

# APPLICATION OF KINEMATIC MODELS OF GEODESIC NETWORK FOR SURVEYING SETTLEMENT DYNAMICS OF A SUSPENSION BRIDGE DECK

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## ABSTRACT

Specific character of the engineering geodesy is based, among other things, upon the complexity and variety of surveying methods used for performing particular tasks. They should be adjusted to the specific conditions of the surveyed object and its' surroundings.

A choice of the adequate methods of survey results handling is equally important for the correct description of the researched phenomenon.

This paper presents a concept for measuring suspension bridge span deflection induced by atmospheric conditions, in particular suspension cable length changes caused by insolation. The survey was performed using modified trigonometric levelling, incorporating monitoring of temporary refraction factor changes considered during bridge span dynamics calculations. Movement parameters of the selected points were calculated using a kinematic model of geodesic network.

Assessment of approximate movement parameters indispensable for the method application was made with an assumption of a static model of realised observations.

## 1. OBJECT DEFORMATION DYNAMICS

Load bearing deck is very sensitive to external factors, which are the cause of various, predominantly elastic deformations. They produce a wide range of vibration frequencies. Fast period vibrations of high frequency and small amplitude are caused mostly by traffic or vibrations of suspension bridge cables induced by gusts of wind. Long-period vibrations of lower frequency and greater amplitude are influenced by the wind or movement of heavy vehicles that appear irregularly on the bridge. Completely different in character is the impact of temperature changes, which in the case of sunny days might cause a diurnal bridge deck oscillations. This phenomenon is triggered by the change in length of the cables. It can be stated that the periods of vibration and amplitudes for these events will meet the condition:

$$\delta t_1 < \delta t_2 < \delta t_3 ; \delta h_1 < \delta h_2 < \delta h_3$$

In reality we always deal with the sum of these phenomena Fig. 1 presents the survey results obtained from three control points on the Siekierkowski bridge deck located furthest away from the pylons, therefore structurally vulnerable to vibrations caused by various external factors.

Interpretation of the graphs is as follows:

1. Each point was observed for approx. 14 hours
2. For each point 45 observations were made (points on the graph show momentary high point) and correct v deviation from the trend line is shown on Figure 2.

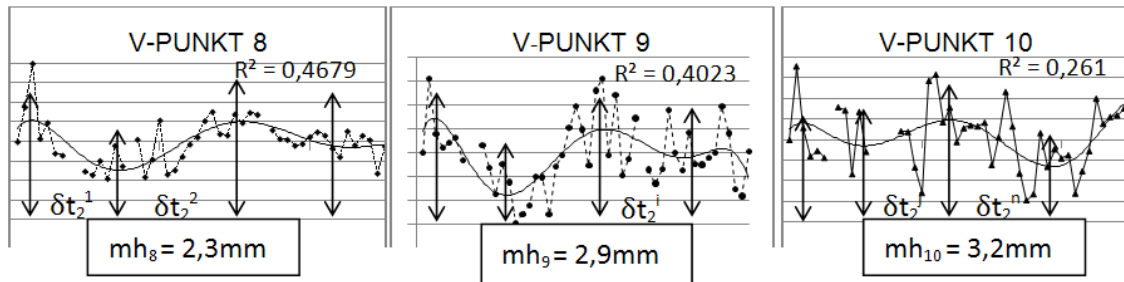


Fig. 1. Graphical presentation of measurements results.

1. It is difficult to interpret registered instantaneous point locations, as the vibrations of high frequency. Observation moments were random therefore did not register temporary maximum. However, they reflect the nature of these vibrations burdened with an additional survey accuracy error. Of course, the smaller the intervals between the moments of the registration status of the object that the results will be closer to reality.
2. The  $\delta t_1$  oscillation frequency is similar on each of the graphs, but this is due to the frequency of observation. Clearly, the amplitude of vibration  $\delta h_1$  and is greater for points further away from the pylon.
3. Combined on charts  $mhi$  values are calculated for the results of the measurement of deviations from the trend line shown in Figure 3.
4. Trend lines shown on graphs suggest, that bridge deck indicates also oscillations of much lower frequency (drawings show that  $\delta t_{2i}$  is 4 hours approx.) and smaller amplitude than momentary vibrations.
5. Relatively lower indicators  $R^2$  for individual trend lines suggest that results interpretation presented in point 5 need not to be true. It's verification would require additional control measurements.

## 2. SURVEY METHOD

The object of the experimental survey was an attempt to observe the diurnal movements of the Siekierkowski Bridge in Warsaw. Described below partial results of the survey indicate difficulties encountered during the task.

Survey was executed using trigonometrical levelling. Diagram 2 show locations of the control points. Measurement position was located on the existing base post, which survived since the bridge construction. All measurements were executed in July 2010, with Leica 1202 instrument.

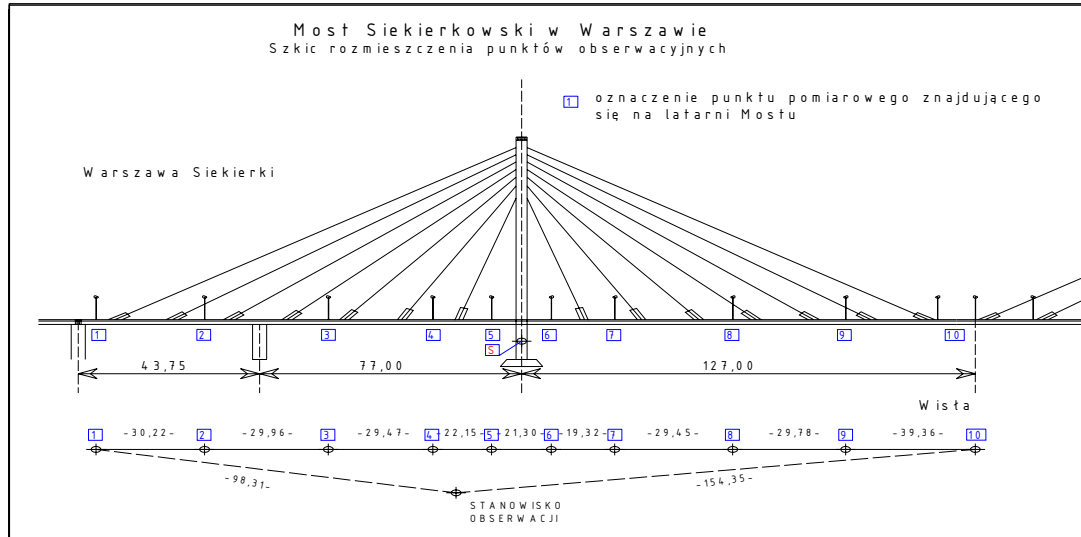


Fig. 2. Control points locations diagram.

From 5am until 7pm 8 series of measurements were performed in 2h intervals. In each of the series 10 control points were observed in 3 rounds. Survey was performed on a warm and sunny day with recorded temperatures ranging from 18°C in the morning to 38°C in the early afternoon. For each of the observations the exact moment of measurement was recorded. Average duration of each of the measurement series was 45 min. approximately.

### 3. STATIC MODEL

For the first stage of the measurement results evaluation it was assumed that the approximate solution will be made using static model. Adoption of such a model results that during one measurement cycle the motion of the control point was considered as insignificant. Under this assumption to calculate control points height, an average vertical angle from 3 measurement rounds was used.

$$\alpha_{n\acute{s}r}^k = \frac{\sum \alpha_n^{L1k} + \sum \alpha_n^{P1k} + \sum \alpha_n^{L2k} + \sum \alpha_n^{P2k} + \sum \alpha_n^{L3k} + \sum \alpha_n^{P3k}}{12}$$

Height of the n point in the k series equals:

$$H_n^k = H_0 + d_n \times \sin \alpha_{n\acute{s}r}^k,$$

where  $H_0$  is height of the rotation axis of the instrument telescope

Table 1. Deflection/subsidence values calculated using static model

9 July 2010 – Deflections [mm]										
Godz.\pkt.	1	2	3	4	5	6	7	8	9	10
5:00	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
7:00	0,5	-1,3	0,1	0,2	0,9	-0,4	-1,6	-5,7	-10,7	-21,3
9:00	0,3	-1,5	0,3	0,3	2,2	-0,3	-7,1	-14,2	-20,8	-35,7
11:00	-2,6	-4,7	-2,3	-0,8	3,5	-0,3	-8,0	-21,7	-33,9	-42,9
13:00	-4,7	-6,2	-4,2	-2,3	4,2	0,1	-9,5	-29,2	-39,8	-47,0
15:00	-4,4	-7,1	-5,4	-2,9	5,3	0,1	-10,7	-27,8	-41,6	-49,6
17:00	-5,7	-6,7	-6,6	-3,7	4,6	-0,3	-13,0	-27,3	-38,3	-45,8
19:00	-3,3	-6,2	-5,7	-2,6	5,3	0,3	-9,8	-25,2	-39,1	-53,7

Table 1 summarises the values of the subsidence recorded at different control points, whereas Figure 3 shows a graphical interpretation of bridge deck deflection, assuming a static model for task solution.

Presented results of the suspension bridge deck subsidence indicate a range of interesting phenomena, which were observed. Detailed interpretation of the results is not a subject of this publication. More specific assessment of behaviour of the test structure can be found in (Kornatowska, Miksa 2011).

Presented graphs clearly show that the bridge deck settles with increasing temperature throughout the day and points further away from the pylon demonstrate increased subsidence. Graphs of point's height changes were supplemented with trend lines (polynomial 2-degree) and indicate that the movement of points is not uniform.

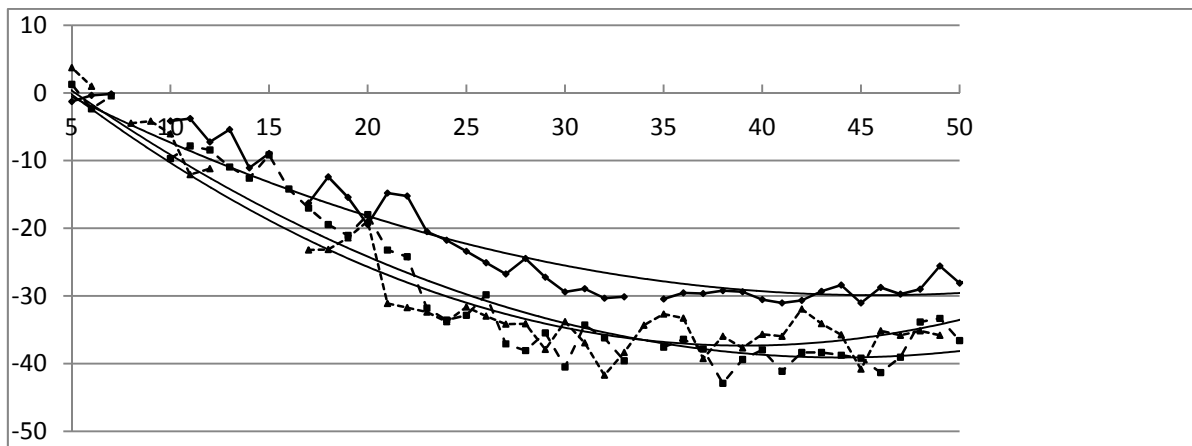


Fig. 3. Height alteration of control points 8, 9 and 10.

In order to create the kinematic model it was assumed that the movement of points is uniformly accelerated motion.

Such defined static model of calculations allow for a simple assessment of the accuracy. Average error of the angle designation can be calculated on the basis of the deviations from the main value, which is in line with the adopted network model.

$$m \propto_{n \dot{s}r}^k = \sqrt{\frac{[vv]}{n-1}}$$

Such a determined value  $m \propto_{n \dot{s}r}^k$  in a given measurement cycle is affected not only by the accuracy of the measurement as a function of the instrument used and surveyor skills, but also the previously mentioned factors that cause the vibrations of the surveyed structure.

While building a model for this task, it can be assumed that in successive cycles of measurements influence of external conditions on the given point will be similar, of course if there is no significant change to these conditions. In this case, the average error of the designation of the vertical angle can be determined on the basis of measurements in all cycles.

$$q = \sum_{i=1}^9 \frac{[vv]_i}{n-1} ; m \propto_n = \sqrt{\frac{q}{8}}$$



$$vH_k^{t_2} + H^{t_2} = H_k^0 + dH_k + (t_2 - t_0) \times V_k + \frac{(t_2 - t_0)^2}{2} a_k$$

$$vH_k^{t_i} + H^{t_i} = H_k^0 + dH_k + (t_i - t_0) \times V_k + \frac{(t_i - t_0)^2}{2} a_k$$

where:

$t_i$  – moment of the observations

$t_0$  – adopted time reference

$H_k^{t_i}$  – observation to the point k made at the time  $t_i$

$vH_k^{t_i}$  – observation amendment

$H_k^0$  – approximate k point height

$V_k$  – k point velocity

$a_k$  – k point acceleration

These equations allow for calculation of the dynamics parameters and initial height of the given point at the moment  $t_0$ .

Table 3 summarizes the results of alignment/compensation using the kinematic model for the four control points exposed to the most dynamic changes of position. The value uncertainties and especially their average errors indicate that the designated parameters of movement points very well describe the actual behaviour of the object.

Table 3. The results of the kinematic model alignment

Point no	7	8	9	10
$m_0 =$	0,9mm	2,1mm	4,1mm	5,8mm
$v \pm m_v =$	(2,0±0,2)mm/h	(5,6±0,40)mm/h	(8,5±0,7)mm/h	(8,6 ±1,0)mm/h
$a \pm m_a =$	-(0.17±0,02)mm/h <sup>2</sup>	-(0.49±0,1)mm/h <sup>2</sup>	-(0.76±0,1)mm/h <sup>2</sup>	-(0.75 ±0,1)mm/h <sup>2</sup>
$dh \pm m_{dh} =$	-(8,6±0,5)mm	-(22,9±1,2)mm	-(34,3±2,3)mm	-(35,4±3,2)mm

Fig. 3. Shows an image of the Siekierkowski Bridge deck deflection between 5am and 7 pm.

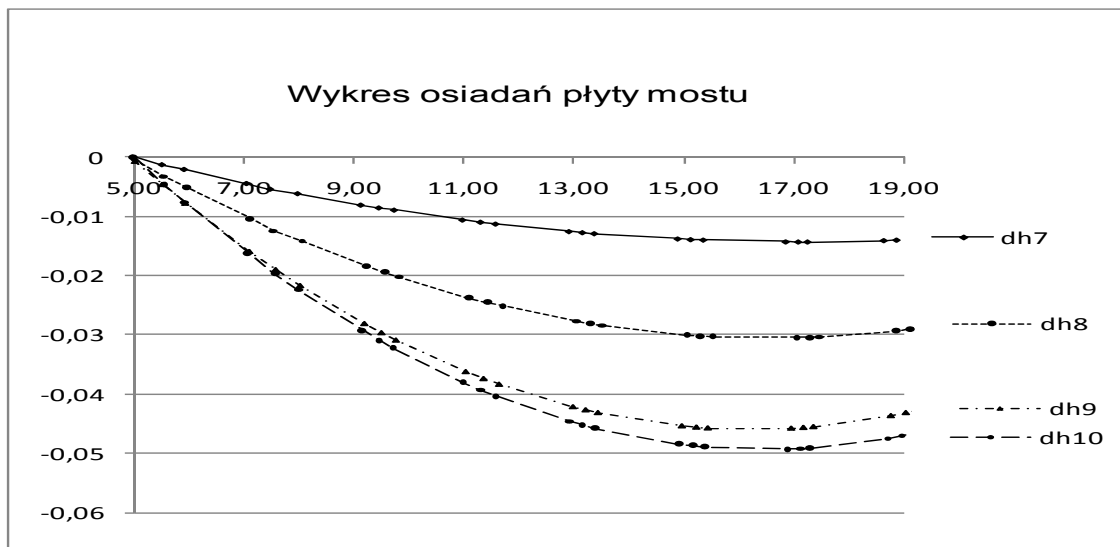


Fig. 3. Subsidence of the control points of the Siekierkowski Bridge.

As the day of the experiment was sunny and very warm, the results presented on the graph clearly show until which point there was an elongation of the ropes, which was the main cause of the bridge deck subsidence.

Figure 4. shows a graph of  $m_F = \sqrt{F^T [A^T A] F}$  function.

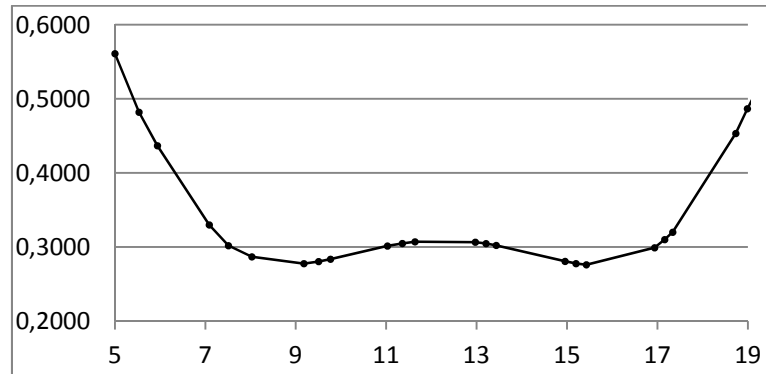


Fig. 4. Graph of average observation error after equalization.

The average error of any point after equalization can be calculated from the dependence  $m_{Hk} = m_0 \times m_F$ .

## 5. CONCLUSIONS

adopted measurement method using trigonometric levelling, though laborious proved successful and allowed to register studied phenomenon of diurnal oscillation of the bridge deck,

studying presented graphs, it is clear that the obtained results are affected by high frequency vibrations, but they have not been significantly deformed,

recorded the movement of points further away from the pylon are unexpectedly large and during executing the measurements reached as much as 50mm,

approved method for obtaining of the measurement results adopting the kinematic model can be considered valid, in order to research fully the impact of the diurnal temperature changes on the bridge deck oscillations more measurements in different conditions would be required

executed experimental survey covered only approximately half of the day, in order to examine the phenomenon fully the measurements should be continued during the night performed measurement technology allowed for true multiple, but independent determination of the various control points, if such studies were to be pursued would be advised to consider opportunities to combine observations of individual points.

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