TRANSITION CURVES SHAPING IN A ROAD ENGINEERING

Ryszard J. Grabowski
Technical University in Białystok

INTRODUTION

The theory of shaping a transition curves which have employment in road and railroads is object of consideration in professional literature still. Can seem, that solutions existing in the form circled arch, clothoid and cubical parabola do not make fields for new research and investigation. It is possible to say, that everything it make in this area already and it is possible to regard this topic as historic. However, it turns out, that it does not know science limitations. Ideas of new solutions emerge still in range of satisfying by it a different criteria. From time to time knowledge turns out useful for research in this topic from relational domains greatly.

In literature concerning roads as a transition curve applies clothoid from straight line to circled arch most often, in railroads – cubical parabola. In railroads are applied small angels of switch in final point of transition curves it causes that differences between ordinates for the same abscissas are small. In other words graph of clothoid and cubic parabola approximately are the same.

Using of clothoid as a transition curve is motivated by proportional growth of curvature respect of length of arch. By constant speed of vehicle on this curve it is revealed at constant speed of turn of steering-wheel circle. They let's notice, that in practice time of riding totals amount to several seconds for transition curve, it does not proceed movement with constant speed, but mostly, it is delayed movement. Practical research of vehicle trail indicates [1], that they do not cover with theoretical curvature. Driver subjectively choose path within road strip, which depends on speed with that vehicle move, mental state and abilities of drivers, abilities of curvature reading etc. therefore, argumentation of behavior of constant speed turn of steering-wheel circle is soundly doubtful on clothoid .

Going out opposite these doubts, in professional literature of the object, new solutions appeared. Proposals of application smooth transition curves deserve particular note. They characterized for diagram of curvature on whole passage: straight line-clothoid-arc of a circle as a smooth transition curve (Fig. 1, see $P_S E_S$). The first scientific papers was published by Auberlen , Goldner , Grabowski, Kobryń.

Almost all existing proposals of new solutions were based on search of curvature in function of natural parameter granting condition smoothness. Subsequently the curve is expressed in rectangular coordinates system. Concept will be brought closer in the present work also, more rarely applicable, creation new solutions in the form of evident function y=f(x). The use of new computational techniques allows to the uses of new solutions about the more folded form.

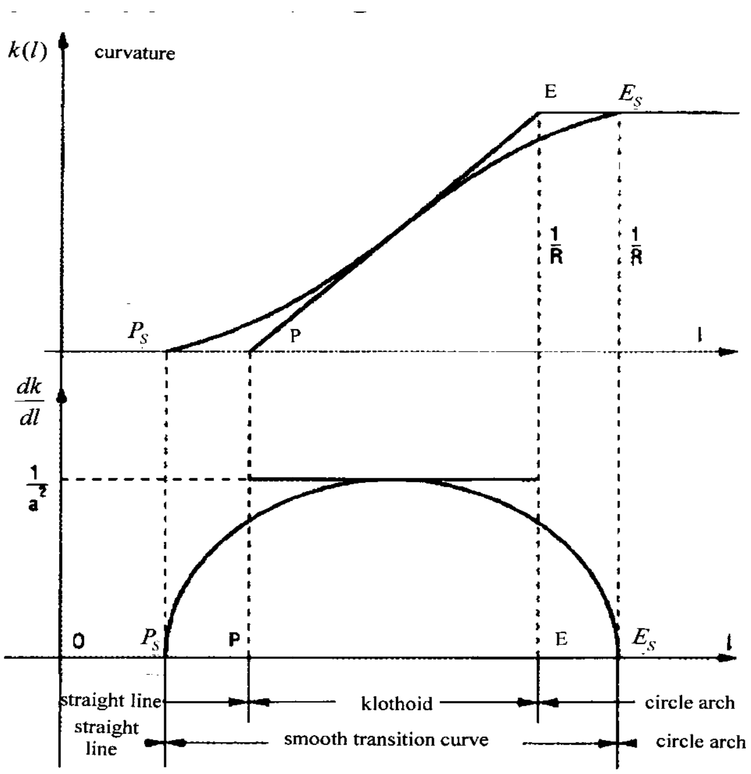

Fig. 1. Curvature graph of clothoid and smooth transition curve.

1. FORMING OF TRANSITION CURVE CURVATURE AS FUNCTION OF NATURAL PARAMETER

The majority of the cases as a proposition forming curve is assumed curvature k(1) as a function of natural parameter 1 which is a length of curve. The function k(l) creates easy capability to analyzing of curvature schedule, which owns great importance in order to given proposal could find practical employment. Besides, marginal conditions are not hard for realization on ends of passages concerning value curvature. The curvature is defined as follows

$$k = \frac{1}{r} = \left|\frac{du}{dl}\right| \tag{1}$$

Where: r- radius of curvature means, u- angle of tangent to a curve, l- length of a curve Above-mentioned relationship taking advantage, we will calculate angle of tangent to a curve from formula

$$u = \int k(l)\, dl \tag{2}$$

For the realization of a curve in terrain is necessary to express it in rectangular coordinates. Relationship among differential elements of increases of coordinates and it is length of arch is as follows

$$dx = \cos u \, dl \tag{3}$$
$$dy = \sin u \, dl$$

Expanding above-mentioned trigonometric function in power series and integrate we have

$$x = \int_0^L (1 - \frac{1}{2!}u^2 + \frac{1}{4!}u^4 - ...)dl$$
$$y = \int_0^L (u - \frac{1}{3!}u^3 + \frac{1}{5!}u^5 - ...)dl \tag{4}$$

As a example we consider smooth transition curve of Goldner, which curvature is described by formula

$$k(l)=\frac{1}{R}\left[3\left(\frac{l}{L}\right)^2-2\left(\frac{l}{L}\right)^3\right]$$

so, according to formula (2) angle of tangent to a curve amount to

$$u=\frac{l}{R}\left[\left(\frac{l}{L}\right)^2-\frac{1}{2}\left(\frac{l}{L}\right)^3\right]$$

Take into consideration above formula in formulas (4) we have

$$x=L-\frac{23}{1008}\frac{L^3}{R^2}+...$$

$$y=\frac{3}{20}\frac{L^2}{R}-\frac{253}{137280}\frac{L^4}{R^3}+...$$

Goldner curve was expressed in rectangular Cartesian coordinates system parametric equation as function of parameter L – length of curve.

2. TRANSITION CURVES IN THE FORM y=f(x)

Searching for a solution in which exist form is evident function y=f(x) put before researcher harder task. So to say, exist form in this case is final form presented in point 2. In order to such solution was accepted for road must satisfy criteria concerning marginal condition as well schedule of curvature. They let's quote as example behind author of polish scientific work published in 1984 [2]. In this work is search for a solution as polynomial of lowest degree

$$y=f(x)=\sum_{n=0}^{n=k}a_nx_n \qquad \textbf{(5)}$$

Where: a_n **number coefficients of polynomial , n=0,1,2,3,….k; y-ordinate of a arbitrary point about abscissa x in rectangular coordinate system.**

According to requirements of geometry of routes and dynamics condition of traffic movement, continuity of diagram of curvature transition curve will get, if curvature will amount to zero in the initial point

$$k\,(P)=0 \qquad \textbf{(6)}$$

and $\frac{1}{R}$ **in the final point E**

$$k(E)=\frac{1}{R} \qquad \textbf{(7)}$$

where: R- radius of circle.

Smooth condition of the transition curve graph demands derivatives with respect to natural parameter 1 equal zero at points P and E curve.

$$\frac{dk}{dl}(P)=0 \qquad \textbf{(8)}$$

$$\frac{dk}{dl}(E)=0 \qquad \textbf{(9)}$$

Research carried in [2] at behavior of condition (6 – 9) for accepted coordinate axes which axis ox covers with rout direction give negative result. Solutions owned unsuitable

schedules of curvature form the point of view of road employment. As it happens, that is possible to appoint solutions in this range granting requirement practice, if determine transition curve will be located in rectangular coordinates system oxy turned about angle u_p in initial point P, in respect axis ox previous coordinates system. Angle of rotation is equal of angle which creates tangent to graph of a curve in final point E in respect axis ox of original coordinates system.

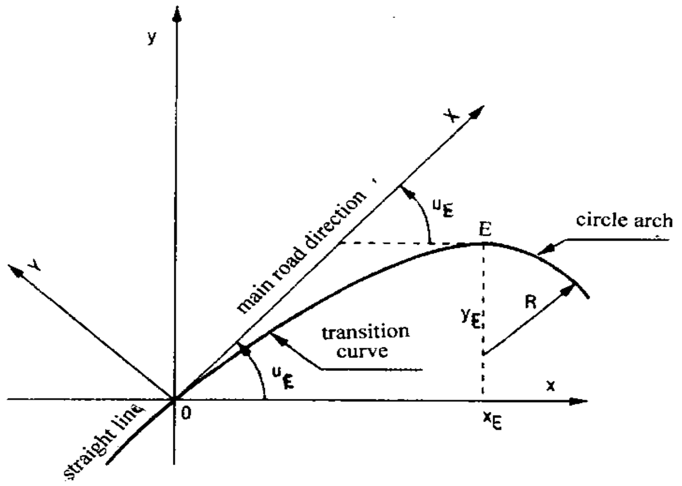

Fig. 2. Transition curve in coordinates system.

According to Fig. 2 initial point P=(0,0) covers with the beginning point of coordinates system, so

$$f(x_P = 0) = 0 \qquad \textbf{(10)}$$

whereas angle of tangent to curve at this point about abscissa $x_p = 0$ is equal u_p, in analytical form means, that

$$f'(x_P = 0) = tgu_P \qquad \textbf{(11)}$$

Angle of tangent to curve at final point E about abscissa $x = x_E$ is equal zero, this is

$$f'(x = x_E) = 0 \qquad \textbf{(12)}$$

Because solution is searched as formula (5), then curvature is calculating form formula

$$k(x) = \frac{f''(x)}{\left\{1 + [f'(x)]^2\right\}^{3/2}} \qquad \textbf{(13)}$$

Then (6) is satisfied, if initial point P about abscissa $x = x_p = 0$

$$f''(x_P = 0) = 0 \qquad \textbf{(14)}$$

Realization requirement (8), equivalent according to [2] condition $\frac{dk}{dx}(x_P = 0) = 0$

Demand, so for x=0

$$\frac{dk}{dx} = \frac{f'''[1 + (f')^2] - 3f'(f'')^2}{\sqrt{[1 + (f')^2]^5}} = 0 \qquad \textbf{(15)}$$

Comparing to zero nominate of above mentioned formulation for zero after taking into consideration (11) and (14) we will receive

$$f'''(x_P = 0) = 0 \qquad \textbf{(16)}$$

Realization in final point transition curve curvature demands according to (12) after provide for (11) realization of the condition

$$f''(x = x_E) = -\frac{1}{R} \qquad \textbf{(17)}$$

Implementation smoothness condition (9) in final point of curve come dawn to realization of equation (15) by $x = x_E$, which is satisfied after taking into consideration (12) and (17), if

$$f'''(x = x_E) = 0 \qquad \textbf{(18)}$$

Searching for solution in the form (5) lowest degree, i.e. at k=6 in rectangular coordinate system oxy, according to fig. 2, form requirements (10), (11), (14) and (16) follows that

$$a_0 = a_2 = a_3 = 0, \qquad a_1 = tgu_p \qquad \textbf{(19)}$$

Taking into consideration calculated above coefficients in function (5) by k = 6 and marginal conditions (12), (17) and (18), we have

$$\begin{bmatrix} 4x_E^3 & 5x_E^4 & 6x_E^5 \\ 6x_E^2 & 10x_E^3 & 15x_E^4 \\ 2 & 5x_E & 10x_E^2 \end{bmatrix} \begin{bmatrix} a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} -tgu \\ -\frac{1}{2R} \\ 0 \end{bmatrix} \qquad \textbf{(20)}$$

Solution of above mentioned equation , we will receive

$$a_4 = \frac{1}{Rx_E^2} - \frac{5tgu_P}{2x_E^3}, \qquad a_5 = -\frac{7}{5Rx_E^3} + \frac{3tgu_P}{x_E^4}, \qquad a_6 = \frac{1}{2Rx_E^4} - \frac{tgu_P}{x_E^5}$$

Taking into consideration calculated coefficients $a_0, a_1, \ldots, a_6$ for function (5) by k = 6, we have

$$y = f(x) = \frac{x_E^2}{R} A_1 + x_E\, tgu_P A_2 \qquad \textbf{(21)}$$

where: $A_1 = t^4 - \frac{7}{5}t^5 + \frac{1}{2}t^6$, $A_2 = t - \frac{5}{2}t^4 + 3t^5 - t^6$ **whereas:** $x \in <0; x_E>$, $t = \frac{x}{x_E} \in <0;1>$

Function (21) after introducing $C = \frac{R\, tgu_P}{x_E}$ **, we can describe in the form**

$$y(x) = \frac{1}{R} Cx + \frac{2 - 5C}{2x_E^2} x^4 - \frac{7 - 15C}{5x_E^3} x^5 + \frac{1 - 2C}{2x_E^4} x^6 \qquad \textbf{(22)}$$

Form investigation curvature schedule in [2] follows designed elements R, u_p and x_E should satisfied inequality

$$\frac{5}{3} R\, tgu_P \le x_E \le \frac{5}{2} R\, tgu_P \qquad \textbf{(23)}$$

In order to received solution could have practical employment. There is basic requirements project of a smooth polynomial transition curve (21) which can the following form also

$$0{,}4 \le \frac{R\, tgu_P}{x_E} \le 0{,}6 \qquad \textbf{(24)}$$

Particularly investigation are extreme which function (21) has the form admissible values parameter C and central value C = 0,5, for which function (21) has the form

$$y = f(x) = \frac{x_E^2}{R}(A_1 + 0{,}5A_2) = \frac{x_E^2}{R}\left(\frac{1}{2}t - \frac{1}{4}t^4 + \frac{1}{10}t^5\right) \qquad \textbf{(25)}$$

From formula (13) follows, that for small angels tangent to a curve, we can assume $k(x) = |y''(x)|$. It is allowed to orientate about diagrams of curvature presented solutions. It is possible to find detailed discussion of this diagram in work [3]. There it results form consideration carried , that for C=0,4 curve (22) is characterized very gentle speed of increase of curvature change in the surroundings of initial point form zero to maximum

for $x=\frac{2}{3}x_E$ **and very fast decrease to zero in interval** $<\frac{2}{3}x_E;x_E>$**. Similarly for C=0,6 speed of change curvature increases very quickly form zero in point P to maximum value for** $x=\frac{1}{3}x_E$**, and next slowly diminishes to zero in the final point E (Fig. 3). Changes of parameters in range described by inequality (24) give indirect schedules of curvatures whereas for C=0,5 graph for curvatures is symmetric (Bloss curve, see Fig.3).**

It is proper to note, that Kobryń in 2002 year [4] has served solution smooth transition curve about horizontal tangent in point P in the form (5) by k=6. Marginal conditions concerning initial point are analogous gave above but for final point E are as follows $f'(x_E)=tgu_E$, $f''(x_E)=\frac{1}{R}$, $f'''(x_E)=0$.

Taking into account above mentioned condition and conducting gave above analogues consideration we have

$$y=\frac{x_E^2}{R}(1+tg^2u_E)^{3/2}A'_1+x_E tgu_E A'_2=\frac{x_E^2}{R\cos^3 u_E}A'_1+x_E tgu_E A'_2 \qquad \textbf{(26)}$$

where: $A'_1=-t^4+\frac{7}{5}t^5-\frac{1}{2}t^6$ **and** $A'_2=\frac{5}{2}t^4-3t^5+t^6$

The principal project condition for solution (26) has the form $0{,}4\le\frac{R\sin u_E}{x_E\cos^4 u_E}\le 0{,}6$.

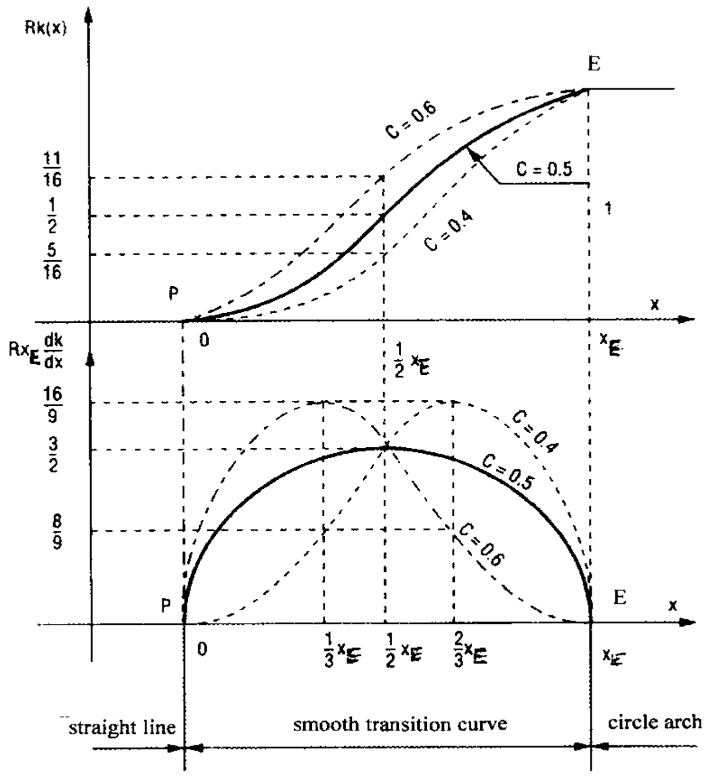

Fig. 3. Graph of curvatures and the speed of their changes for the family transition curves (22).

3. FINAL REMARKS

It is proper to underline distinctly, that there is main advantage of presented solution (22) not only, that it is expressed in the form of evident function but, that it presents dependent on parameter C (0,4≤ C ≤ 0,6). We have family of solution permitting fit curvature schedule on transition curve for requirements of practices. As it happens, that suggested in professional literature Bloss solutions cover with solution (22) by C=0,5. Solution (22) was subjected analysis in work [5] in range of usefulness for practical employment on railways. In light of scientific theoretical and practical research in this work, taking into consideration dynamic interaction between vehicle and railroad track, it is ascertained, that "most favorable for employment on polish state railways is smooth polynomial transition curve for C=0,6" (it is of transition curve selected form family solution presented in this paper for project parameter C=0,6 in formula (22). There is curve, which maintains condition of smoothness but it is quickly deflected in horizontal surface form straight direction in touch point at passage form straight line into transition curve. It belongs to check off, that they were researched by Koc [5] most widespread in professional literature transition curve i.e. parabola of 3 degree, sinusoid, cosine, Bloss curve and other. Advantage of solution (22) resists on capability of optional forming of schedule of curvature on whole length of arch through selection of project parameter form 0,4 for 0,6 C. Planner has no one in project work available solution but family solutions (Fig.3).

We see form consideration quoted, that forming of solution whit assistance of evident function is more difficult, because curvature is expressed as non-quantifiable function (13), however, it supplies capability in this case in forming of curvature more considerably. Investigative barriers have lost in the form of computational problems on meaning in view of general employment in practice of computer for accounts.

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