APPLICATION OF THE AUTOREGRESSION ALGORITHM FOR MOVING OBJECTS' TRACK PREDICTION

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1. INTRODUCTION

Loss of the signal is a very important problem during determination a moving object's position in a real time using GPS. Lack of connection between a receiver and satellites can be caused by many different reasons e.g. tunnels, trees (ground vehicles) or change of orientation of an aircraft connected with a change of a flight direction. The process of signal searching and another initialization, which is made to solve an ambiguity, can last for several seconds. During this time a user cannot determine his current position. Such interruption can be negligible for cars or ships but it may be significant for flying objects, as they change their position very fast – it can cause a serious danger.

In navigation, there are many different methods of position determination without using satellite systems. Ones of the most important are inertial systems, which operate without any technical support from outside a vehicle.

This paper contains some tests of the statistical analysis of time series describing a vehicle movement, which were done to check the usefulness of the autoregression method to predict those pieces of track, for which a receiver cannot determine a position due to the lack of the signal. The prediction is based on data (plane coordinates X, Y) collected before loss of the signal from satellites.

2. MEASUREMENTS AND DATA COLLECTION

Collection of data - X, Y coordinates describing in a discrete form the route covered by a car (antenna phase centre) - was done in July 2007 in Grybów. Three, nine-channelled, L1/L2 satellite receivers Trimble 4700 and MicroCentered L1/L2 antennas were used. Two antennas were placed on a car's roof in a distance of about 1 meter (fig. 1), and a third one on the roof of the Training Centre of Warsaw University of Technology in Grybów.

Fig. 1. Antennas placed on a car's roof.

The receivers determined their positions (antennas' positions) in ETRF89 at 1 second intervals. The determination of relative positions for all three pairs of receivers was done in postprocessing using Trimble Total Control Software (elimination of observing, instrumental and environmental errors). The plane coordinates *X*, *Y* in '2000' datum (zone with central meridian 21 degrees) were obtained as an outcome.

Time series of coordinates *X*, *Y* were describing routes of antennas phase centres (moving receivers) related to the base receiver and the route covered by one moving antenna related to the second moving antenna (the shape of this route is similar to a circle due to the constant distance between antennas). The next step was to reject those observations, for which the distance between two antennas significantly differed from the mean value (fig. 2), which was caused by measurement disturbances, poor Dilution of Precision (DOP) etc.

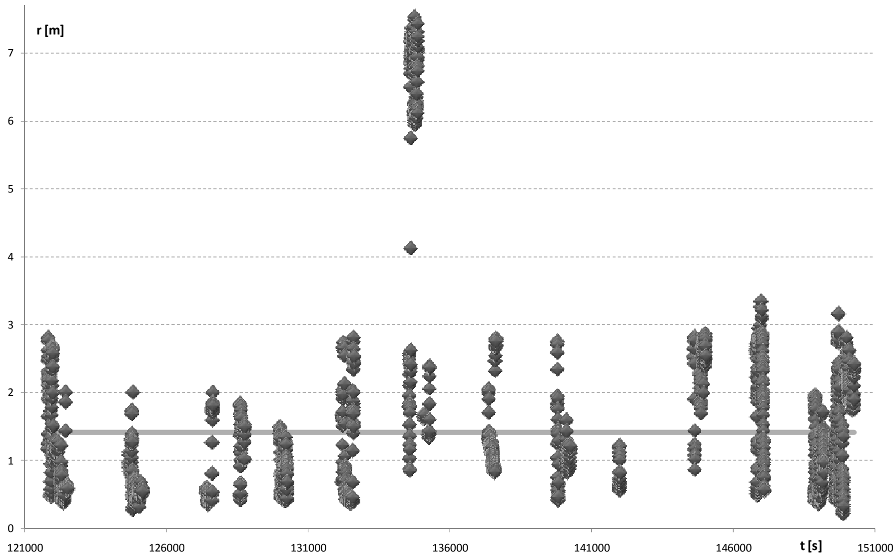

Fig. 2. The distance between two moving receivers
(centre phase of antennas) as a function of time.

Several complete pieces of the track (diverse regarding a shape) were chosen from the rest of the observing material. All of them were transformed to the local datum. The accuracy of the coordinates was estimated at a few centimetres in the local coordinate system (about 1 meter shift in '2000' frame due to an error of the base receiver position's determination). The criteria for sections selection depended on their shape.

For those pieces of track, which had a shape similar to a straight line, double differences values were of the same order of magnitude. Curved sections derivatives were analyzed to find local extremes or points of inflection.

The plane coordinates *X* and *Y*, describing in a discrete way individual pieces of the track, were treated as time series and analyzed separately on the assumption that there is no correlation between them (despite the fact that such correlation exists).

The values *X* and *Y* as functions of time were considered as a random signal, which means that we know only the general statistic rules according to a signal changes in time (Lyons, 1999). The main assumption refers to a fluency of a vehicle motion on analyzed section (no sudden curves or way of motion change). The differences of coordinates between the following determination (X_n-X_{n-1}…X_2-X_1; Y_n-Y_{n-1}…Y_2-Y_1) depend on a shape of the track and a speed of vehicle as well. The analyzed sections were travelled with a constant speed (uniform motion) or a constant accelerate (uniformly accelerated motion). The two-dimensional time series (*X*, *Y*) describing a shape of every section are the outcome of the measurements. The track was travelled only once, so we have only single realization of a random process. The coordinates as a function of time are non-stationary time series - the parameters such as mean value or variance changes in time. Lack of data was simulated on every analyzed piece of the track.

3. CALCULATIONS

The autoregression method (AR) is suitable for relatively short time series prediction (minimum 50 values of data), that is why this method was chosen for calculations. The most important thing was to lead time series of data to stationarity by double differentiating. This process is equivalent to a strong high-pass filter use (Kosek, 2007).

The moving average (three values in each subset) was used to smooth out time series (to dispose of short-term fluctuations and highlight longer-term trends). The values, which significantly differed from the mean value, were eliminated during this process. The moving average with subsets consisting of three elements was used, because three was an optimal number, which enable disposing of such values, but it did not cause a significant track generalization. The mean value of each time series was subtracted (after this operation the mean value of each time series was zero).

The autocorrelation function was calculated for fifty elements of analyzed time series to find an order of autoregression *P* (the Rovelli-Vulpiani criterion). An order of autoregression contains information about how many ‘previous’ samples are needed to determine ‘next’ values (Rovelli and Vulpiani, 1983). The calculations were based on the formulas:

$$P = \frac{\pi}{2} \sum_{k=1}^{N-1} \frac{|c_k|}{c_0} \qquad (1)$$

where:

c_k means autocovariance (correlation between stochastic process and the same process shifted in time) calculated using the formula:

$$c_k = \frac{1}{N} \sum_{t=0}^{N-k} x_t x_{t+k} \qquad (2)$$

c_o means variance of this time series (for an expected value 0):

$$c_0 = \frac{1}{N}\sum_{t=0}^{N} {x_t}^2 \tag{3}$$

The quotient c_k/c_o stands for autocorrelation (autocovariance after a normalization).
The autoregression coefficients were determined using the Burg method (minimalization of forward and backward prediction error and recurrent Levinson-Durbin algorithm) as a function of this part of time series, on which the prognosis was based, and an order of an autoregression. Missing values of time series consisting of double differences of *X* and *Y* coordinates were determined using the formulas (Box and Jenkins, 1976):

$$\begin{aligned} \hat{x}_{N+1} &= \hat{a}_1 x_N + \hat{a}_2 x_{N-1} + \ldots + \hat{a}_M x_{N-M-1} \\ \hat{x}_{N+2} &= \hat{a}_1 \hat{x}_{N+1} + \hat{a}_2 x_N + \ldots + \hat{a}_M x_{N-M-2} \\ &\ldots \\ \hat{x}_{N+L} &= \hat{a}_1 \hat{x}_{N+L-1} + \hat{a}_2 \hat{x}_{N+L-2} + \ldots + \hat{a}_M x_{N-M-L} \end{aligned} \tag{4}$$

During the next step, the determined values of time series were transformed into time series of coordinates *X*, *Y*. The results were compared with coordinates received from the measurements.
The prognosis were done for a dozen or so pieces of the track (for different numbers of samples *N* taken for the autoregression order *P* determination). The error of prediction stands for differences between predicted values and values from the measurements.

4. SELECTED RESULTS – PREDICTION

4.1. Section 6

The length of section 6 (fig. 3) was enough to make two predictions (‘a’ and ‘b’) with different beginning points. This piece of the track contains the bend with a significant radius of a curvature (the vehicle did not significantly change the way of moving). Prognosis ‘a’ refers to such part of the track, whose shape is similar to a straight line. The value of a speed was changing fluently from 12 to 19 m/s for prognosis ‘a’ and from 10 to 19 m/s for prognosis ‘b’. Prognosis ‘a’ was done on the basis of 56 known values, prognosis ‘b’ – 76 values.

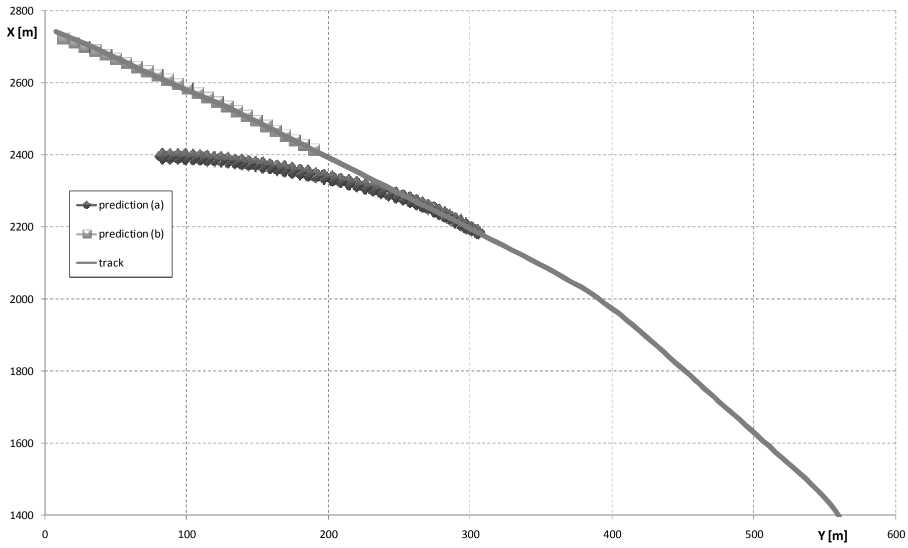

Fig. 3. Section 6 and its prognosis.

The results of two predictions for the same section are considerably different. The beginning of prediction ‘a’ was in the middle of the bend. Despite slight curvature of the bend, prognosis gave a continuance of this bend with fluent increase of the curvature. The beginning of prognosis ‘b’ did not belong to the bend, so its result was better (fig. 4 and 5).

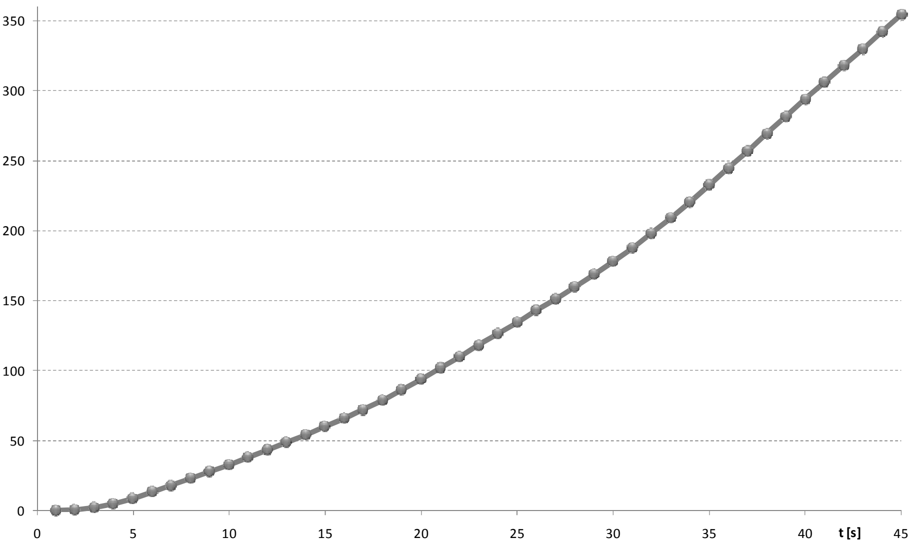

Fig. 4. Errors of the following points of prognosis ‘a’ of section 6
(differences [m] between the values from the prediction and values from the measurements).

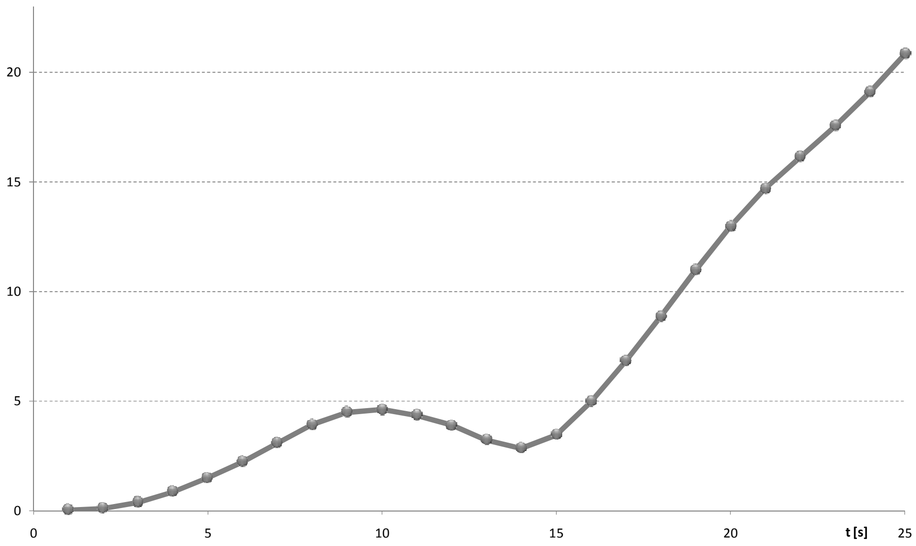

Fig. 5. Errors of the following points of prognosis ‘b’ of section 6
(differences [m] between the values from the prediction and values from the measurements).

The values of errors of following predicted points (prognosis ‘b’) increase to 4.6 m, after that they decrease to 2.8 m and rise again. The track determined by the autoregression method crosses the real track of a car, but the value of an error is higher than 0, because the point of intersection refers to different moments of time for the measured and predicted track (the speed of a vehicle differs from the speed determined from the prognosis).

4.2. Section 12

Two predictions (‘a’ and ‘b’) with beginning in different points were made for this section. The vehicle’s speed varied from 19 to 21 m/s. Prognosis ‘a’ was made on the basis of 50 known values, prognosis ‘b’ – 65 values.

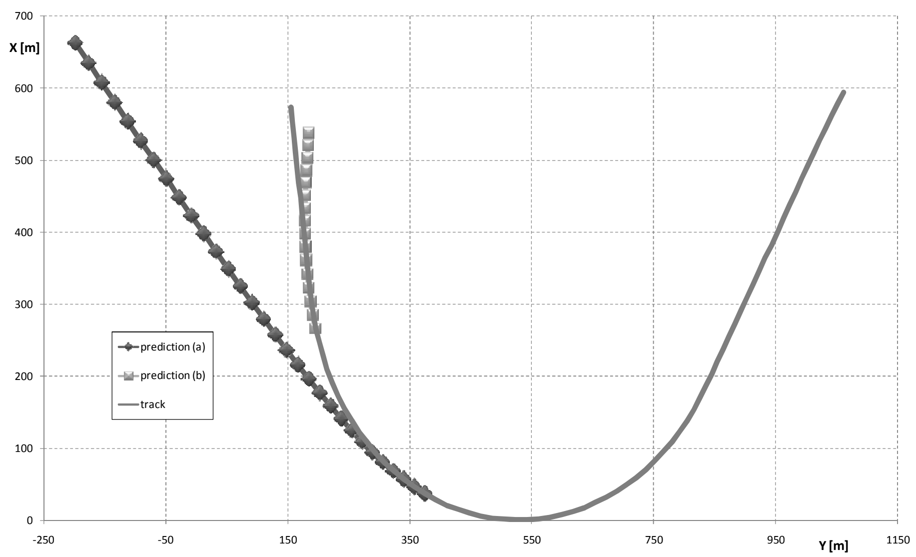

Fig. 6. Section 12 and its prognosis.

Prognosis ‘b’ described missing the part of the track very well only for some period. The predicted line has a significant curvature as a result of a curvature of this part of the section on which prognosis was based, while the track from measurements is similar to a straight line in its further part.

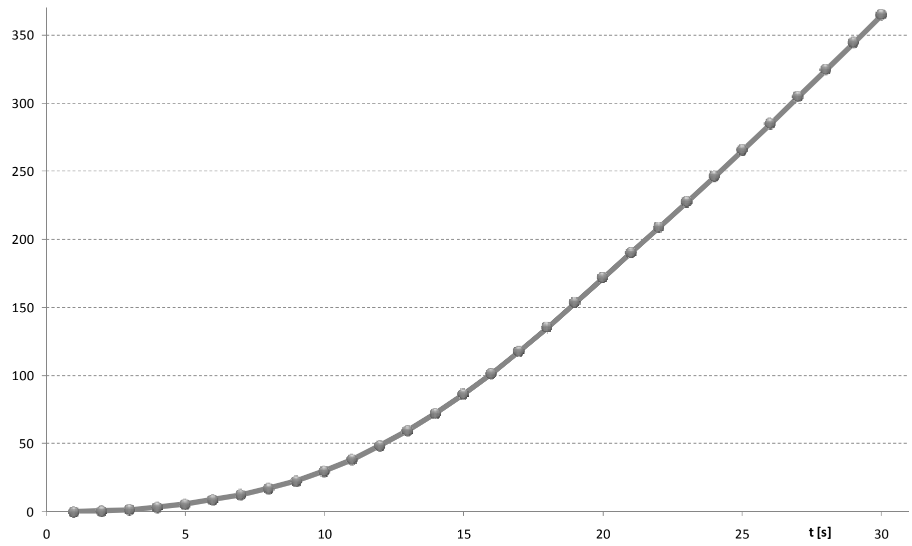

Fig. 7. Errors of the following points of prognosis ‘a’ of section 12
(differences [m] between the values from the prediction and values from the measurements).

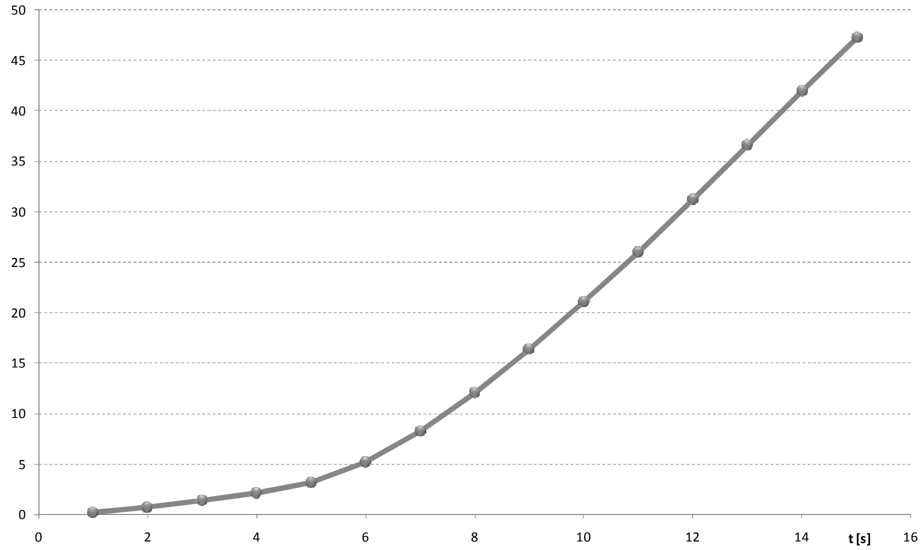

Fig. 8. Errors of the following points of prognosis ‘b’ of section 12
(differences [m] between the values from the prediction and values from the measurements).

4.3. Section 13

Two predictions (‘a’ and ‘b’) with the beginning in different points were made for section number 13. The speed was about 19 m/s, but it decreased to 11 m/s at the end of this part of the section, on which prognosis ‘a’ was based (the reduction of a speed in the middle of the bend). After that, the speed increased again up to 14 m/s and this value remained to the end of this part of the section, on which prognosis ‘b’ was based. Prognosis ‘a’ was made for 62 known values, prognosis ‘b’ – 81 values.

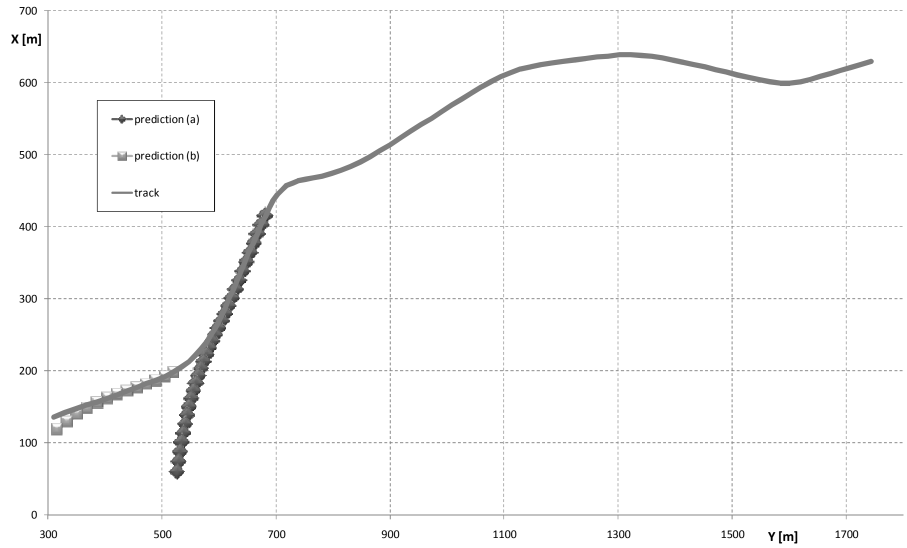

Fig. 9. Section 13 and its prognosis.

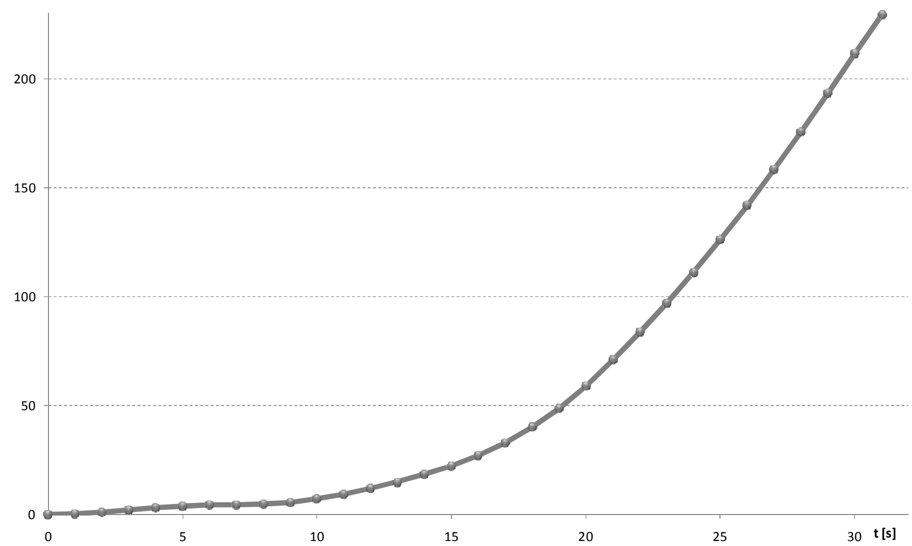

Fig. 10. Errors of the following points of prognosis ‘a’ of section 13 (differences [m] between the values from the prediction and values from the measurements).

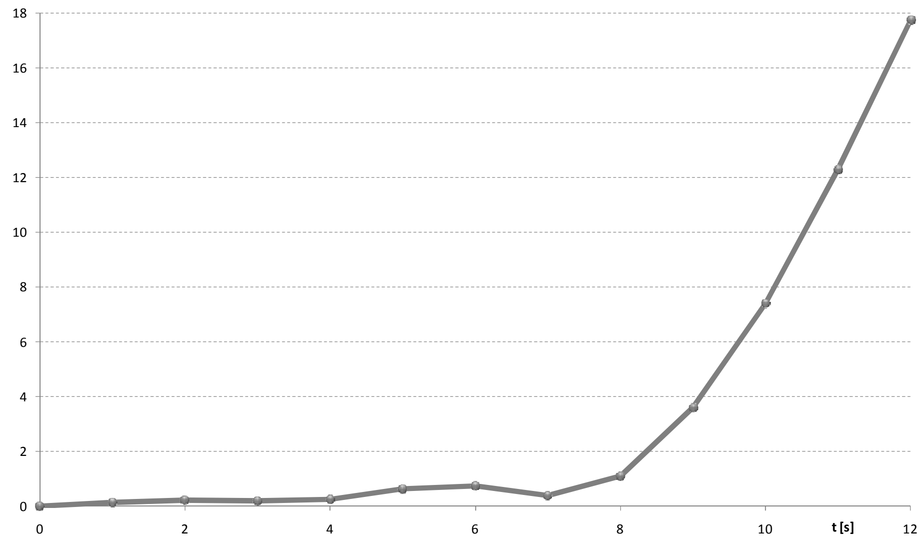

Fig. 11. Errors of the following points of prognosis ‘b’ of section 13
(differences [m] between the values from the prediction and values from the measurements).

4.4. Section 15

The speed of the vehicle on the section, which was used for prognosis ‘a’, varied from 13 to 19 m/s and it stabilized at the value of 18 m/s at the end of the section. 50 known values of time series were used for prognosis ‘a’, 64 for prognosis ‘b’.

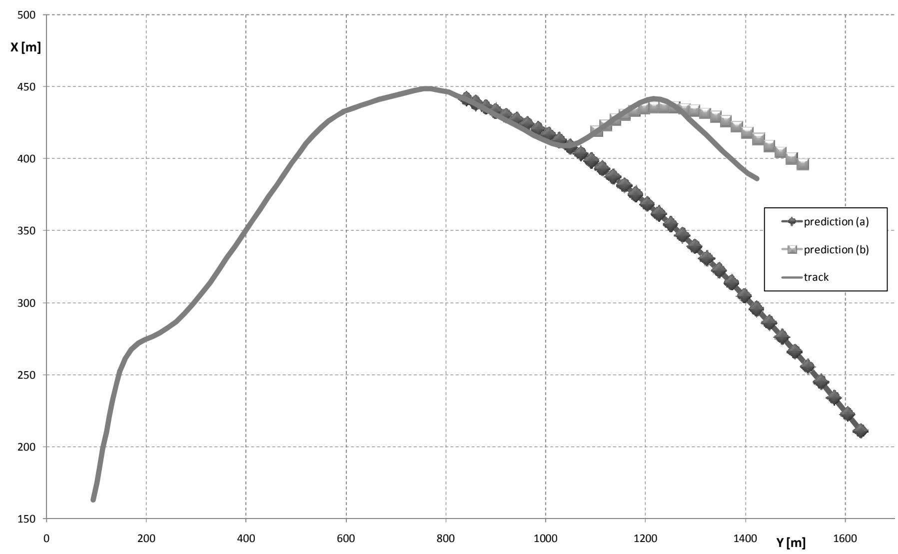

Fig. 12. Section 15 and its prognosis.

Despite the intersection of the predicted and measured trajectory, the prediction error is higher than 0 for this point, because of the difference in speed of the vehicle on this section and the speed that has been calculated from the prognosis.

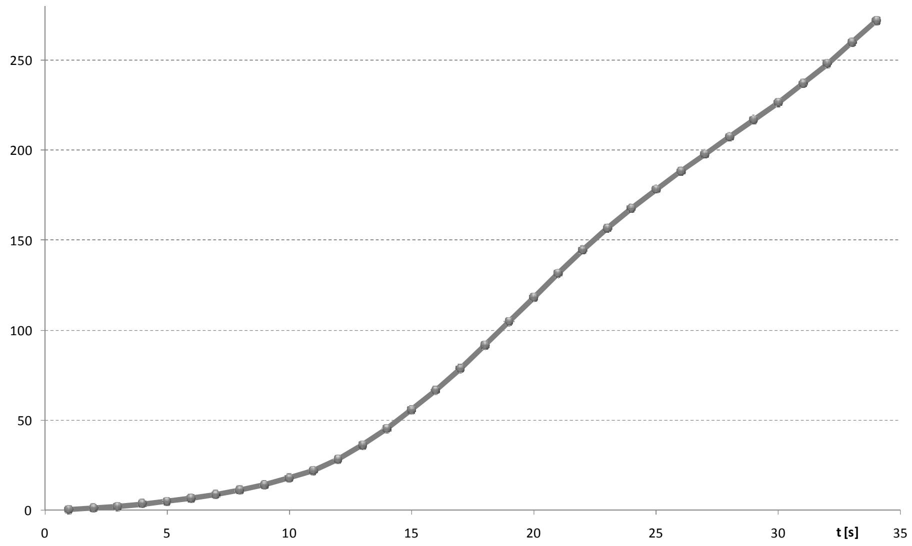

Fig. 13. Errors of the following points of prognosis ‘a’ of section 15
(differences [m] between the values from the prediction and values from the measurements).

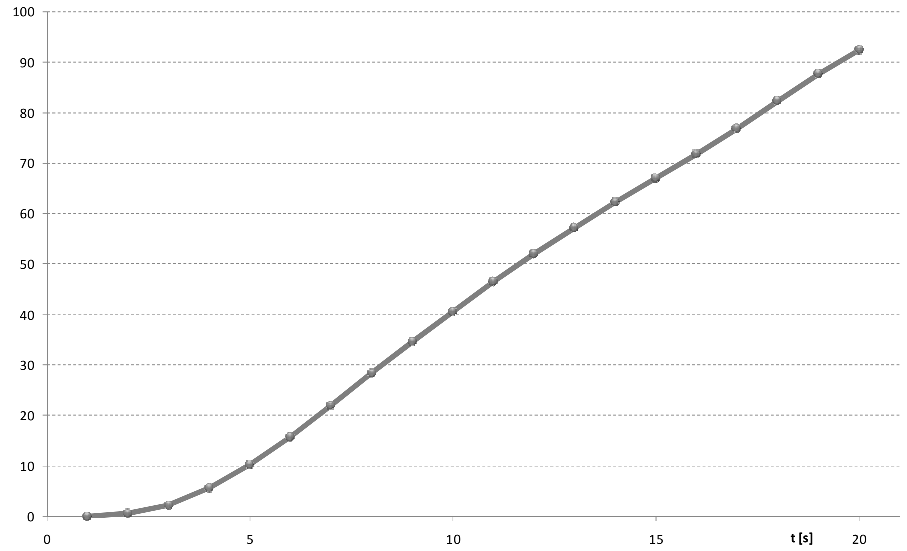

Fig. 14. Errors of the following points of prognosis ‘b’ of section 15
(differences [m] between the values from the prediction and values from the measurements).

For every section values S_a, S_b, S_c were calculated. They signify the length of a track that has been travelled from the beginning of a prediction to the point, in which the prognosis’ error is higher than 10 cm, 1 m and 5 m (the width of a road). For those sections, whose shape is similar to a straight line those values amount to:

S_a = 22 m
S_b = 61 m
S_c = 146 m,

and for sections with significant curvature:

S_a = 18 m
S_b = 57 m
S_c = 118 m.

Those values were calculated as an average from all the results for the analyzed pieces of the track.

5. SELECTED RESULTS - INTERPOLATION

Additionally, an interpolation by forward and backward prediction for a few appropriately long sections was made. The predictions were also made using the autoregression method. Such way of data making up is possible only in a postprocessing (impossible in real time because of a lack of further data). The mean prognosis was determined by forward and backward prediction – weighted average has been used (in a function of a distance from the point of an intersection). The trajectory determined using the autoregression method was compared with a curve described by a polynomial of degree 2.

5.1. Section 18

The speed of the vehicle on this part of the section, which was used for forward prediction was increasing uniformly (from 8 up to 19 m/s). The backward prediction was made on the basis of the section, on which the vehicle's speed was constant (about 20 m/s). 72 known values of time series were used for forward, 62 for backward prognosis.
The prognosis gave very good results – the maximum distance between the predicted line and the trajectory taken from measurements was 5 meters (for the same point the distance between the trajectory and the second degree curve was 91 meters).

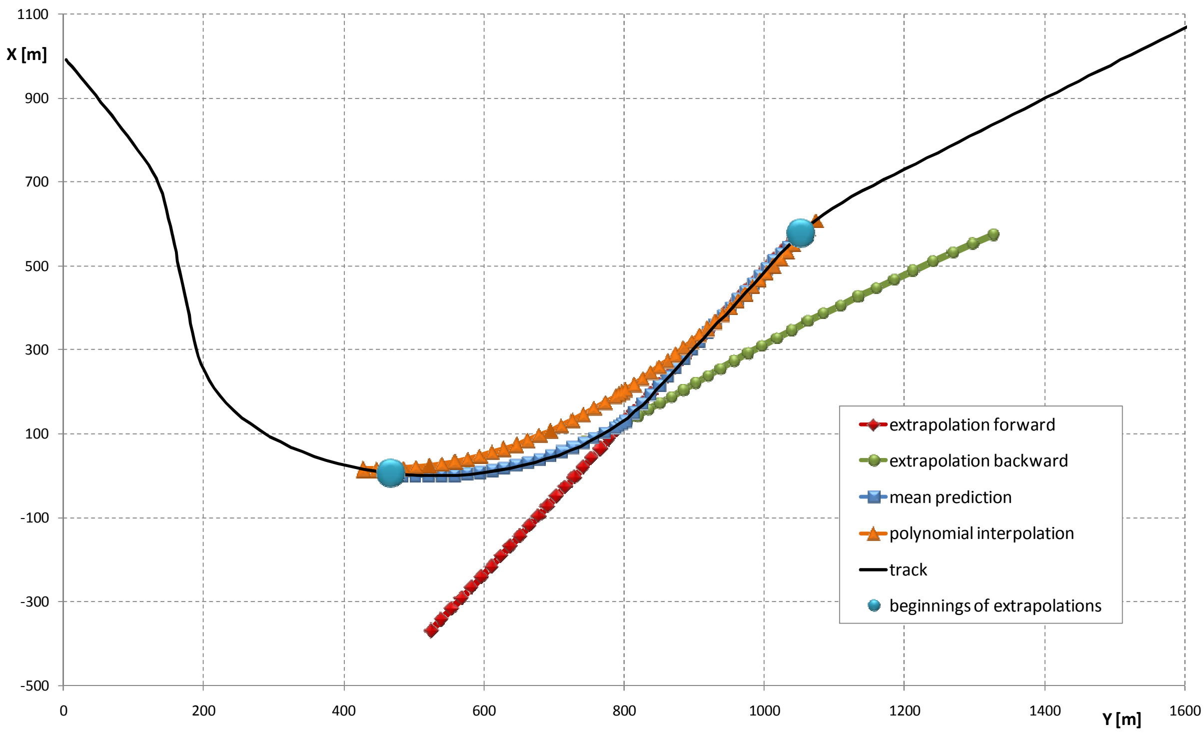

Fig. 15. The comparison of interpolation by the autoregression method with interpolation using second degree polynomial (section 18).

6. CONCLUSIONS

- The autoregression method of a prediction of a track gives good results for short pauses in a connection between a receiver and satellites for a fluent motion of a vehicle.
- Prediction accuracy depends on the shape of an analyzed piece of track and on the position of the point, in which prognosis begins.
- Sections with a minor curvature are easier to predict than the ones with a significant curvature (lower prediction errors).
- The errors of prediction are caused both by errors of time series of coordinates X and time series of coordinates Y prediction.
- In most cases the prediction errors increase in a linear or a parabolic way.
- The biggest errors concern situations, when a vehicle unexpectedly changes the way of motion.
- The very important factor, which has an influence on the results' deterioration, is a non-uniform speed (or change of acceleration) of a moving object (different coordinates' increases in the same period of time) – time series should be transformed into time series with a constant time step.
- The combination of the autoregression method of prediction with other ground methods of position determination (e.g. inertial) could cause a significant outcomes' improvement.
- The interpolation by forward and backward prediction gives very good results, but they depend on the shape of the section. The autoregression method is better in describing missing part of the track than the interpolation based on the second degree polynomial.

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