UTILIZATION OF THE GPS OBSERVATIONS, GEOID AND PRECISE LEVELLING IN THE REALIZATION OF THE HEIGHT SYSTEM IN POLAND

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1. INTRODUCTION

This paper was inspired by a review of heights systems completed recently in Europe and numerous other countries. In 1994 Subcommission on Continental Networks for Europe (EUREF) proposed a new adjustment and enlargement of the United European Levelling Network to Eastern Europe. The final effect of these works was adjusted continental European levelling network named UELN -95/98 which heights were delivered all states participating in the project (Lang and Sacher,1996).

Simultaneously were conducted works over European Vertical Reference Network (EUVN). The aim of this project was integration of GPS, levelling network and tide gauge data in the static height system. The practical realization of the UELN and EUVN networks made possible to establish relationships between the UELN network with the initial point in Amsterdam (Normaal Amsterdams Peil) and the national (local) height systems. In the result of realizing the UELN and EUVN projects Subcommission EUREF defined the European Vertical Reference System 2000 (EVRS) (Ihde and Augath, 2000).

This paper starts with the fundamental definition of the various height systems classifying them into two groups: geometrical and physical/natural height systems. It simultaneously reviews the various vertical datum surfaces for these height systems and their practical realization. Then the influence of the Sun and Moon (permanent tide) on geoid (the potential of gravity), the levelling measurements and on the physical surface of the Earth was shortly explained in order to take into account the tide in a uniform way in height measurements. After that was described how the permanent tide was taken into account in the Polish precise levelling network, in GPS network (EUREF, POLREF) and in the gravimetric geoid. Finally the conception of the new height system in Poland was introduced.

2. HEIGHT SYSTEMS AND DATUMS

A height system is an one dimensional coordinate system used to define the metric distance of some point from a reference surface along a well defined line. Basically, there are two classes of height system. Ones that ignore the Earth's gravity field and thus use straight line, and those that are naturally linked to the equipotential surfaces and plumb lines of the Earth's gravity field and thus follow curved paths. The latter are of most practical and intuitive use.

In recent years, some authors (e.g. Kumar, 2005) have become proponents of purely ellipsoidal height systems, which neglect the effect of gravity. Clearly, these are unsuitable for any application that involves fluid flow in any way, among other reasons (e.g., Vaniček, 1998). However, there are cases where the use of ellipsoidal heights alone may suffice, such as the vertical component of an airborne mapping project or marine navigation where the body of a GPS-navigated ship has to clear seabed depths expressed in ellipsoidal heights (FIG Guide, 2006).

In Poland, ellipsoidal heights can refer to the old east European Krassowski ellipsoid or to the Geodetic Reference System 1980 (Moritz, 1980) reference ellipsoid, which is identical to the World Geodetic System 1984 (WGS84), reference ellipsoid (NEVIA, 2004) at the 0.1 mm level. Importantly, Krassowski and GRS80 ellipsoidal heights are not equal, and can differ for the same point up to as much as ~35m because of the different size, shape and orientation (regional or geocentric) of these ellipsoids. As such, Polish users dealing with ellipsoidal heights need to know what reference surface they apply: the Krassowski or GRS80/WGS84. Later, it will be shown that the datum is also essential information that must accompany ellipsoidal heights.

Natural or physical height systems come in several forms, depending principally on the treatment of gravity and thus the curved path over which the one-dimensional metric distance (height) is defined. They also depend on the choice of the reference surface used, though this is not as noticeable as it is for the ellipsoidal heights since the maximum differences are of 2 m.

Once the height system is selected and the appropriate corrections are applied to levelling observations, it is necessary to carry out a least squares adjustment of the corrected height differences so as to minimize the impact of random errors. Ideally, the adjustment should be performed on the geopotential numbers or height systems that have the theoretical zero misclosures.

The results of the least squares adjustment form the “final ” height values of all benchmarks. These adjusted heights and benchmarks define the vertical datum.

Obviously, the vertical datum will be different depending on the choice of height system and reference surface adopted.

This adjustment gives in a vertical geodetic datum the heights of the benchmarks at a particular epoch of adjustment. In Polish vertical datum Kronstadt 1960 and Kronstadt 1982 were implicitly assumed to refer to a single epoch.

The selection of the height system used in a vertical datum can be somewhat arbitrary, and seems to have depended on the proponents of a particular height system in each country.

Associated with the selection of the height system is the selection of the compatible reference surface on which the height is zero. Recall that the orthometric height system uses the geoid, the normal height system the quasigeoid, and the ellipsoidal height system the reference ellipsoid. The latter further depends on the use of a local or global ellipsoid as well as when the ellipsoid was defined. For instance, there are the GRS67 (IAG, 1967) and GRS80 (Moritz, 1980) global geocentric reference ellipsoids with different geometries (as well as different normal gravity fields). The choice of reference ellipsoid is simple to make, but the epoch of the adjustment to form the ellipsoidal height datum still needs to be defined, which will be described later. In the case of the quasigeoid and geoid, these datum levels have to be observed indirectly, which will be described next.

The ellipsoidal height h from GPS observation can be transform into the height H referred to the mean sea level (with the accuracy of fractions of millimetres) using formula:

$$H = h - N \tag{1}$$

where N is geoid-ellipsoid separation computed from gravimetric data.

This relationship is true when the influence of the Sun and Moon (permanent tide) on the geoid (the potential of gravity), the levelling measurements and on the physical surface of the Earth is treated in a uniform way. Therefore it is necessary to remind the basic information from the theory of tides.

3. THE PERMANENT TIDE

The gravitational force of the Sun and the Moon cause tides not only in the ocean but also in the lithosphere. In this respect the earth acts as an elastic body and undergoes the deformations. The deformations are periodic or permanent in time. All these deformation have influence on the gravimetric and levelling measurements (Ekman,1989).

The tidal potential of the point P on the surface of the earth is the function of the zenith angle of the celestial body and can be written down in entirely sufficient way using the formula:

$$W(P)=\frac{GMr^2}{d^3}\frac{1}{2}\left(3cos^2 z-1\right)+... \tag{2}$$

where G is the gravitation constant , M is the mass of the celestial body (Sun or Moon), r is the geocentric distance of the point P and d is the geocentric distance to the Sun or Moon. Neglecting in equation (2) the terms of the higher order we commit small mistake of the order 0.03%.

From the course of spherical astronomy it is known, that:

$$cos\,z=cosh\,cos\,\delta\,cos\,\varphi+sin\,\delta\,sin\,\varphi \tag{3}$$

The zenith angle z can be expressed as the function of declination δ and hour angle h of celestial body and latitude φ of the point P on the surface of the ground. Putting (3) into (2) we will receive the well known Laplace'a tidal formula:

$$\begin{aligned}W(A)&=D(cos^2\varphi\, cos^2\delta\, cos\,2h+\\ &sin\,2\varphi\, sin\,2\delta\, cosh+\\ &3(sin^2\varphi-\frac{1}{3})(sin^2\delta-\frac{1}{3}))\\ &=D(S+T+Z)\end{aligned} \tag{4}$$

where

$$D=\frac{3}{4}GM\frac{r^2}{d^3} \tag{5}$$

is Doodson's tidal constant[1].

In equation (4) terms S, T and Z are well known sectorial, tesseral and zone harmonics. Both S as and T are the functions of the hour angle h and so they change in the time quickly. S explains the phenomenon of the existence of the semi diurnal tide ($cos2h$) which maximum falls out on the equator and disappears on poles. T is responsible for the existence of the diurnal tides ($cosh$) which have their maximum on the width ± 45^o and zero value on poles and equator.

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[1] $2.63\times10^5[m\times mGal]$ **for the Moon and** $1.21\times10^5[m\times mGal]$ **for the Sun.**

The term T is the function of the declination of the celestial body and is responsible for long term tides, in the case of Moon for tide about 14 days period and in the case of Sun for tide about six months period. On latitude ± 35.27^o this term assume zero value and on poles the value of this tide is maximum.

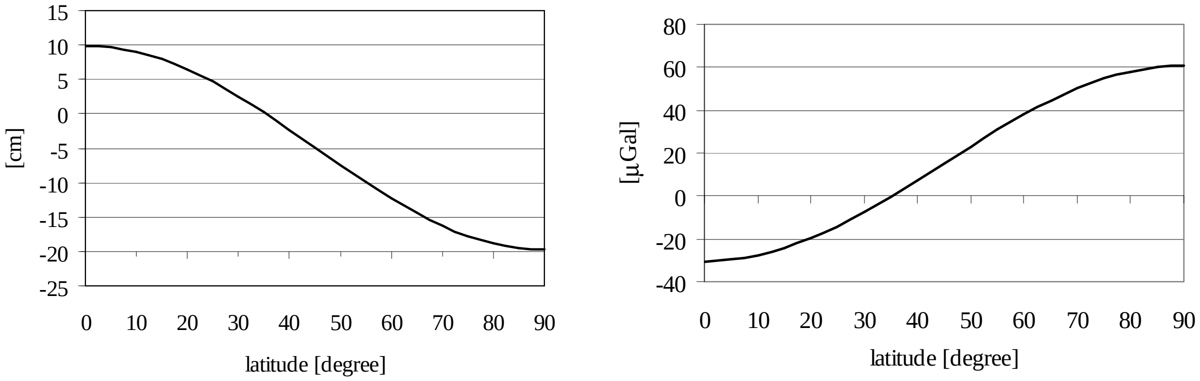

Fig. 1. Geoid deformation and gravity change due to permanent tide (McCarthy, 1992).

Mean value of S i T tides is equal zero. It means that:

$$\overline{W_{S,T}} = \frac{1}{2\pi}\int_{0}^{2\pi}(S+T)\,dh = 0 \tag{6}$$

In the opposition to terms S and T the zonal term Z is not equal zero. If we express the declination in the function of the ecliptic latitude ε and ecliptic longitude l we obtain (assuming in the first approximation ε as the constant):

$$\begin{aligned} \overline{W_2} &= \frac{1}{2\pi}\int_{0}^{2\pi} W_2 dl = \frac{D}{2\pi}\int_{0}^{2\pi} Z(\varepsilon,l)dl = \\ &= D\left(\frac{1}{2}sin^2\varepsilon - \frac{1}{3}\right)\left(3\,sin^2\varphi - 1\right) \end{aligned} \tag{7}$$

This tide is called *the permanent tide* and it was discovered by G. H. Darwin in 1899. This tide is function of latitude of the place of observation only. Geoid deformation and gravity change due to permanent tide are shown on Fig. 1.

3.1. Tidal deformations of the earth crust

From the equation (4) it is seen that the tidal potential is the variable potential in the time and it causes that the earth crust deforms periodically. The vertical oscillations of the earth crust are shown on Fig. 2 for period of 10 days in Cracow and they come to 25 cm maximally.

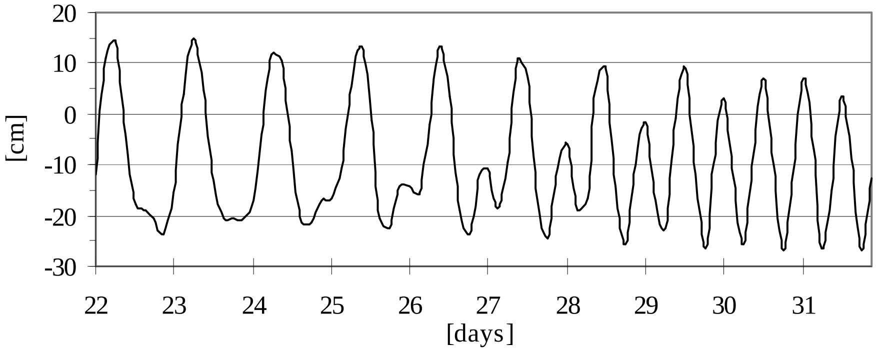

Fig. 2. Vertical oscillations of the earth crust in Cracow
(from September 22 to October 1, 1997).

The Earth reacts on the changing in the time tidal forces as the elastic body, however in the case of the uniform tide the Earth assume the hydrostatical equilibrium. The radial deformations of the Earth can be describe by the Love number κ_1 which in the case of the solid Earth is $\kappa_1 = 0$. In the case of the real Earth the value of the Love number depends how long the tidal forces act on the Earth.

In a first approximation the identical value of the Love number will be used for all oscillations being present in the tidal spectrum. Upon such assumption the radial deformations of the Earth crust are following:

$$\Delta r = \frac{\kappa_1 W_2}{g} \tag{8}$$

By analogy the horizontal deformations of the Earth crust are described by the Shida numbers *l*. In the case of the liquid Earth *l* = 0.23, and in the case of the solid Earth *l*= 0, however in the case of the real Earth *l* is in the interval about 0.08. The horizontal deformation in the northern direction is described by the equation:

$$\Delta s_\varphi = \frac{\ell}{g}\frac{\partial W_2}{\partial \varphi} \tag{9}$$

The deformation in the eastern direction by equation:

$$\Delta s_\lambda = \frac{\ell}{g \cos\varphi}\frac{\partial W_2}{\partial \lambda} \tag{10}$$

From equations (9) and (10) one can determine the vector of the horizontal deformation of the Earth crust for the given place.

3.2 Tidal deformations of equipotential surface

This paragraph is indispensable to explain farther considerations relating to the height system. Tidal deformations of the Earth crusts change the gravity potential of the Earth about $W' = \kappa_2 W_2$ where κ_2 is second Love number.

So now the geoid deformation in the vertical direction caused the change of the potential will be:

$$\Delta N_1 = \kappa_2 \frac{W_2}{g} \tag{11}$$

if only the potential of the Earth is taken under the consideration. However, if additionally the potential of tidal forces is included then the geoid deformation will be:

$$\Delta N_2 = (1 + \kappa_2)\frac{W_2}{g} \tag{12}$$

Mean value of equation (11) gives zero geoid, however from average of equation (12) we receive mean geoid. Removing all deformations we receive non tidal geoid (Fig. 3).

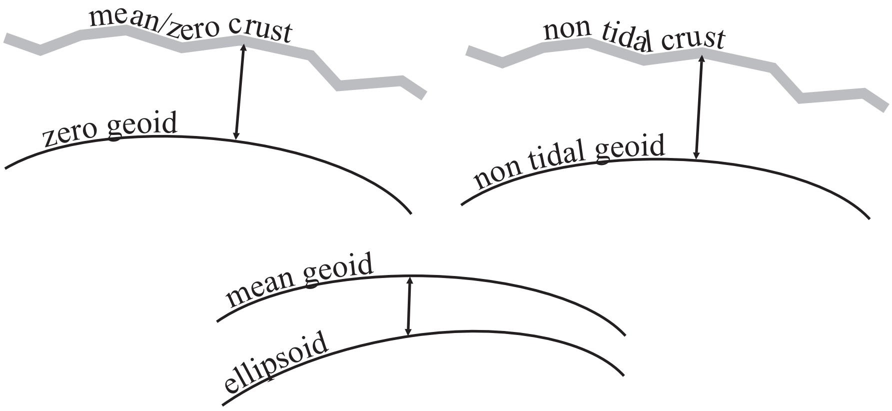

Fig. 3. Various kinds of surfaces discussed in the paper.

Summarizing we can say that non tidal geoid is a geoid computed from gravity data from which the <u>whole</u> tidal effects were removed. Mean geoid is a geoid computed from gravity data from which all tidal effects were removed <u>except</u> the permanent tide.

Zero geoid is a geoid computed from gravity data from which tidal attraction is eliminated whereas the permanent tidal deformation is retained.

We can say by analogy, that the non tidal Earth crust this such the crust from which periodic and permanent tidal deformations were eliminated and while permanent deformations were left in zero crust. There is no analogy in the case of the Earth crust from mean geoid because the mean crust is identical with zero crust.

4. HEIGHT AND GRAVIMETRIC SYSTEMS IN POLAND

Four campaigns of the precise levelling (Wyrzykowski, 1988), (Łyszkowicz and Leończyk, 2005), were conducted up to the present moment in Poland

- first campaign in years 1926 -1937,
- second campaign in years 1947 - 1958,
- third campaign in years 1974 – 1982,
- fourth campaign in years 1999 2003.

The result of these campaigns was established of a vertical datums called in the present work: datum 1939, datum Kronstadt 1960 and datum Kronstadt 1982. The results of the adjustment the last campaign temporary were called datum Kronstadt 2006 (Gajderowicz, 2007). The surfaces of the reference systems were defined variously. The results of first and second campaigns of the precise levelling were not corrected for tide, however the whole tidal effect was removed from the results of third and probable from the fourth campaign. This means that the mean geoid is the reference surface of vertical system 1939 and Kronstadt 1960 however in Kronstadt 1982 and Kronstadt 2006 the reference surface is non tidal geoid. The differences between these reference surfaces on the territory of Poland are from -0.8 up to 2.5 cm. The biggest differences occur if we compare mean and zero reference surfaces (Fig. 4).

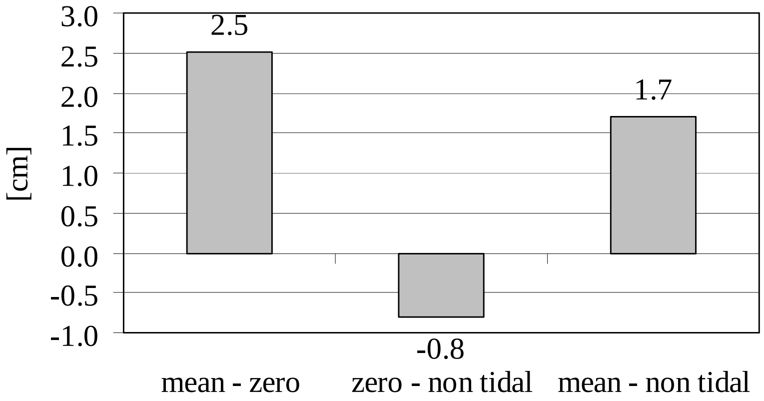

Fig. 4. Differences between the mean, zero and non tidal levelling reference surfaces on the territory of Poland.

4.1. The mean level of the Baltic sea

Not disturbed sea level should agree with geoid. In reality the oceanographic and meteorological factors produce deformations of the mean sea level and such deformations are called the topography of the sea (sea surface topography, SST).

SST can be determined using the precise levelling (Łyszkowicz A.,1995), or from GPS measurements. In order to correctly determine SST the heights of mean sea level should be referred to the mean geoid.

Because the last height system in Poland is referred to the non tidal geoid therefore in order to investigate Baltic SST, the heights of this system should be transformed to the mean geoid.

In Poland tidal stations are located almost along the same parallel so the corrections between non tidal and mean geoid they are small and do not exceed two millimetres. Because values of SST along the south coast of the Baltic sea are few centimetres (4 - 5 cm) therefore the corrections to the mean geoid cannot be neglected

4.2. Vertical crust movements in Poland

The investigation of the vertical movements has the long tradition in Poland. The first map of the vertical movements was published by T. Wyrzykowski in 1971 (Wyrzykowski,1971). Last map of vertical movements (Fig. 5) elaborated by K. Kowalczyk is based on the data from the third and fourth levelling campaigns (Kowalczyk, 2006).

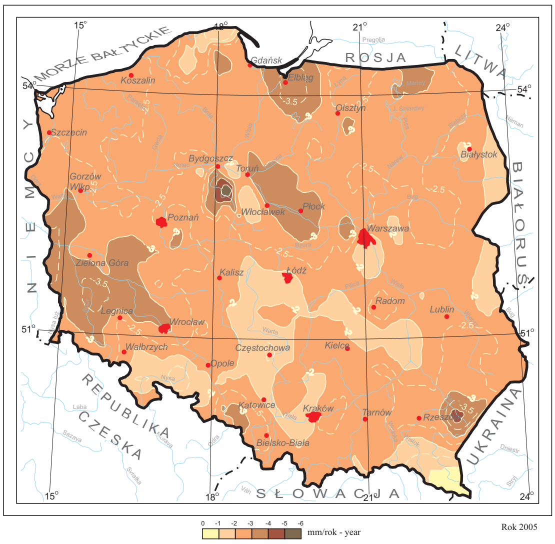

Fig. 5. Land uplift on the territory of Poland (Kowalczyk, 2006).

The last study of the vertical movements of the earth crust in Poland shows crust movements in the interval from -3.5 to 0.5 mm/year. Taking into account eustatic changes (estimated on 1 -1.5 mm/year) it gives the value of vertical movements in the range from -2 to +2 mm/year.
As it is known the vertical datums Kronstadt 1982 and Kronstadt 2006 are referred to different surfaces. That is why one should add corrections to values determined by Kowalczyk to get more probable values of vertical movements.
Gravimetric geoid in Poland. First gravity network in Poland was established and adjusted in 1964 based on 18 first order and 122 second order gravimetric points. The network was measured by the pendulum gravimeters and their accuracy is few ± 0.01 μms^{-2}. Then in 1999 a new gravimetric network called POGK99 was established. The network consisted of 12 absolute gravity points and 363 points measured by LaCoste&Romberg and Scintrex CG3M gravimeters. Accuracy of adjusted gravity points does not exceed ±0.10 μms^{-2} (Kryński, 2007, p.97). After additional relative and absolute measurements the network was adjusted again giving the new reference system called POGK04. In mentioned networks the all tidal effects were removed from gravity data <u>except</u> the permanent tide therefore the gravity is simply mean gravity. Differences between the mean gravity and non tidal and zero gravity for territory of Poland does not exceed 20 µGal (200 μms^{-2}), which produce difference in geoid up to 8 cm (see Fig. 7)

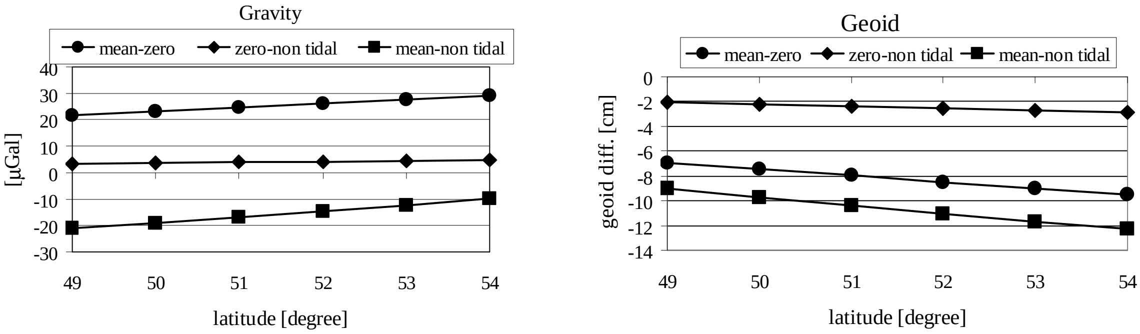

Fig. 6. Differences between mean, zero and non tidal gravity and geoid
on the territory of Poland.

In order to determine the orthometric or normal heights from GPS observations, additionally the geoid-ellipsoid separations are needed with as high as possible accuracy. The gravimetric geoid depends on the kind of gravity data used in computations. Since all tidal effects were removed from Polish gravity data except the permanent tide therefore the gravity is mean gravity and computed gravimetric geoid is mean geoid.

GPS networks in Poland. The results of the GPS campaigns in Poland are ellipsoidal heights between the ellipsoid GRS80 and non tidal crust. This results from the fact, that in all presently available GPS computer programs the deformations caused by permanent tide are eliminated. For example in the Bernese program equation 6 from work (McCarthy,1992) is applied in order to remove all effects caused by the permanent tide (procedure TIDALD). Also in definition of the system ITRF the deformations caused by the permanent tide are removed. In Poland maximum difference between mean/zero and non tidal crust is 1.6 cm and cannot be neglected.

5. THE PROPOSAL OF UNIFICATION OF EXISTING HEIGHT SYSTEM IN POLAND.

In Poland levelling the last two levelling networks are refereed to non tidal reference surface.

The invariable in the time part of the lunar-solar tide should be in the uniform way taken into account in levelling and GPS measurements as in geoid calculation. GPS software deliver coordinates reduced to the non tidal crust. The last two levelling networks refer to non tidal reference surface, ellipsoidal heights from GPS observation are refereed to non tidal crust while gravimetric geoid is mean geoid.

Differences between various geoids/crust amounts up to 10 cm on the territory of Poland (see Fig. 6). We can easily convert all quantities to their non tidal values using the following formulas (Ekman, 1989):

$$\begin{aligned} \Delta H_m - \Delta H_n &= 0.296\gamma\left(sin^2\varphi_N - sin^2\varphi_S\right) \\ N_m - N_n &= (1+k)\left(0.099 - 0.296 sin^2\varphi\right) \\ \Delta h_m - \Delta h_n &= -0.296h\left(sin^2\varphi_N - sin^2\varphi_s\right) \end{aligned} \qquad (13)$$

The first formula is used to convert height differences of the mean crust above the non tidal geoid to height differences of the non-tidal crust above the mean geoid and is appropriate for treating levelling.

The second formula converts the mean geoid heights above the ellipsoid to the non-tidal geoid heights and third formula converts height differences of the mean crust above the ellipsoid to the height differences of the non-tidal crust. In each case, the Love numbers used in the original non tidal calculations must be applied, regardless how close to or far from reality they are.

Described in the present work considerations concerning definition of the vertical system so that it would be possible combine GPS, precise levelling and gravimetric geoid leads to following postulates:

- the zero point is NAP (Normaal Amsterdams Peil),
- the heights are normal heights (resolution nr.2,Subcommission EUREF, Ankara 1996),
- permanent tide is completely removed so the crust refers to the non-tidal crust and the geoid is non-tidal geoid,
- the normal heights should be reduced from the national system epoch to the epoch e.g. 2000 using the land uplift relative to the geoid (i.e. the sum of the apparent uplift and eustatic rise of the sea level). The absolute uplift which can be obtained with GPS, can be converted to this “levelled” uplift by subtracting the rise of the geoid.

6. CONCLUSIONS

There is strong need for establishment in the future of the uniform national height system because of more and more high accuracies gained by the GPS technique. This technique in the contrast to classic geodetic measurements, with easiness cross the borders of countries. I am conscious that the present work does not exhaust all problems connected with creation of the new height system as it does not also give the exact algorithms of creation of this system. The aim of the present work is to draw attention on the complexity of the problem connected with the definition of the height system. The proposed system is the first step in the inclusion of Poland to the new European EVRS height system and to global height system, too. For final version of the new global height system we will have to wait surely not long.

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