the feed material as a set of particles [7] and the other is to simulate it as layers [8].

The simulation method applied here can be described as a simplified particle-like approach. The course of the screening process and its efficiency was analysed on the basis of the speed of transporting the body with the mass of 45 g along the screen. The mass of 45 g was assumed to be the value corresponding to the average mass of a vegetable being screened. Thus, the interactions between particles of feed material are neglected while the relations between the particle and the sieve are the focus of attention.

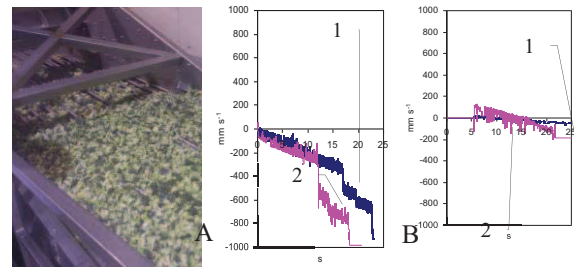


Fig.17. Variations in the feeding speed of the screened material in the $0x$ (A) and $0z$ (B) directions for excitation frequency of 10 Hz (1) and 13 Hz (2)

It can be noted that at the sieve vibration frequency of 10 Hz the movement of raw material is smoother than at the frequency of 13 Hz. The velocity fluctuations in the direction perpendicular to the sieve axis are smaller (Fig.17B), similarly as for material feed velocity along the sieve axis. In consequence, the efficiency of the screening process is also worse.

The sieve is a thin-walled structure. From the point of view of its strength the value of stress in cross-sections of steel plates forming the sieve frame is important. In the examined structure many material discontinuities were present, particularly near the joints. It can be assumed that it was the effect of both resonance vibrations in the $0z$ direction at the exciters' rotational frequency of 16.5 Hz and asynchronous operation of exciters.

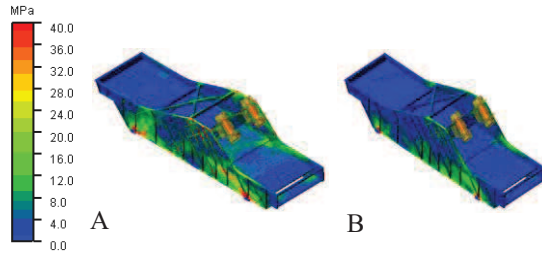


Fig. 18. Stress maps in sieve framework plates at excitation with the frequency of 10 Hz for asynchronous (A) and synchronous (B) operation of exciters

As it can be seen from stress map shown in Fig.18, in extreme asynchronous state, i.e. when only one exciter is operating, the reduced stresses in some areas of the sieve structure exceed the value of 40 MPa, while during synchronous operation such level of stresses occurs only in points of fixing the vibrating isolators to the sieve frame. It is understandable that with the increase of excitation frequency the values of stress will rise. With the stress cycle varying with time the above can lead to damage of the sieve structure.

5. CONCLUSIONS

Oscillations of numerous machines founded on vibration isolators can be analysed by considering these machines as vibrating systems composed of stiff bodies supported with elastic-damping elements featuring specified stiffness and damping values. Usually, the free vibrations frequency of the structure is sufficiently high to avoid excitation of such vibrations by the moving elements of the machine. The properly selected disc-type vibration isolator has the ability of damping vibrations in many directions. For vibrating screens, where for proper operation of the sieve its gravity centre should draw an ellipse or circle in vertical plane, the specially designed vibrating isolators are used which allow for substantial displacements of the element founded on them. The product of the ROSTA Company is one of examples of such vibration isolators. It features low stiffness in the plane parallel to the plane of gravity centre trajectory and high stiffness in the direction perpendicular to this plane. In consequence, the sieve free vibration frequency in the direction of high stiffness of a support is also high.

Operation in conditions of free vibrations adversely affects both the strength of sieve's structural elements and the efficiency of screening process. The system can be pulled out of resonance by attaching additional mass to it. However, such countermeasure is of limited effectiveness, since attaching mass as high as 500 kg does not lead to noticeable reduction of free vibration of a sieve, as it was confirmed during examination of the machine during its operation. In the case being considered it was observed that asynchronous operation of exciters

is also a source of high-amplitude vibrations of the sieve. The sieve driving mechanism was designed so that the exciters' discs with the same unbalance should rotate in opposite directions synchronically, starting from the lowest position of the gravity centre. The exciters may operate synchronically provided that the values of resistance to motion of their rotors are identical. As it is practically impossible, the vectors of the disc unbalance in the Oz direction do not mutually compensate. At the excitation frequency close to the free vibration frequency very high amplitudes of vibration occur. Upon reduction of the excitation frequency, this phenomenon is limited.

Numerical simulations lead to the conclusion that an increase in the excitation frequency causes an increase of material travel speed but also enhances the effect of feed material fluctuation in the direction perpendicular to the sieve axis, which obstructs the movement of material being fed. Therefore, with free vibrations in the direction perpendicular to the sieve axis the efficiency of the screening process is limited and also the value of stress in the sieve frame cross-sections increases. However, the main reason for sieve frame breakages and fractures is damage of one of the exciters. Such damage has occurred several times during operation of the vibrating screen, causing damage of the sieve frame in many points and therefore, reduction of its stiffness. In consequence, the free vibration frequency of the sieve decreased near the excitation frequency and the system returned to work in resonance conditions again. Only when the excitation frequency was significantly reduced, the amplitude of the sieve vibration velocity was lowered.

Self-synchronisation of exciters may occur only when the values of their rotors' resistance to motion are identical. However, practice shows that this is very difficult to achieve. As a satisfactory solution for the concurrency of the exciters forced synchronisation can be considered. To this end, the frequency converters used in exciters' drives should be equipped with a controller for the process of equalizing the phase angle between rotating vector of the disc unbalance. The system for continuous measurement of relative angular displacement between the unbalance vectors must be equipped with phase markers. When the measurement of this angle shows that it is not equal to zero, the follow-up system commands the converter which drives the exciter with lower rotational speed to increase this speed until the angular difference between the disc unbalance vectors is eliminated.

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