CALCULATION OF VESSEL SPEED AND ACCELERATION VECTORS FROM GPS/DGPS MEASUREMENTS

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ABSTRACT

Position coordinate measurements are an intrinsic feature of navigation. These measurements include points on vessel's trajectory and trajectory derivatives: speed vector and acceleration vector. Due to the fact that systematic and random errors occur in both mathematical model of the navigation process and in specific measuring models, there is no full congruity of the results of measurements performed by various navigational instruments and systems. This work attempts at comparing trajectories, speeds and accelerations obtained by these` navigational devices and systems. The results may be used in examining the measurement reliability and the performance correctness of navigational equipment and system. The comparison of various sources of information also enables detection and identification of systematic errors. Consequently, mathematical models of particular phenomena and processes can be verified.

INTRODUCTION

The simplest concept of navigation may be understood as a section of 'knowledge on steering a vessel, methods of its position determination and choice of a track' [5] or – process of steering a vessel (vehicle) during its movement from one place to another. Another approach defines navigation as "science of methods and means of obtaining information on vessel position and movement and steering it from one point in the time-space to another point along a preset trajectory" [2]. Without going into further details describing navigation as a science and art we can state that the movement of navigational object is essential in navigation. If there is movement, we have to deal with a trajectory (track), speed and acceleration. Therefore, the basic feature of navigation is the fact it is connected with the movement of material bodies and with steering their movement.

Vessels' trajectory, speed and acceleration are interrelated. We do not use hydrodynamic equations of ship movement in practical navigation as some forces affecting the movement cannot be measured [1]. That is why we make relevant navigational measurements: points on a trajectory – observed positions (position

systems), speed components – courses and length of speed vector (compasses, logs) and acceleration components (accelerometers).

1. QUANTITIES DESCRIBING VESSEL MOVEMENT

The description of a body movement requires that the manner in which all its points are moving should be given. In general, the movement of a given body may be divided into two components: progressive motion and rotary motion. Progressive motion is such that all the points of a body move in the same way; the straight line joining any two points of the body travels parallel to itself. Rotary motion is such that all the points draw circles that lie on planes parallel to one another, with the centres of these circles lying on one straight line referred to as the axis of rotation. The motion of real objects is often a combination of the two motions – linear and rotary. Motion can be uniform, when it is described by an equation linear relative to time or it is non-uniform (variable), when it is described by an equation non-linear relative to time.

Vessel's movement is mostly described by kinematical equations of material point movement. Sometimes, however, such a description is not sufficient, particularly while maneuvering in areas whose sizes are comparable with vessel's size.

Let us specify the notions connected with vessel movement. Due to the character of vessel movement (in two- or three-dimensional space) we will be mainly using vector functions. Let us introduce the following notations and definitions [6]. Let *T* denote space (axis) of time, $T \supset I = (t_1, t_2)$ - fixed time interval.

Definition 1

The trajectory (track) of a material point in the interval *I* we call an image of the set *I* with vector representation

$$T \ni t \mapsto \mathbf{x}(t) \in E^3, \tag{1}$$

i.e. set $\mathbf{x}(t) \subset E^3$, E^3 – Euclidean space. We usually assume that $E^3 = R^3$. In marine (surface) navigation we reduce the space to two dimensions (ellipsoid, spherical or plane coordinates). The form of a function describing a trajectory in a given coordinate system is called the movement equation. In order to determine the movement of a material point we have to know the representation (1). This can be [8]:

- **vector equation of a curve,**
- **parametric equation of a curve,**
- **curve equation in the orthocartesian system.**

The movement equations should be twice differentiable, which results from Newtonian dynamics.

Definition 2

The speed vector (speed) of a material point at the moment *t* is called a derivative:

$$\mathbf{v}(t) = \frac{d\mathbf{x}(t)}{dt}. \tag{2}$$

Speed is a vector tangent to a trajectory (curve).

Definition 3
Acceleration is called a derivative of speed relative to time, i.e. the second derivative of the trajectory relative to time

$$\mathbf{a}(t)=\frac{d\mathbf{v}(t)}{dt}=\frac{d^2\mathbf{x}(t)}{dt^2}. \qquad \textbf{(3)}$$

Besides, higher derivatives of trajectory are also known in physics. The third derivative of a trajectory (first derivative of acceleration) is called the jerk (j) [7]. Higher derivatives are referred to as snap (s), crackle (c), pop (p). These notions were introduced as terms of the expansion of strong nonlinearity of movement [4], e.g. in the Hubble telescope control.
With the parametric representation of trajectory $I \ni t \mapsto \mathbf{x}(t) \in E^3$ we can introduce a new parameter defined by this equation

$$S(t)=\int_{t_1}^{t}\|\dot{\mathbf{x}}(\tau)\|d\tau, \qquad \textbf{(4)}$$

where the symbol $\|\cdot\|$ denotes a metric in the space E^3. The number $S(t)$ (length of trajectory $\mathbf{x}([t_1,t])$) expresses a track covered by a material point from the moment t_1 to the moment t. The function $[t_1,t]\ni t \mapsto S(t) \in R^1$ is a differentiable function where for any $t \in (t_1,t_2)$ this equation can be written

$$\frac{dS(t)}{dt}=\|\dot{\mathbf{x}}(t)\|=\|\mathbf{v}(t)\|. \qquad \textbf{(5)}$$

The derivative $S'(t) = V(t)$ describes speed. As the derivative $S'(t)=\|\mathbf{v}(t)\|=V(t)$ is non-negative, the function $S=S(t)$ is non-decreasing. If $\dot{\mathrm{x}}(t)$ does not disappear at any point of the interval (t_1,t_2), then there exists an inverse function $t=t(S)$. In this case we can parametrize points of a trajectory by a variable

$$[0,S(t_2)]\ni S \mapsto \mathbf{x}(t(S)) \in E^3. \qquad \textbf{(6)}$$

This is a natural description of a curve representing the trajectory of a material point. The following relations exist between speed and track:

$$V(t)=\frac{dS}{dt} \text{ and } S(t)=\int_{t_0}^{t}V(\tau)d\tau. \qquad \textbf{(7)}$$

2. COMPARISON OF NAVIGATIONAL DGPS, DR AND IMU MEASUREMENTS

Navigational position systems where time is discrete (in a DGPS receiver Δt ~ 1 second) measure coordinates of points on a trajectory burdened with measurement errors and movement disturbances (plus errors of time measurement). Besides, we do not know the form of a trajectory function (1). We disregard Doppler methods of measurement in receivers of navigational satellite systems, in which relative speed is measured (speed of receiver relative to transmitter). Therefore, using a DGPS we can determine speed, making use of the definition of mean (resultant) speed) [3]:

$$\mathbf{v}_{\acute{s}r}(t_2,t_1)=\frac{\Delta\mathbf{x}(t)}{\Delta t}=\frac{\mathbf{x}(t_2)-\mathbf{x}(t_1)}{t_2-t_1}. \qquad (8)$$

Let us note that the mean speed is a function of t_1 and t_2. In a general case the mean speed is different from an instantaneous speed. In uniform motion only the mean speed is constant and equal to the instantaneous speed.
It should also be noted that in navigational positioning systems we determine trajectory points from only one point attached to the vessel – receiver antenna. That is why position coordinate measurements do not describe ship movement as a solid body. For a two-dimensional (2D) position two antennas should be used, (or, additionally, data from dead reckoning navigation should be included), while in the case of a three-dimensional position (3D) – three antennas would be needed.
Accelerations in positioning systems are calculated as averaged values of the mean acceleration. This results from the fact that mean acceleration is expressed by this relation:

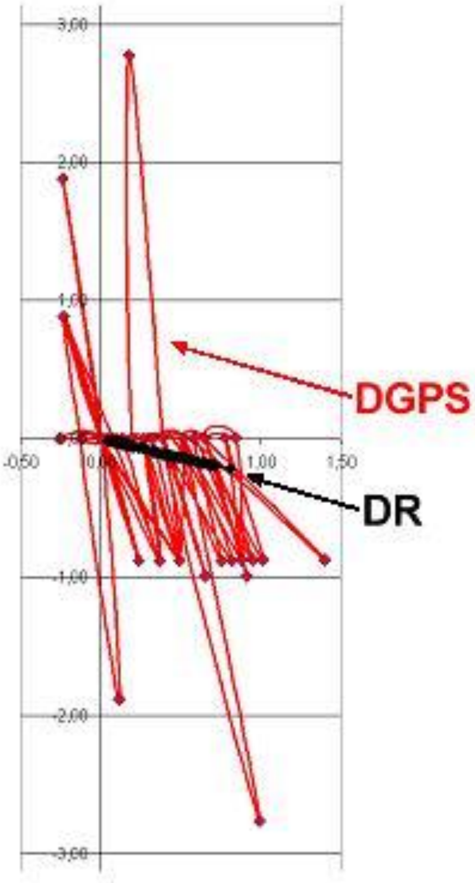
$$\mathbf{a}_{\acute{s}r}(t_2,t_1)=\frac{\Delta\mathbf{V}(t)}{\Delta t}=\frac{\mathbf{V}(t_2)-\mathbf{V}(t_1)}{t_2-t_1}. \qquad (9)$$

Mean acceleration is also a function of moments t_1 and t_2. However, the numerator of the formula (9) includes instantaneous values of speed, and a DGPS yields mean values – formula (8). Bearing in mind that the speed mean value is an arithmetic mean of instantaneous speeds and assuming that $\Delta t = t_{i+1} - t_i$ = const., after transformations we obtain the arithmetic mean of two mean accelerations, i.e.

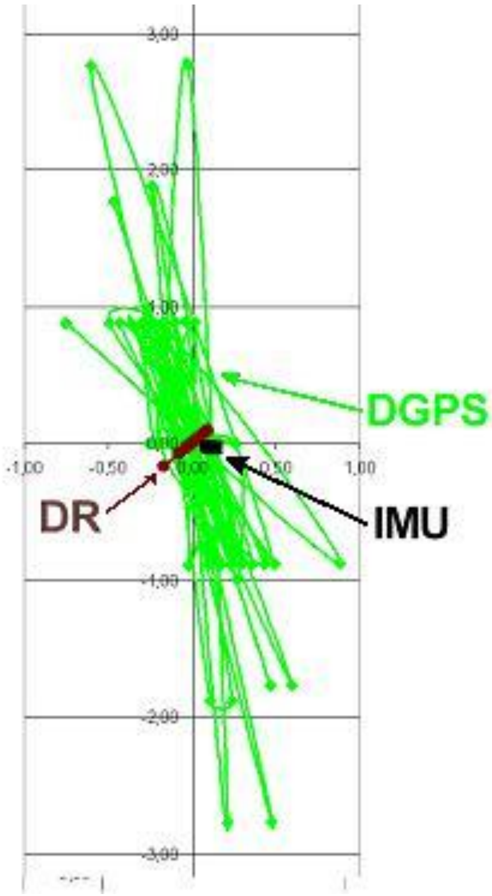
$$\bar{\mathbf{a}}_{\acute{s}r}=\frac{1}{2}(\mathbf{a}_{\acute{s}r}(t_3,t_2)+\mathbf{a}_{\acute{s}r}(t_2,t_1))=\frac{\mathbf{V}_{\acute{s}r}(t_3,t_2)-\mathbf{V}_{\acute{s}r}(t_2,t_1)}{\Delta t}. \qquad (10)$$

The vessel speed module, i.e. (instantaneous) speed is measured by means of a log, while speed direction by a compass. These measurements feature both random errors and essential systematic errors. Logs measure an instantaneous speed, as compared to navigational positioning systems, as the measuring frequency is about 8 Hz, whereas acceleration measured by a log is a mean value expressed by the relation (9).
Instantaneous acceleration is measured by accelerometers and compasses (direction). In this case the major source of errors is in the drift of accelerometers.
The graphic charts below show a comparison of measurements and calculations of speeds and accelerations. Figure 1a compares the vessel's mean speed calculated from

its position coordinates with instantaneous speeds obtained from a dead reckoning (DR) system, consisting of a log and gyrocompass. Figure 1b, in turn, presents a comparison of acceleration calculated from DGPS measurements, DR with instantaneous acceleration measured in an IMU inertial converter coupled with a gyrocompass.



a)



b)

Fig. 1. Comparison of navigational measurements: a) speed determined from DGPS and DR (log – gyrocompass); b) accelerations determined from DGPS, DR (log – gyrocompass) and an IMU – gyrocompass system

As it can be seen in the charts, calculated mean values differ substantially from instantaneous values. In general it is obvious. However, one should expect high deviations of the instantaneous values from the mean value, while in fact it is the opposite. This indicates poor accuracy of the measurements used for calculating the mean values of speed and acceleration.

Figure 2 presents an approximated trajectory of a vessel obtained from DGPS measurements by a third-order polynomial. The chart visibly shows the phenomenon of lower precision of coordinates measurements that resulted from discretization. A better approximation, particularly concerning turning of a vessel, is obtained by using the mean ARMA (Fig. 5.), but for better perception the analytical form of the trajectory is

more convenient. Figure 3 depicts a comparison of the mean speed calculated from raw DGPS measurements and a smoothened trajectory.

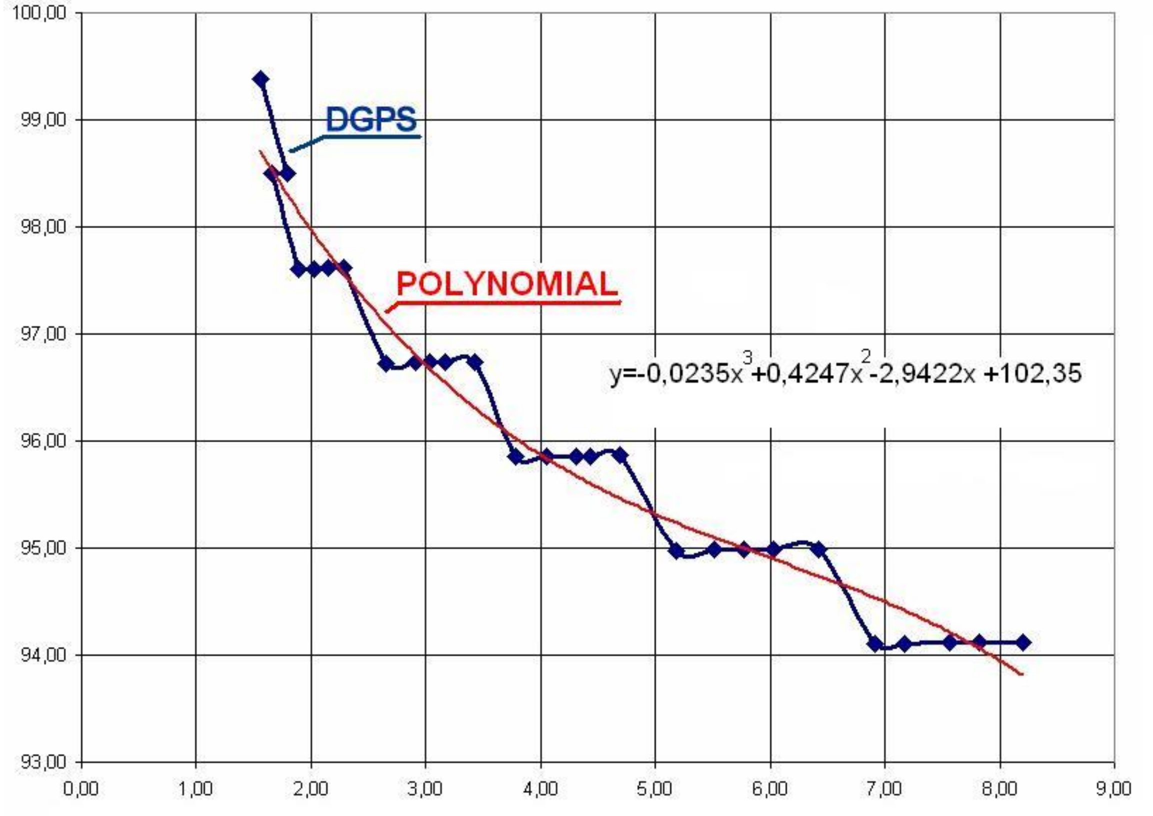


Fig. 2. Approximation of a trajectory (DGPS) by a third-order polynomial

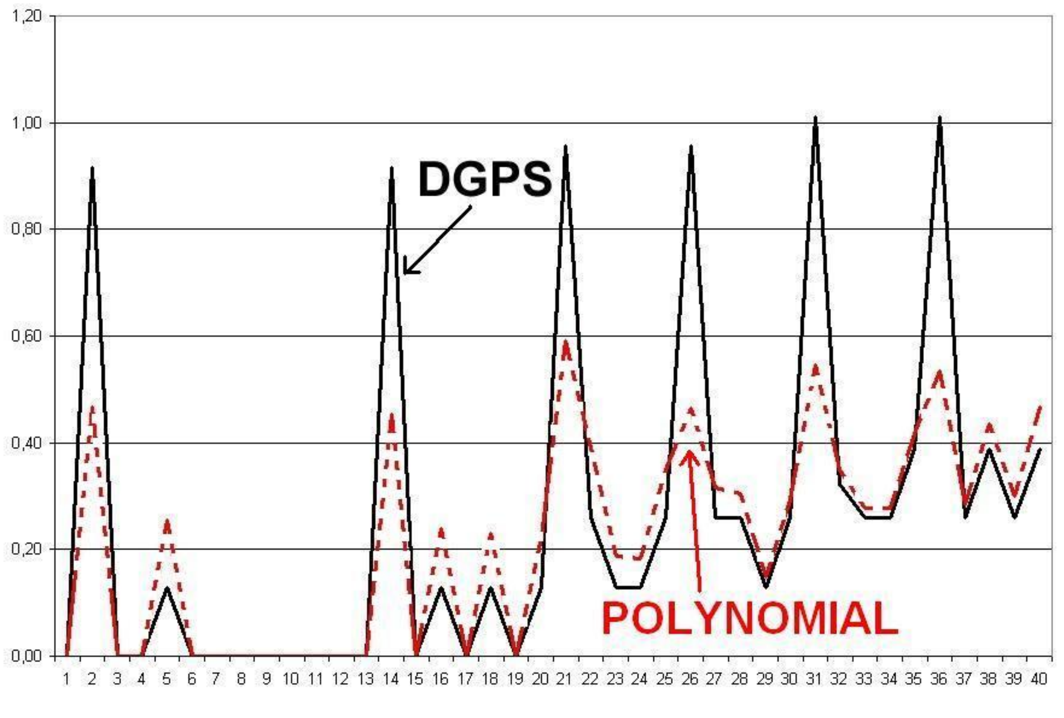


Fig. 3. Comparison of speeds determined from raw DGPS measurements and Those smoothened by a third-order polynomial.

Figure 4 illustrates hodographs of speeds of a vessel making a full turn, obtained from dead reckoning (DR) and the mean speed from DGPS. High peaks of the mean speed (DGPS) are clearly visible; besides, at the bottom of the diagram we will see a systematic error (drift) of log measurements. This is the consequence of the lack of measurements of the transverse component of ship's speed.

For navigational purposes trajectory derivatives can also be calculated – speed and acceleration. The inverse procedure is also performed, i.e. after integrating speed we obtain a trajectory (DR). It can also be obtained after twice integrating of accelerations (INS). The above mentioned Figure 5. presents a comparison of trajectories obtained from raw DGPS measurements with those calculated from speeds in dead reckoning navigation, the mean speed determined by a polynomial and the mean ARMA.

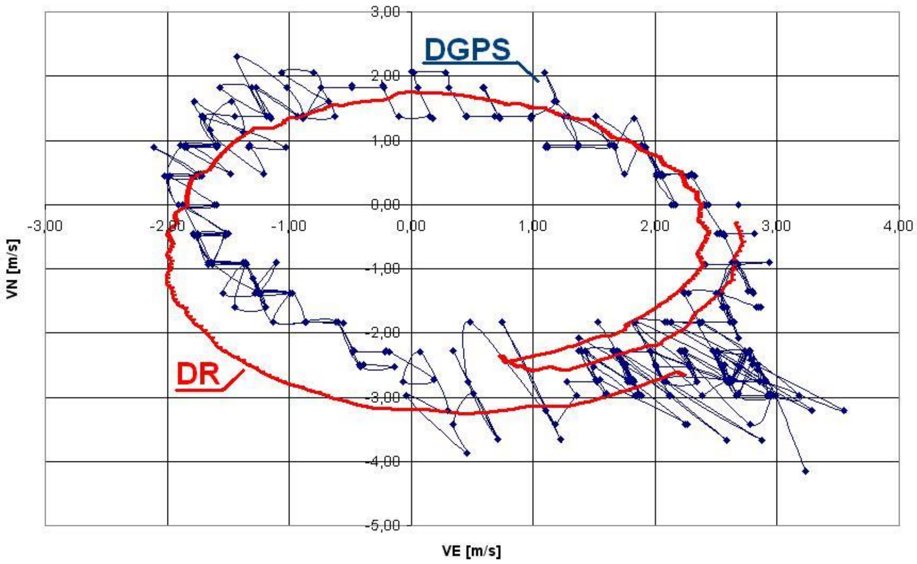


Fig. 4. Comparison of hodographs of speeds during vessel's turn from DGPS and DR (log-gyrocompass)

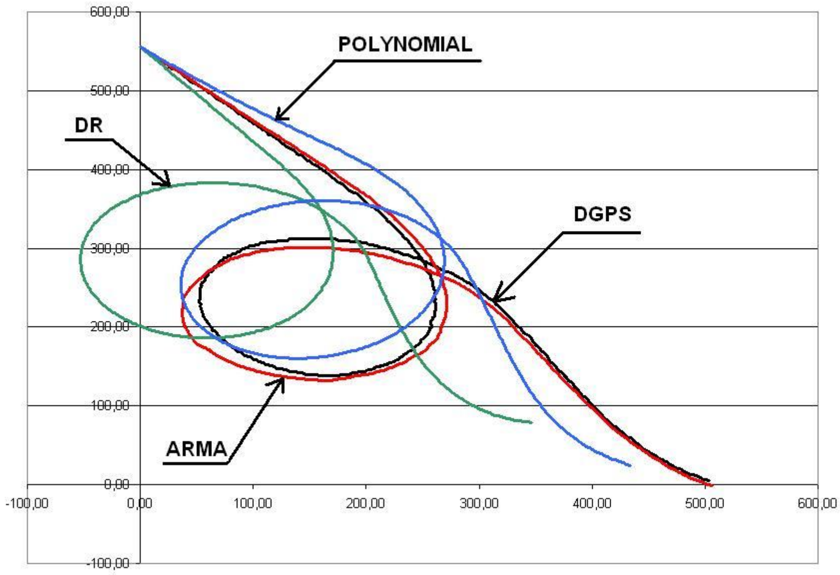


Fig. 5. Comparison of trajectories from DGPS, DR, smoothened by a polynomial and the mean ARMA

As we can see, the most deviated trajectory from the real one (raw DGPS measurements) is the DR trajectory. This results from the previously mentioned drift (systematic error). On the other hand, deviations from the trajectories calculated with the use of mean speed (approximation by a polynomial and the mean ARMA) result from a specific inertia of the mean value as compared to the instantaneous speed value.

SUMMMARY

As presented above, the differences between mean values of speed and acceleration calculated from points on a vessel's trajectory or log measurements are affected by a number of factors that normally are not considered. These can be included in the following conclusions:

- **interval of measurement discretization Δt in (D)GPS has a significant effect on the accuracy of calculated speeds and accelerations (only low frequency components are reproduced, which results from Shannon's theorem),**
- **accuracy and precision of coordinate determination have a similar effect,**
- **too long a period of smoothening also leads to false results, e.g. on a ship's full turn,**
- **with one DGPS receiver the ship's trajectory reproduced is not that of a solid body; this is particularly important when the antenna is located outside the hull axis of turn or the axis is variable,**
- **measurement integrating systems ((D)GPS and DR/IMU) provide very good results of estimation of trajectory, speed and acceleration.**

It turns out that, basically, presently used measurement methods – DGPS receivers and DR navigation and inertial devices – they may complement, but not replace each other [4]. Experiments with determining speed from Doppler DGPS measurements have been promising [8].

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