RHEOLOGICAL MODELS APPLIED IN DETERMINING DEFORMATION OF THE SURFACE

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1. INTRODUCTION

To assess technical condition of a building, the cause of possible construction damage should be found. A complex set of stresses in constructions results from a range of factors such as land subsidence, loads (wind and seismic ones, caused by land vibrations, unbalanced loads of longitudinal and cross bearing walls), temperature (applying materials of various coefficient of thermal expansion), shape irregularities, diversity of material (cooperation of a wall and a head of different resilience, cooperation of old and new walls, cooperation of a wall with a reinforced concrete frame). Those factors may occur together or independently. When analysing the mechanism of damage formation, it may be stated that land deformations are their most common cause.

Mine exploitation is one of the deformation causes, and it has had a very adverse influence both on the land surface and its development. The underground mining has influence on changes to water relations, land displacements and land tremors. Changes to water relations entail water blocks or partial land drainage and continuous deformations connected with that: inclinations, curvatures, horizontal displacements and distortions, as well as discontinuous deformations: craters, steps, and cracks [1] Land deformation occurs also in the surroundings of deep excavations for new objects (underpasses, buildings with underground multilevel parking lots, underground train stations). Loss of escarpment stability caused by the ground works and changes to the level of groundwater makes another example land deformation [2]. It should also be added that each newly-built construction causes deformations of the surrounding land when it subsides.

The following paper is to present theoretical basics of rheological models that are applied when determining the function of land subsidence.

2. RHEOLOGY

The aim of rheology is to examine the influence of a load on work of various materials considering also of the duration of such a load. The name rheology originates from Greek words *rheo* (flow) and *logos* (science). Sometimes rheology is treated as an independent field of science that encompasses such special issues as: resilience theory, plasticity theory, or mechanics of viscous liquids. Models of the aforementioned ideal materials are treated as special cases of a more general rheological model. Such a division is a result of the interdisciplinary significance of rheology.

Materials exhibiting rheological properties are subjected to the same general laws of mechanics as the rest of materials. Differences in their mathematical description lie in formulating appropriate constitutive equations which include an additional independent variable: real time.
Rheology is of a huge practical significance in numerous fields of technology, including construction technology, for rheological properties are exhibited by construction materials and lands. Those properties become visible to various degrees depending on the type of material and working conditions of any given material or construction.

3. RHEOLOGICAL MODELS

Simple structural models, the aim of which is to interpret fundamental properties of materials in terms of physics, are used in the literature on rheology. A spring is a model of an elastic material that subjected to the Hooke's law. As a model of viscous liquid it is possible to consider a silencer, presented as a perforated piston moved in a cylinder filled with viscous liquid. As a result of the applied force, the silencer performs a movement, velocity of which is proportional to the amount of the force. A parallel combination of an elastic element and a viscous one forms a model of viscous-elastic material (so called Kelvin-Voigt material). The parallel combination of elements means that deformations of those elements are equal to each other, the stress is, however, divided into two parts:

$$c = c_s + c_l \tag{1}$$

where indexes *s* and *l* stand for the elastic and the viscous element respectively.
The constitutive relations for the particular elements are as follows:

$$c_s = E \cdot \varepsilon_s = E \cdot \varepsilon \tag{2}$$

$$\sigma_l = \eta \frac{\partial \varepsilon_l}{\partial t} = \eta \frac{\partial \varepsilon}{\partial t} \tag{3}$$

where *E* and *n* are the module of elasticity and the coefficient of viscosity respectively. Equations (1- 3) provide a constitutive equation describing the model of Kelvin material for a one-dimensional case:

$$\sigma = E\varepsilon + \eta \frac{\partial \varepsilon}{\partial t} \tag{4}$$

If the history of stress is known, the deformation corresponding to that stress may be calculated by solving differential equation (4) with an appropriate initial condition. In the case considered, the partial derivative in equation (4) may be substituted with a regular derivative for here only real time *t* is an independent variable.

The general integral of homogeneous differential equation (4) is:

$$\varepsilon = A \exp\left(-\frac{E}{\eta} t \right) \tag{5}$$

where *A* is constant.

The form of the particular integral depends on the shape of the stress function. Let us consider a case where the stress history is provided with the following formula:

$$c(t) = p_0 + p_1 t \qquad \textbf{(6)}$$

where p_0 and p_1 are numbers. By replacing, it may be verified that the particular solution to equation (4), where the stress is provided by equation (6), is as follows:

$$\varepsilon = \frac{p_1}{E} t + \frac{1}{E}\left[p_0 - \eta \frac{p_1}{E} \right] \qquad \textbf{(7)}$$

A solution to that equation is made by the sum of general (5) and particular (7) integrals. Constant *A* is calculated from the initial condition.
Now, let us consider a special case of stress, when p_1=0. It is equivalent to applying the stress of =p_0 in the initial moment, which is kept as a constant quantity for t>0. As the initial condition, (t=0)=0 is accepted. Then, the deformation is given by the following equation:

$$\varepsilon(t) = \frac{p_0}{E}\left[1 - \exp\left(-\frac{E}{\eta} t \right) \right] \qquad \textbf{(8)}$$

where:

$\frac{p_0}{E} = \varepsilon$ **when** $t - \propto$ **,** $\varepsilon(t) = s(t), \varepsilon = s$ **.**

$$s(t) = s\left[1 - \exp\left(-\frac{E}{\eta} t \right) \right] \qquad \textbf{(9)}$$

where *s(t) – approximated settlement* , *s – measured settlement.*

4. EXAMPLE OF FOUNDATIONS SETTLEMENT

The example presents the settlement of benchmarks located in the building at Uniešcie. The configuration of subbase layers is determined on the basis of the geological drillings performed before the building has been built and is shown in the table 1. Notice a 30-cm peat layer which ill significantly influence the settlement of the structure. The settlement of the building was observed between 1972 and 2008. The observed settlement cases, approximated with function (9) have been shown on the basis of three chosen benchmarks.

Table 1. Results of geological drillings

Elevation m n.p.m.	Ground
+1,0 +0,7	surface soil
+0,7 -4,1	medium sand
-4,1 -7,1	alluvial deposit
-7,1 -7,4	Peat
-7,4	sandy loam

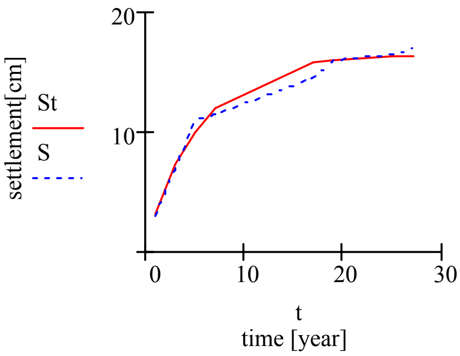

Fig. 1. Benchmark 1 settlement diagram.

Standard deviation of approximation:
$_{app}$=0.806

Average values and standard deviations of estimated variables:

S_k=16.44 **$_{sk}$=0.52**

=0.1807 **=0.0300**

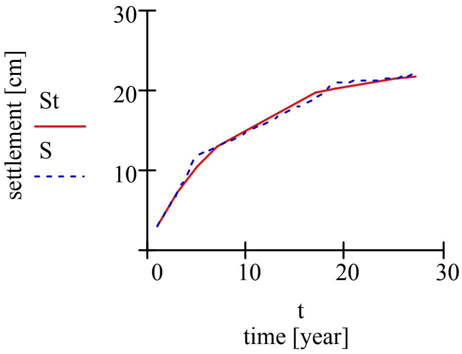

Fig. 2. Benchmark 2 settlement diagram.

Standard deviation of approximation:
$_{app}$=0.772

Average values and standard deviations of estimated variables:

S_k=22.40 **$_{sk}$=0.80**

=0.1200 **=0.0170**

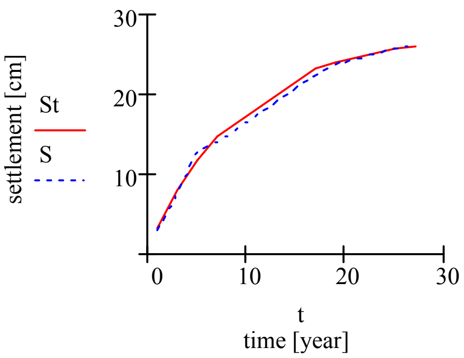

Fig. 3. Benchmark 3 settlement diagram.

Standard deviation of approximation:
$_{app}=0.769$

Average values and standard deviations of estimated variables:
$S_k=27.51$ $_{sk}=0.98$
$=0.1057$ $=0.0130$

5. RECAPITULATION

The issues of land modelling are very complex and specific. There is no universal model that would equally consider all properties of the material. Depending on the accepted theoretical model, various models of displacement may be obtained. The following paper has presented theoretical basics of rheological models that are applied when describing vertical land displacements.

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