# SEASONAL MODULATION OF THE TIDAL WAVES 

Janusz Bogusz ${ }^{1)}$<br>Magdalena Klek ${ }^{2)}$<br>${ }^{1)}$ Department of Geodesy and Geodetic Astronomy, Warsaw University of Technology<br>${ }^{2)}$ Institute of Geodesy and Cartography, Maria Sklodowska-Curie Warsaw Academy

## 1. INTRODUCTION

This presentation shows the results of studies on the modulation of tidal waves at station Jozefoslaw. Method based on elliptical regularization of tidal gravimetric data was presented. To facilitate the comparison of data for individual waves, the original results of observation analyses were adjust under the assumption that the graph depicting the modulation-dependent movement of the end of the vector representing parameters of the tidal waves is an ellipse (Chojnicki, 1996). In case of regular modulation the end of the vector representing parameters of the tidal wave changes in the time equal to the period of this modulation. The analysis of observations was performed by the classical method based on the least-squares principle. The data covered the time period of 3 years tidal observations made at Jozefoslaw Observatory using LaCoste\&Romberg ET26 gravimeter. This presentation contains also research on changes of the modulation caused by the ocean and atmosphere.

## 2. DATA

The gravimetric data was collected in Astro-Geodetic Observatory in Jozefoslaw from 2002 to 2005. The following chart presents these observations (units - nm/s^2).


## 3. DATA ADJUSTMENT

The data was divided into 1 -month parts and analysed separately using classical manner based on the lest squares method (Chojnicki, 1970) using Eterna 3.4 software (Wenzel, 1999). For each month amplitude $A$ and phase shift a was obtained. Additionally the data was corrected for the atmosphere and ocean in order to recognize how these two fluids affect seasonal modulations. The data from ocean correction was taken from Onsala'a service for ocean loading (www.oso.chalmers.se/~loading) using GOT00 model. The atmospheric correction was calculated using atmospheric data collected at the Observatory and single regression coefficient determined during previous researches (Bogusz, 2000).

## 4. APPROXIMATION

Parameters of the tidal wave may be presented as a vector whose amplitude is $\mathbf{A}$ and the direction is equal to the phase shift a. Due to modulation of amplitude and phase, the end of the vector would draw a closed curve which, depending on the degree of regularity of modulation, would more or less approximate an ellipse. The diagram of the movement of the vector's end, disregarding the much larger pattern of the whole vector, is an optimal manner to present the modulation.
One of the three coordinate systems can be used to present the effect of modulation as defined above (Chojnicki, 1996):

1. Polar coordinates $A, \alpha$;
2. Modified polar coordinates system $A$, a, with perpendicular axes and parallel $a=$ const lines ( $A \gg 0$ );
3. Orthogonal coordinates system $x=A \sin \alpha, y=A \cos \alpha$.

In the present work, the third of these was used. The following formulae were applied to determine the x and y coordinates:

$$
\begin{align*}
& x_{i}=A_{m} W_{i} \sin \alpha_{i}-A_{m} W_{m}  \tag{1}\\
& y_{i}=A_{m} W_{i} \cos \alpha_{i}-A_{m} W_{m}
\end{align*}
$$

where $W$ is an amplitude factor and the subscript $\mathbf{m}$ denotes mean value for 12 months. The introduction of mean values $A_{m}$ and $W_{m}$ results in location of the origin of coordinates in the middle of the graph.
Determination of the regularity of modulation of tidal waves might provide a valuable clue to direction of research which could bring elucidation of the causes of the phenomenon. It could be assumed, for instance, that in the case of regular modulation, the end of the vector representing parameters of the tide wave would draw an ellipse in the time equal to the period of this modulation. Such an ellipse can be determined by the least squares method from 12 months analyses. To this, two groups of correction equations should be constructed, separately for the $x$ and $y$ coordinates, using the general form of parametric equation of the ellipse. The correction equations $v$ would be the following:

$$
\begin{align*}
& v_{x i}=D_{x} \cos \left(\alpha_{0}+\omega t_{i}\right)+E_{x} \sin \left(\alpha_{0}+\omega t_{i}\right)+C_{x}-x_{i}  \tag{2}\\
& v_{y i}=D_{y} \cos \left(\alpha_{0}+\omega t_{i}\right)+E_{y} \sin \left(\alpha_{0}+\omega t_{i}\right)+C_{y}-y_{i}
\end{align*}
$$

where $C, D$ and $E$ are coefficients determined during adjustment, $\alpha_{0}=0$ is the initial phase of the ellipse parameter, $\omega=30^{\circ}$ is the coefficient of the parameter, $t$ is the parameter $(t=1,2, \ldots, 12), i$ is a subscript denoting the months.
After applications of the least squares method to correction equations, the unknown coefficient $C, D$ and $E$ are determined and used for construction of the regularization ellipse based on points whose coordinates are calculated from formulae:

$$
\begin{align*}
x_{i} & =D_{x} \cos \left(\alpha_{0}+\omega t_{i}\right)+E_{x} \sin \left(\alpha_{0}+\omega t_{i}\right)+C_{x}  \tag{3}\\
y_{i} & =D_{y} \cos \left(\alpha_{0}+\omega t_{i}\right)+E_{y} \sin \left(\alpha_{0}+\omega t_{i}\right)+C_{y}
\end{align*}
$$

Geometric interpretation of coefficients $C, D$ and $E$ is not necessary for determination of a regularized ellipse. It is worthwhile to notice that $C_{x}$ and $C_{y}$ are coordinates of the centre of the ellipse, and $D$ and $E$ are functions of the magnitude of semi-axes of the ellipse $a$ and $b$.

$$
\begin{align*}
& a=\sqrt{D_{x}^{2}+E_{x}^{2}} \\
& b=\sqrt{D_{y}^{2}+E_{y}^{2}} \tag{4}
\end{align*}
$$

## 5. DISCUSSION

All figures representing regularized ellipsis for individual waves. It concern they observation executed on years 2002-2004. The most approximated shape and position of ellipsis are to wave $\mathbf{J} 1$. It is possible to notice, that middle of ellipse changes his position from - 1 until 1.All three ellipsis are in this case elongated relevant $\mathbf{x}$ axis. Only ellipse which was arise based on date from 2004 is light turn in regard to ellipsis made based on data from easily years. It is proper to remark in case observations from 2003 and 2004 position of ellipses is very closed. It differs orientations and shape, although in some cases this parameters are so close. In case of 01 wave, shows, that elipse-2003 is oriented longer semi-axis according to $y$ - axis, however elipses-2004 longer semi-axis has situated parallel to $\mathbf{x}$-axis. In case ellipses made for observations made in years 20032004 this is the biggest differences in orientation of ellipses. For this wave ellipses-2002 and ellipses-2004 has closed shape while ellipses -2002 and 2003 has closed orientation. In case of S2, M2 and N2 waves it is possible to notice that ellipses determined for 2003 year has definitely shorter big semi-axes. For M2 wave ellipses 2002 is closed to circle but we can seen small differences between length semi-axes and this longer semi axes has orientations similar to orientation of the same semi-axes ellipses 2003. Similar situation, with regard in orientation and shape, we have in case S1 wave. Ellipses 2002 is closed to circle, and a little longer big semi-axes is oriented just as the others one. With regard to S 2 wave ellipses- 2002 except short twist big semi-axes for shape not much differ to the other, but position her middle, like the other cases, is different in reference to rest ellipses. In case orientation of big semi-axes the biggest differences we can see for N2 wave. Big semi-axes ellipses 2002 is situated like little semi axes in ellipses 2003 and 2004, this semi-axes is rotated over 90 grades, besides there is much longer. Also situation of its middle definitely differ to another.

## REFERENCES

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O1 wave - 2002


O1 wave - 2003


O1 wave - 2004


O1 wave - 2002 with atmosphere correction


O1 wave - 2003 with atmosphere correction


O1 wave - 2004 with atmosphere correction


O1 wave - 2002 with ocean correction


O1 wave - 2003 with ocean correction


O1 wave - 2004 with ocean correction



S1 wave - 2002


S1 wave - 2003


S1 wave - 2004


S1 wave - 2002 with atmosphere correction


S1 wave - 2003 with atmosphere correction


S1 wave - 2004 with atmosphere correction


S1 wave - 2002 with ocean correction


S1 wave - 2003 with ocean correction


S1 wave - 2004 with ocean correction


S2 wave - 2002


S2 wave - 2003


S2 wave - 2004


S2 wave - 2002 with atmosphere correction


S2 wave - 2003 with atmosphere correction


S2 wave - 2004 with atmosphere correction


S2 wave - 2002 with ocean correction


S2 wave - 2003
with ocean correction


S2 wave - 2004 with ocean correction

