PROPOSAL FOR NEW STRATEGY IN PRECISE POSITIONING

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ABSTRACT

In this paper a new approach for GNSS carrier-phase data processing is presented. This approach is based on some properties of Ambiguity Function Method. New algorithm ensures the condition of integer ambiguities without necessity of computing it explicitly. The condition of "integerity" of the ambiguities is ensured through inserting condition equations in the functional model of adjustment problem. An appropriate, differentiable function for the condition equations is proposed. Some numerical problems connected with new approach were resolved using variable linear combinations of GNSS signals in cascade adjustment algorithm.

1. INTRODUCTION

GNNS data processing is connected with a non-standard adjustment problem. In this problem one group of parameters has an integer nature and that fact has to be taken into account. Since the earliest years of GPS system, different approaches to solve this problem have been developed. Recently LAMBDA method is regarded as the most efficient (Teunissen,1995). All these methods require the stage of the ambiguity resolution (AR). The values of the ambiguities are computed explicitly. This, cause the necessity of cycle slip detection and repairing. There is one group of methods (Ambiguity Function Method - AFM) which does not require to deal with cycle slip problem.

These methods take advantage of special properties of the chosen periodic functions, which have known values for the integer arguments (Counselman and Gourevitch, 1981; Remondi, 1984; Remondi, 1990; Mader, 1990; Han and Rizos, 1996).

In this paper, a new approach for GNSS carrier phase observations processing is proposed, which is based on some properties of AFM. The new method ensures the condition of parameter "integerity" without necessity of the additional stage of the integer ambiguity search. It takes advantage of linear condition equations in the functional model. As a result of using the condition equations the ambiguity parameters are eliminated from the adjustment, although the condition of "integerity" of the ambiguities is ensured in the results.

In the paper the formula of function for the condition equation is proposed. Some numerical problems were solved by using linear combinations of GPS signals in cascade adjustment algorithm.

2. OBSERVATION EQUATIONS

For each double differenced (DD) carrier phase observable one can form the following simplified equation (Leick, 2004; Hoffman-Wellenhof et al., 2008; Teunissen and Kelusberg, 1998):

$$\Phi + v = \tfrac{1}{\lambda}\rho\left(X_c\right) + N \tag{1}$$

where:

- **– DD carrier phase observable (in cycles)**
- **– signal wave length**
- **v – residual (measurement noise)**
- **X_c – receiver coordinate vector**
- **(X_c) – DD geometrical range**
- **N – integer number of cycles (DD initial ambiguity)**

To simplify the equations, the propagation delay terms are omitted. Each term in equation (1) is expressed in the units of carrier cycles. There are two groups of parameters in this equation. The first group consists of three real-value receiver coordinates (included in the term) and the second group consists of an integer-value DD ambiguity (N).
Let us rewrite the equation (1) as:

$$\Phi + v - \tfrac{1}{\lambda}\rho\left(X_c\right) = N. \tag{2}$$

This is because of the integer nature of the ambiguity parameter N.

Typically, carrier phase measurement accuracy is of about 0.01 cycle (Hofmann-Wellenhof et al., 2008). Thus the residual values should be much less than half a cycle. Hence the condition of "integerity" of the ambiguities will be fulfilled if:

$$\Phi + v - \tfrac{1}{\lambda}\rho = \mathrm{round}\left(\Phi - \tfrac{1}{\lambda}\rho\right). \tag{3}$$

or

$$v = \mathrm{round}\left(\Phi - \tfrac{1}{\lambda}\rho\right) - \left(\Phi - \tfrac{1}{\lambda}\rho\right). \tag{4}$$

where *round* is a function of rounding to the nearest integer value (Cellmer, 2008). LSA procedure requires linearization of the right side of the equation (4), hence, an auxiliary variable s is introduced:

$$s = \Phi - \tfrac{1}{\lambda}\rho. \tag{5}$$

Therefore, the right side of the equation (4) can be shown as a new function proposed by author:

$$\Psi = \mathrm{round}(s) - s = -\tfrac{1}{\pi}\mathrm{arctg}\left[\mathrm{tg}(\pi s)\right]. \tag{6}$$

The function must be differentiable so it may be linearized through, e.g., Taylor series expansion. The derivative of is:

$$\frac{\delta\Psi}{\delta X_c} = \frac{\delta\Psi}{\delta s}\frac{\delta s}{\delta X_c}. \tag{7}$$

With

$$\frac{\delta\Psi}{\delta s} = 1, \text{and} \frac{\delta s}{\delta X_c} = -\frac{1}{\lambda}\frac{\delta\rho}{\delta X_c}. \qquad (8)$$

Hence:

$$v = \frac{1}{\lambda}\left(\frac{\delta\rho}{\delta x}dx + \frac{\delta\rho}{\delta y}dy + \frac{\delta\rho}{\delta z}dz\right) + \text{round}\left(\Phi - \tfrac{1}{\lambda}\rho\right) - \left(\Phi - \tfrac{1}{\lambda}\rho\right), \qquad (9)$$

where: dx, dy, dz are elements of the unknown parameter vector X. The residual equations (9) are formed for each of n DD carrier phase observations. Then the system of these equations is solved with LSA method. General formula of the residual equations can be shown in the following form:

$$V = \tfrac{1}{\lambda} AX + \ , \qquad (10)$$

with:

$$X = [dx,dy,dz]^T \qquad (11)$$

$$A = \begin{bmatrix} \frac{\partial\rho_1}{\partial x} & \frac{\partial\rho_1}{\partial y} & \frac{\partial\rho_1}{\partial z} \\ \frac{\partial\rho_2}{\partial x} & \frac{\partial\rho_2}{\partial y} & \frac{\partial\rho_2}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial\rho_n}{\partial x} & \frac{\partial\rho_n}{\partial y} & \frac{\partial\rho_n}{\partial z} \end{bmatrix} \qquad (12)$$

$$\Delta = \text{round}\left(\underline{\Phi} - \tfrac{1}{\lambda}\underline{\rho}\right) - \left(\underline{\Phi} - \tfrac{1}{\lambda}\underline{\rho}\right) \qquad (13)$$

where:

- **V – residual vector (n×1),**
- **X – parameter vector (increments to a priori coordinates vector X_0),**
- **A – design matrix (n×3),**
- **Δ – misclosures vector (n ×1),**
- **n – number of DD observations.**

3. ADJUSTMENT PROBLEM

Adjustment problem can be formulated as:

$$V^T PV = \min \qquad (14)$$

where:

- **V – residual vector, equation (11),**
- **P – weight matrix.**

The solution of that problem is found as the following parameter vector:

$$X = - \ (A^T PA)^{-1} A^T P \qquad (15)$$

together with its covariance matrix:

$$C_X = \sigma_0^2 \ ^2 (A^T PA)^{-1} \qquad (16)$$

The ambiguity parameters do not occur in the derived adjustment model. Nevertheless, the above formulation gives results that fulfill the condition of integer ambiguities. Therefore, there is no need to deal with, e.g., cycle slip effects.

4. LINEAR COMBINATIONS OF L_1 AND L_2 SIGNALS WITH INTEGER AMBIGUITY

In case of equations (10) written for phase observations on single frequency (GPS L_1), the objective function of the LSA procedure (14) has many local minimums (Figure 1). Figure 1 presents example of LSA objective function for *x* and *y* coordinates only with *z* coordinate held fixed. The final correct solution depends on sufficiently close approximation of parameter values X.

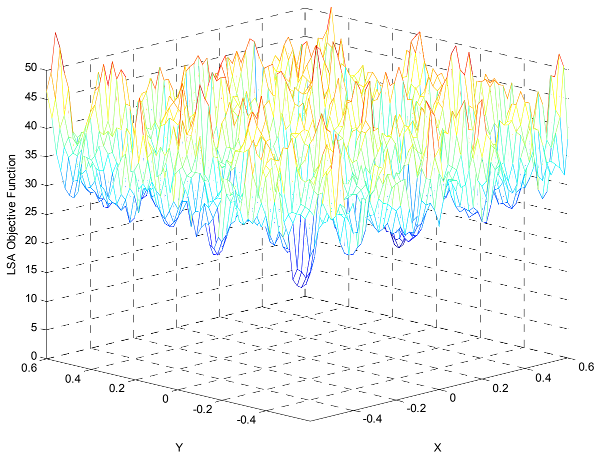


Fig. 1. LSA objective function (with *z* coordinate fixed to its reference value).

In the case of insufficient quality of the parameter vector approximation, the solution is found in any local minimum instead of the global one (where the actual correct solution exists). In order to solve this problem, different linear combinations (LC) of L_1 and L_2 observations with integer ambiguities and longer wavelengths may be applied. The frequencies of the L_1 and L_2 signals are: f_1=1575.42 MHz and f_2 = 1227.60 MHz respectively.

The equation (1) for LC of the two signals can be shown in the following form (Han and Rizos, 1996):

$$+ v = i \;\; _1 + j \;\; _2 + v =$$

$$= i\lambda_1^{-1} \;\; + iN_1 + j\lambda_1^{-1} \;\; + jN_2 = (i\lambda_1^{-1} + j\lambda_2^{-1}) \;\; + (iN_1 + jN_2) \qquad (17)$$

The wavelength, frequency and integer ambiguity of the linear combination can be expressed as:

$$\lambda = \frac{1}{i\lambda_1^{-1} + j\lambda_2^{-1}} = \frac{c}{if_1 + jf_2}, \qquad (18)$$

$$f = if_1 + jf_2 \qquad (19)$$

$$N = iN_1 + jN_2 \qquad (20)$$

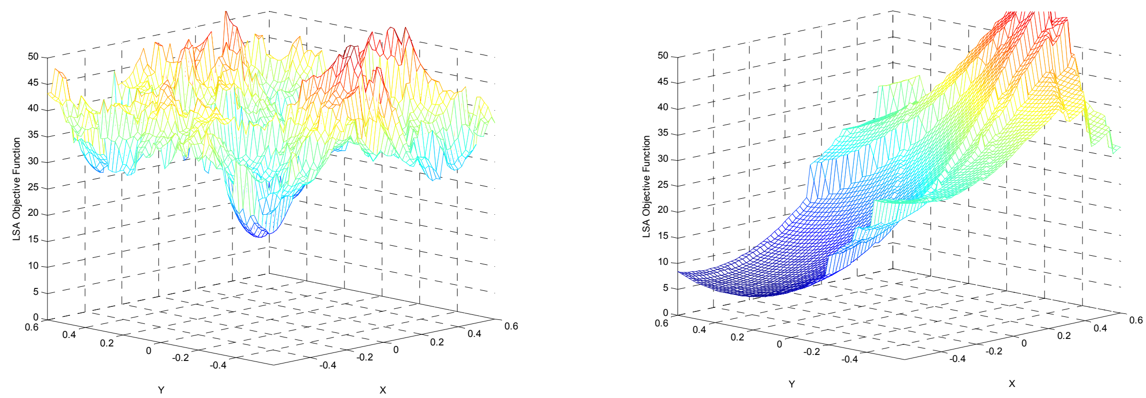
where:

c – speed of light.

If i and j are integer then N must be integer as well.
Table 1 presents the linear combinations used in the method proposed, along with their wavelengths (Han and Rizos, 1996).

Table 1 Linear combinations of L1 and L2 signals having integer ambiguity

LC#	i	j	[m]
1	-3	4	1.6281
2	1	-1	0.8619
3	1	0	0.1903



a) V^TPV for LC=L_1-L_2 **b) V^TPV for LC=-3L_1+4L_2**

Fig. 2. LSA objective function for two linear combinations (with *z* coordinate held fixed to its reference value)

Figure 2 presents the objective function with coordinate *z* fixed to its reference value for the first and the second linear combination from Table 1. Global minimum in the case of the second LS (Figure 2 a)) is visibly located and third LC (Figure 2 b)) has only one minimum. The choice of the above linear combinations was done on the basis of the analyses of theoretical properties of these combinations (Han, Rizos, 1996) and on the basis of large amount of the numerical tests performed.

5. THE ALGORITHM OF THE CASCADE ADJUSTMENT WITH DIFFERENT LINEAR COMBINATIONS

In the computation process the adjustment is performed for observation sets created for the linear combinations. The adjustment is performed successively according to the order listed in Table 1, that is, starting from LC with the longest wavelength.

For each LC the following computation stages are performed:

1) calculating the free term vector:

$$\underline{\Delta}_k = \text{round}\left(\underline{\Phi}_k - \tfrac{1}{\lambda_k}\underline{\rho}\right) - \left(\underline{\Phi}_k - \tfrac{1}{\lambda_k}\underline{\rho}\right) \quad (21)$$

where:

k – LC index according to Table 1,

$_k$ – wavelength of the k^{th} LC.

2) calculating the parameter vector:

$$X = -\ _k(A^TPA)^{-1}A^TP\ \ _k \qquad (22)$$

3) updating the design matrix A and the vector of double differenced geometrical distances _ to provide better a priori parameters for the adjustment of the next combination.

It is confirmed on the basis of the tests performed so far that, there is no need to update both A and $\underline{\rho}$ at stage 3 of the algorithm. It is enough to update $\underline{\rho}$ only according to formula:

$$\underline{\ }_k = \underline{\ }_{k-1} + AX_{k-1}, \qquad (23)$$

where:

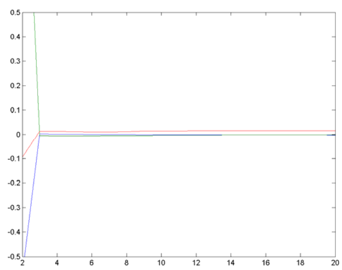
$X_{k-1} = X_{0k-1} + dX_{k-1}$ – parameter vector updated after adjustment of the previous, (k-1), linear combination.

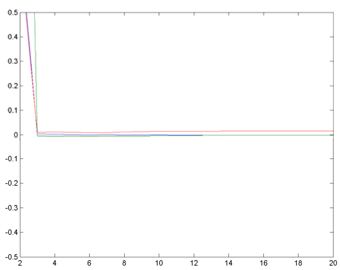
The initial a priori vector X_0 may be a float solution parameter vector defined by formula (11) or DGPS (Differential GPS) solution parameter vector.

6. FIELD TEST RESULTS

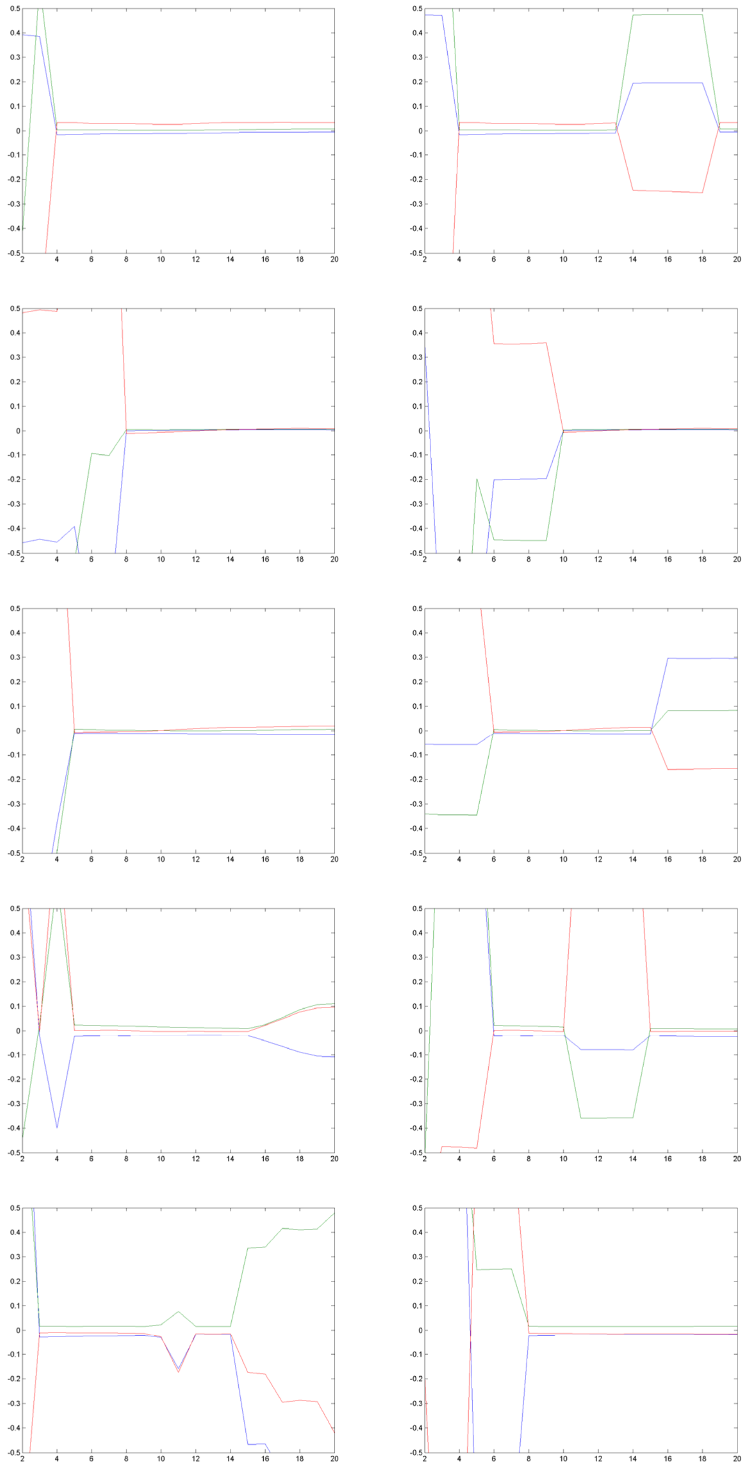
Test surveys were performed on May 8th, 2007, 10:00:00-11:00:00 UT on 25 km baseline, with 30-second sampling rate. One hour data set was divided into six, ten minutes (20 epochs) sessions long. The sessions were processed according to proposed approach and independently according to classical approach using LAMBDA method for integer ambiguity resolution. In each, 10 minutes long session, first solution was obtained after 2 epochs. Next solutions were obtained on the basis of data sets subsequently increased with data from consecutive epochs. "True" coordinates were derived using Bernese software on the basis of 10 hour data set (Dach, 2007).

Figure 3 presents N, E and U component residuals, with respect to "true" position from Bernese. In the first column are results from proposed approach and in the second column results for the same data sets, obtained from classical approach using LAMBDA method for ambiguity resolution. On the right side of the graphs is time of sessions duration.





10:00 – 10:10



10:10 – 10:20

10:20 – 10:30

10:30 – 10:40

10:40 – 10:50

10:50 – 11:00

Fig. 3. N, E and U component residuals with respect to the "true" position from Bernese.

Only in the last and previous to last sessions proposed approach did not give good results. In sessions 1 to 4 solutions gained convergence after 8th epoch.

7. CONCLUDING REMARKS

It was shown, that the new method enables precise GPS positioning without necessity of explicit computation of the carrier phase DD ambiguities, although the condition of their "integerity" is fulfilled. First tests showed high efficiency of the proposed approach. The advantage of the presented method is robustness to cycle slip effect. Extensive numerical tests of the new approach must be carried out using several different data sets.

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