# APPLICATION OF THE AG MEASUREMENTS TO THE TIDAL GRAVIMETER CALIBRATION

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## ABSTRACT

The tidal measurements at the Astro-Geodetic Observatory in Jozefoslaw began in 1993 with LaCoste&Romberg G model. Nowadays the Observatory is equipped with ET-26 spring gravimeter. The use of results depends on correct determination of the scale of tidal records which should have relatively high accuracy. The most frequently used method for calibration of tidal gravimeters with more or less regular drift is comparison to the absolute measurements. The repeated absolute gravity measurements with the FG5 No. 230 gravimeter at the Observatory in Jozefoslaw were used for computing calibration factors of the tidal gravimeters using least-squares fitting. The scales of the tidal gravimeters were determined from 8 2-day absolute campaigns performed between October 2006 and May 2007 with the accuracy of about 10%.

## 1. INTRODUCTION

All relative gravimeters need to be calibrated due to their instrumental specificity. The calibration should contain determination of scale factor and phase lag as well. There are several methods for scale factor determination, e.g. on gravimetric base (mainly for field gravimeters), utilizing movement of the huge mass near the gravimeter (rather difficult to realize), inertial methods (lifts – rather complicated and useless in case of big gravimeters), comparison with theoretical models of Earth tides (mainly for determination of instrumental drift) and comparison with absolute gravity measurement, described in details in this elaboration.

## 2. DATA

Both gravimeters, absolute and relative, measure the gravity but not in the same principles. ET-26 measures relative changes of the gravity using movement of the trial mass hinged on the quartz spring. The measurements are continuous, sampled five times a second and averaged using moving window. FG5-230 measures gravity using free-fall of the trial mass. Measurements are repeated in series consisted of 100 falls, one fall per 10 seconds, one series per one or two hours.
The data used in this project concerns observations collected between October 2006 and May 2007. Eight independent absolute gravity determinations were taken into account.

Table 1. Information about the absolute data.

| No. | Date | Numbers of series |
|---|---|---|
| 1 | 14.10.2006 | 24 |
| 2 | 11.11.2006 | 28 |
| 3 | 02.12.2006 | 14 |
| 4 | 07.01.2007 | 12 |
| 5 | 04.02.2007 | 13 |
| 6 | 03.03.2007 | 27 |
| 7 | 01.04.2007 | 13 |
| 8 | 05.05.2007 | 25 |

The following figure presents data collected during first calibration session in October 2006.

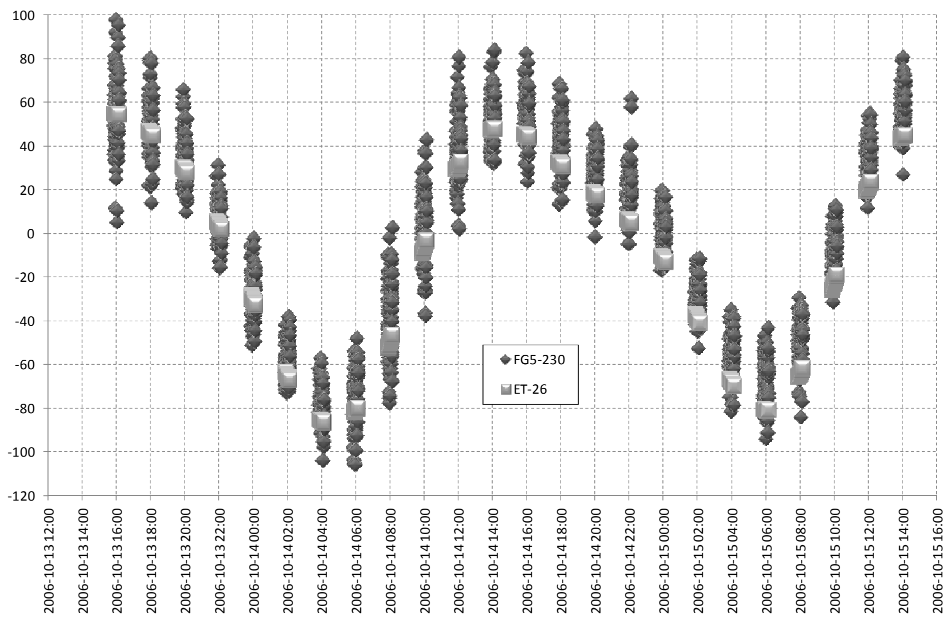


Fig. 1. Absolute (FG5) and relative (ET-26) gravity data [nm/s$^2$].

## 3. CALIBRATION

First of all we have to take a look at the particular series. The Fig. 2 presents observations collected by absolute gravimeter (100 drops) with standard deviations and ET-26 (standard deviation of relative determinations is much smaller – about 4 nm/s^2 – and not seen at the picture). The chart denotes first series on October 13th at 16 pm.

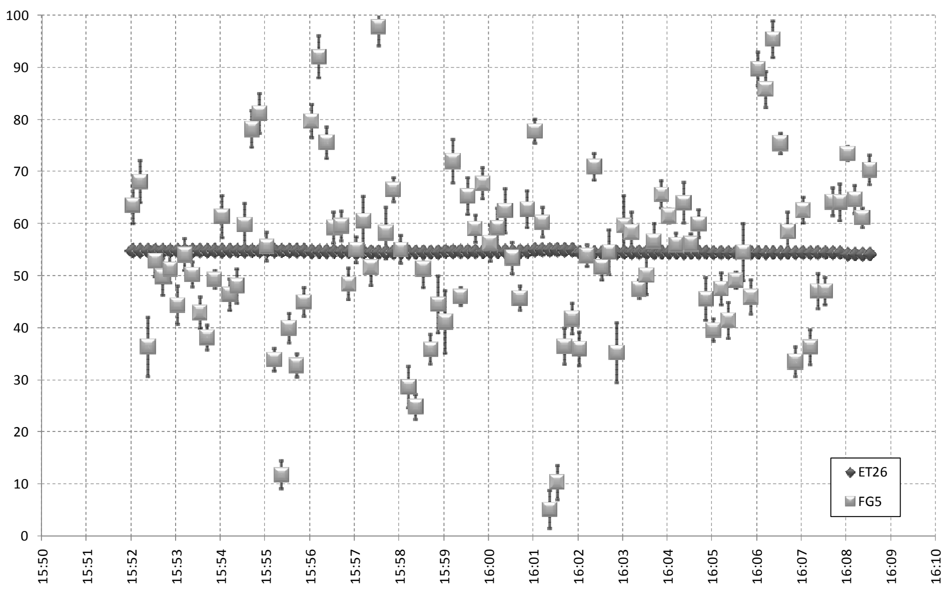


Fig. 2. Comparison between absolute and relative data in one series (100 drops) – [nm/s^2].

The chart in figure 2 illustrates first problem which occurred during calibration. The observations are different by means of accuracy and they are difficult to be compared. Absolute determinations are less precise and much more scattered. Relative determinations are more stable. The same situations appears in each series. It was assumed that mean value (after removing outliers using 3σ criterion) describes gravity for mean time of observation. The next chart presents observations after averaging in particular series.

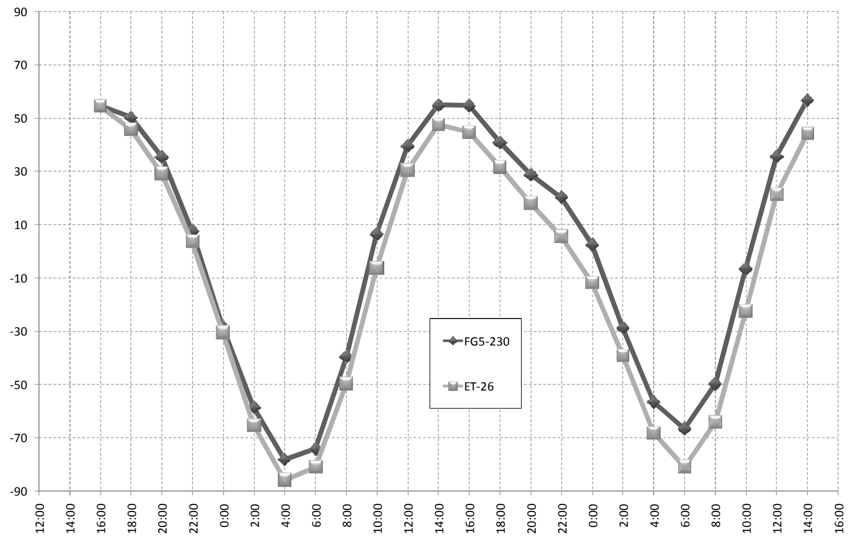


Fig. 3. Comparison between absolute and relative data in particular series after averaging – [nm/s^2].

Next problem that is clearly seen is the existence of a kind of the drift between particular observations. Relative gravimeters always have drift but absolute instrument should be driftless. The easiest method for the drift determination is subtracting from observations theoretical tides and approximation using straight line. The Fig. 4 presents gravity determination together with the theoretical tide calculated using Eterna 3.4 package (Wenzel, 1996) and HW95 tidal potential catalogue (Hartmann and Wenzel, 1995):

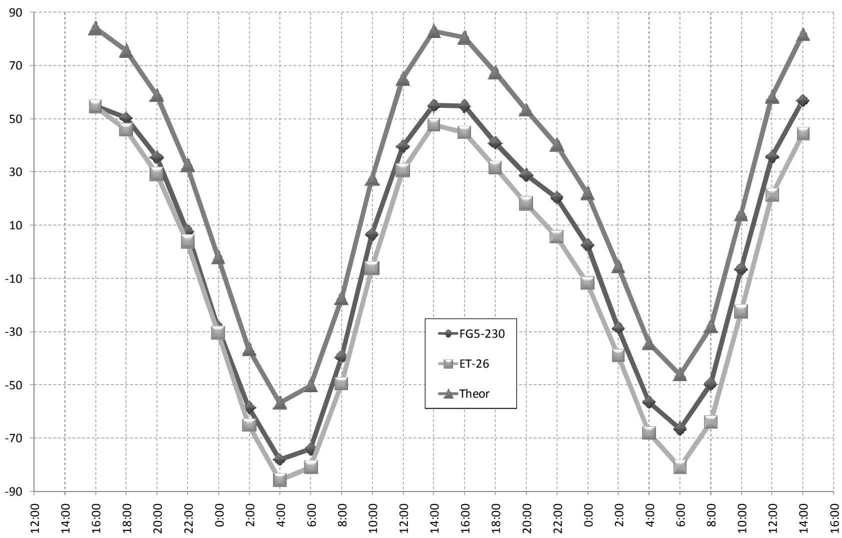


Fig. 4. Averaged gravimetric data with theoretical tide – [nm/s^2].

and differences with linear approximations:

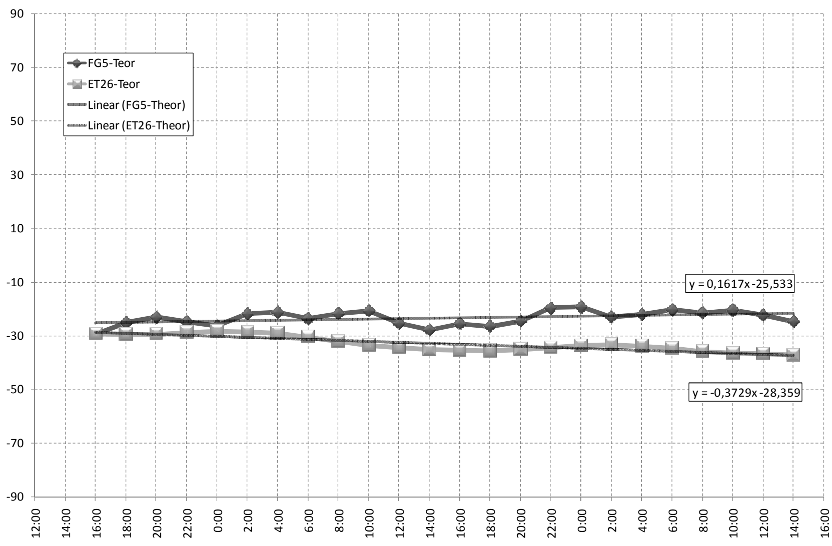


Fig. 5. Linear approximations – [nm/s^2].

After that the observations were restored to their original values using theoretical tides again.

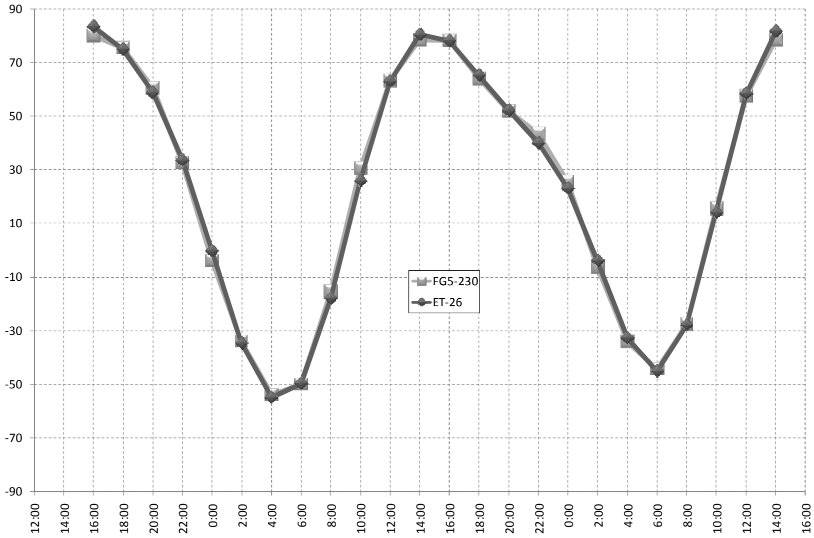


Fig. 6. Gravimetric data after drift elimination – [nm/s^2].

From the charts in figures 5 and 6 arise that constant factor of ET-26 gravimeter is determined almost correctly.

4. PROCEDURE

To characterize properly relations between two curves describing the same phenomenon two parameters were implemented: scale coefficient *k* and shift *s*. Then the relation could be written as follows:

$$a = k \cdot r + s \qquad (1)$$

where *a* and *r* are the observations made by absolute and relative gravimeters respectively. The observation equation *v* would be expressed by the following:

$$v = r \cdot k + s - a \qquad (2)$$

and then the system of the normal equation is created:

$$R \cdot X - A = 0 , \qquad (3)$$

where:
*R* - matrix of coefficients at unknowns;
*A* - matrix of free terms;
*X* - matrix of unknowns.

Now we impose least square condition [*vv*]=min and we obtain:

$$\left(R \cdot R^T\right) \cdot X - R \cdot A^T = 0 \qquad (4)$$

The unknowns are calculated by the conversion:

$$X = \left(R \cdot A^T\right) \cdot \left(R \cdot R^T\right)^{-1} \tag{5}$$

Furthermore for each gravimeter standard deviation is determined:

$$m_0 = \sqrt{\frac{[vv]}{n}} \tag{6}$$

and errors $m_k$ and $m_s$ of the individual parameters.

## 5. RESULTS

Table 2 presents the values of *k* and *s* factors for ET-26 gravimeter.

Table 2. The results of the calibration.

| Date | $k$ | $m_k$ | $s$ | $m_s$ | $m_0$ |
|---|---|---|---|---|---|
| 2006-10-14 | 0,99063 | 0,13104 | 0,22155 | 0,99978 | 2,08869 |
| 2006-11-11 | 1,00274 | 0,09051 | -0,04361 | 0,67684 | 1,51951 |
| 2006-12-02 | 1,00501 | 0,21204 | 0,19057 | 2,01968 | 3,68603 |
| 2007-01-07 | 0,98974 | 0,14963 | 0,26180 | 1,20420 | 2,12461 |
| 2007-02-04 | 1,00126 | 0,15549 | -0,00961 | 1,30204 | 2,24868 |
| 2007-03-03 | 0,99760 | 0,19142 | 0,07186 | 1,53058 | 3,18699 |
| 2007-04-01 | 1,01391 | 0,27308 | -0,69567 | 2,23100 | 3,59218 |
| 2007-05-05 | 1,00379 | 0,09635 | -0,06936 | 0,81179 | 1,78250 |

Finally the calibration factor was determined as follows:

ET-26: 1.00059 +/-0.007911

## 6. VALIDATION

The calculated calibration factor was validated upon the tidal analysis of 2-years tidal data collected in Jozefoslaw Observatory using ET-26 gravimeter. The classical manner utilized least squares method was applied (Chojnicki, 1970). Figure 7 presents amplitude factors for 31 tidal waves and their changes with new calibration factor.

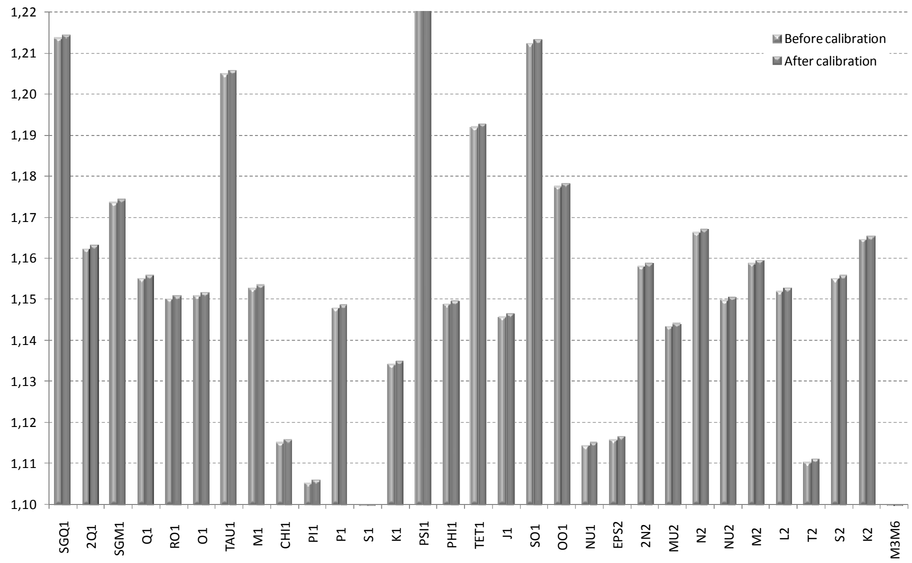


Fig. 7. Amplitude factors for main tidal waves.

From this figure we can point out that the change of amplitude factor is the same for all tidal constituents, which is the main disadvantage of the described method. It is not frequency-dependent.

Second step of validation was the comparison of the main tidal waves O1 and M2 amplitudes to their model values for Jozefoslaw Observatory. Unelastic non-hydrostatic Dehant-Defraigne-Wahr Earth model was used (Dehant et al., 1999).

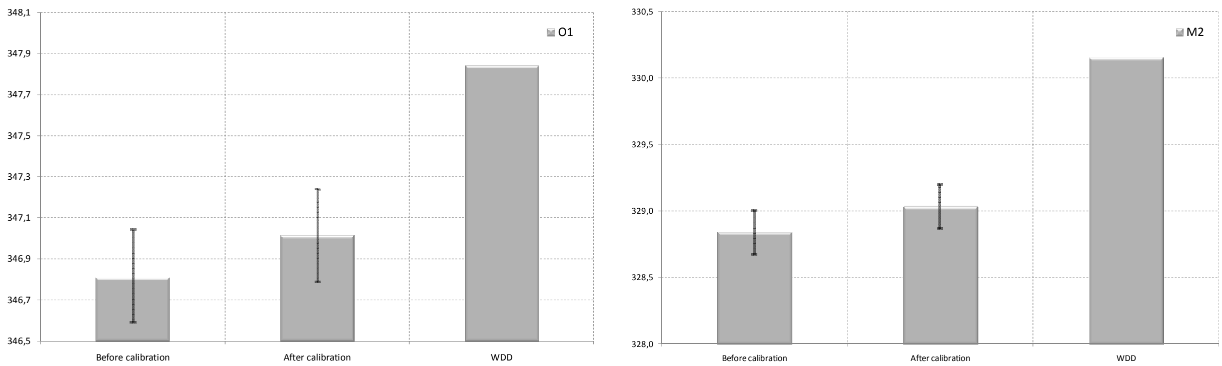


Fig. 8. Comparison of tidal amplitude O1 (left) and M2 with WDD Earth model [nm/s^2].

Following these charts the conclusion of correctness of this calibration was drawn.

## 7. CONCLUSIONS

Recently, the most precise tidal gravimetric measurements are collected using superconducting gravimeters, but spring gravimeters due to their availability are still used at many tidal stations. Their usage is mostly limited by the constant factors determination. The calibration has to be done periodically due to the instrumental characteristic of the relative gravimeters. The method presented in this paper is widely applied to tidal gravimeters, but its disadvantage is that it treats all frequency bands in the same way with single-frequency admittance. But from the other side it is very fast, convenient and cheap for AG owners.

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REFERENCES

1. Chojnicki T. (1970): „Wyznaczanie parametrów pływowych przez wyrównanie obserwacji metoda najmniejszych kwadratów”. Geodezja i kartografia 3/70, Nr 18, PP 152-182.
2. Dehant, V., Defraigne, P. and Wahr, J. (1999): “Tides for a convective Earth”. J. Geophys. Res., 104, B1, pp. 1035–1058, 1999.
3. Hartmann T., Wenzel H.-G. (1995): „The HW95 tidal potential catalogue”. Geophysical Research Letters. Vol.22, no. 24, pp 3553-3556, 1995.
4. Wenzel H.-G. (1996): ”The nanogal software: Earth tide processing package ETERNA 3.30”. Bulletin d'Information des Marées Terrestres (BIM), No. 124, pp. 9425-9439, Bruxelles, 1996.

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