THE ACCURACY ASSESSMENT IN DEAD RECKONING NAVIGATION

Andrzej Banachowicz (1), Grzegorz Banachowicz (2), Adam Wolski (2)

(1) Gdynia Maritime University (Poland)
(2) Maritime University of Szczecin (Poland)

SUMMARY

Dead reckoning is one of the oldest methods of ship's position determination in sea navigation. Its advantages are autonomy and possible integration with other position determination methods, in particular with the statistical model for working out measurements of navigational parameters. Dead reckoning makes use of basic navigational instruments that every ship is equipped with, i.e. the compass and the log. Basically, the method pertains to calculating ship's coordinates for a defined moment of time by means of the integration of ship's speed and acceleration in the case of analogue measurements, or by calculating the increment of the coordinates from differential equations in the case of discrete measurements of speed or acceleration. The present position is determined as a sum of the coordinates increments.

INTRODUCTION

Dead reckoning is one of the methods of the vessel position coordinates determination. The method allows to determine a ship's position at any given moment of time interval for which we know the initial position and the track covered or speed vector (or acceleration vector). The distance covered or speed vector are not functionally determined a priori, they are obtained from measurements. In integrated navigational systems we use measurements of instantaneous speed (or acceleration) for determining present position coordinates. The navigational instruments used for this purpose can be combined in a variety of ways. In Doppler (hydroacoustic) and inertial systems the direction and module of speed (acceleration) vector relative to the bottom are measured. In the other cases the speed vector direction and module are measured relative to the water. The following cases can be distinguished:

1. course measurement by means of a magnetic compass:
- **one-component relative log,**
- **two-component relative log,**
- **two-component absolute log or accelerometers system;**

2. course measurement by means of compasses other than magnetic (gyrocompasses, gyroazimuths, laser etc.):
- one-component relative log,
- two-component relative log,
- two-component absolute log or accelerometers system.

Relative logs measure ship's speed through the water (relative to the water). In navigation logs measuring speed over ground (Earth's non-moving surface) are called absolute logs, which term does not comply with the notion of motion relativity, in this case ship's motion is relative to the bottom. Although this can sometimes lead to misunderstandings, the term is strongly rooted in navigational practice.

The principles of calculating ship position coordinates in dead reckoning navigation and its accuracies will be described as a general case, where navigational parameters measurements are performed relative to the bottom, with differences given for measurements relative to the water. Our considerations, due to the character of measurements, refer to discrete systems. In the case of analogue systems, (nowadays practically not used) the relevant formulas will feature integers instead of summing up.

1. CALCULATING THE COMPONENTS OF SPEED VECTOR

When measuring the ship's speed relative to the bottom (absolute log) we obtain its two components – longitudinal V_x and transverse V_y (Fig. 1). Their vector sum yields the vector of speed over ground V_d (in the local coordinate system related to the ship).

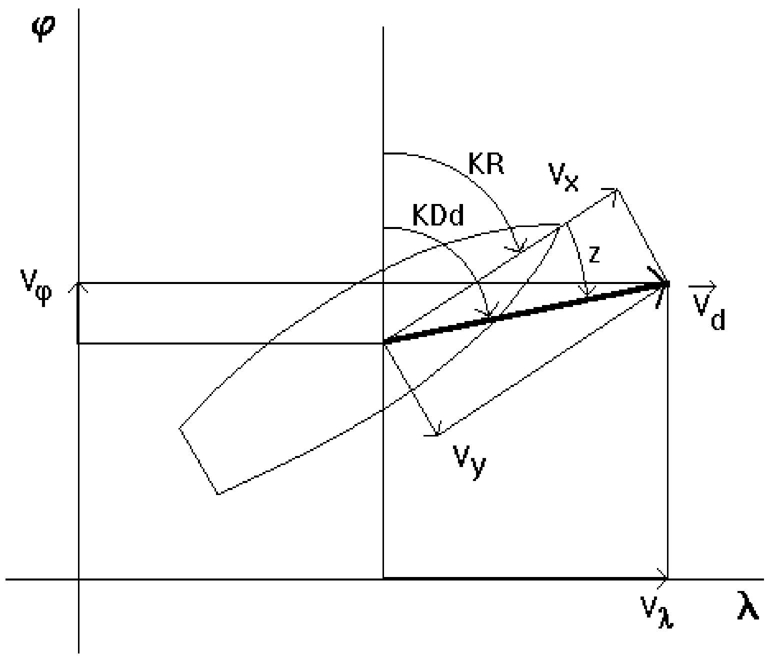

Fig. 1. Ship speed vector components for the absolute log.

The speed vector components (or acceleration) are not directly measured in the global system (related to the Earth). The log is used for indirect measurements of speed vector module, and, having accounted for relevant corrections, its direction is measured by a compass. The speed module (resultant speed) is calculated from this formula:

$$V = \sqrt{V_x^2 + V_y^2}. \tag{1}$$

where:

- **$V = V_d$ for the absolute log,**
- **$V = V_w$ for the relative log.**

The speed vector direction relative to the bottom is determined as the course made good. When a magnetic compass is used, the proper angle (course) is calculated from this formula

$$KDd = KK + d + \delta + z, \tag{2}$$

where:

KK – compass course (measured),

δ - deviation,

d - declination,

z – angle of total leeway.

If the course is measured by compasses other than magnetic ones, then course made good is calculated from the formula below (for a gyrocompass):

$$KDd = K\dot{Z} + p\dot{z} + z, \tag{3}$$

where:

$K\dot{Z}$ – gyrocompass course (or course determined by another compass),

$p\dot{z}$ – total correction of the gyrocompass (or another compass).

The total leeway angle is determined by this formula

$$z = \alpha + \beta, \tag{4}$$

where:

α - drift angle,

β - leeway angle.

The total drift or leeway angle, in turn, is computed using this formula

- **Two-component relative log**

$$\alpha = \text{arc tg}\frac{V_y}{V_x}, \tag{5}$$

- **Two-component absolute log**

$$z = \text{arc tg}\frac{V_y}{V_x}. \tag{6}$$

The components of the vector of speed trough the water will be as follows:

- **meridian**

$$V_{N_w} = V_w \cos KDw, \tag{7}$$

- **parallel**

$$V_{E_w} = V_w \sin KDw. \tag{8}$$

By analogy, we calculate the current vector components:
- **meridian**

$$V_{N_p} = V_p \cos K_p, \tag{9}$$

- **parallel**

$$V_{E_p} = V_p \cos K_p. \tag{10}$$

hence the components of speed over ground are as follows:
a) relative log
- **meridian**

$$V_N = V_{N_w} + V_{N_p}, \tag{11}$$

- **parallel**

$$V_E = V_{E_w} + V_{E_p}, \tag{12}$$

a) absolute log
- **meridian**

$$\boldsymbol{V_N = V_d \,.\, \cos KDd,} \tag{13}$$

- **parallel**

$$\boldsymbol{V_E = V_d \,.\, \sin KDd.} \tag{14}$$

In inertial systems acceleration components are measured, while average speeds are expressed by these formulas:

$$\begin{aligned} V_x &= V_{x(\log)} + a_x \cdot dt, \\ V_y &= V_{y(\log)} + a_y \cdot dt, \end{aligned} \tag{15}$$

where:
a_x **,** a_y **– acceleration vector components,**
dt **– time interval between speed measurements by a log,**
$V_{(logu)}$ **– speed measured by a log.**

The speed components in formulas (7) – (15) are determined in the same speed units as those measured, i.e. knots or m/s. Let us change these units to the appropriate units of the axes, i.e. angular units of the meridian and parallel (angular measure of the geographical coordinates on an ellipsoid). The linear speeds above, found in the SI system, will be expressed in m/s, while angles in radians. Therefore, ship's speed over ground will be expressed in the Earth's coordinate system as, respectively:

- **meridian component (N)**

$$V_\varphi = k_\varphi \bullet V_N, \tag{16}$$

- **parallel component (E)**

$$V_\lambda = k_\lambda \bullet V_E, \tag{17}$$

where:

$$k_\varphi = \frac{1}{R_M} = \frac{\sqrt{\left(1-e^2\sin^2\varphi\right)^3}}{a\left(1-e^2\right)}, \tag{18}$$

$$k_\lambda = \frac{1}{R_N\cos\varphi} = \frac{\sqrt{1-e^2\sin^2\varphi}}{a\cos\varphi}, \tag{19}$$

φ **- geographic latitude,**
λ **- geographic longitude,**
a **– semi-major axis of the Earth's ellipsoid,**
e **– first eccentricity of the Earth's ellipsoid,**
R_M **– radius of meridian curvature,**
R_N **– radius of first vertical.**

2. CALCULATION OF THE POSITION COORDINATES

Coordinates of a dead reckoning position at the moment t_{i+1} is calculated from the following formulas:

- **latitude**

$$\varphi_{i+1} = \varphi_i + V_{\varphi_i}\left(t_{i+1}-t_i\right), \tag{20}$$

- **longitude**

$$\lambda_{i+1} = \lambda_i + V_{\lambda_i}\left(t_{i+1}-t_i\right). \tag{21}$$

The system of equations (20), (21) can be written as a vector equation

$$\mathbf{x}_{i+1} = \mathbf{x}_i + (t_{i+1}-t_i)\mathbf{V}_i\,, \tag{22}$$

where:

$\mathbf{x}=[\varphi,\lambda]^{\mathrm{T}}$ **- state vector (position coordinates),**

$\mathbf{V}=[V_{\varphi},V_{\lambda}]^{\mathrm{T}}$ **- speed vector.**

With the equation (22) we can formally regard calculations of ship's position coordinates as a recurrent sum of two vectors.

3. ASSESSMENT OF DEAD RECKONING POSITION ACCURACY

The accuracy of a dead reckoned position calculated by using the equation (22) is determined according to the following relations [6]. The matrix of coordinates covariance at the moment t_{i+1} equals this sum:

$$\mathbf{P}_{z_{i+1}}=\mathbf{P}_{z_i}+\left(t_{i+1}-t_i\right)^2\mathbf{P}_{V_i}, \tag{23}$$

where:

$\mathbf{P}_z$ **– dead reckoned position covariance matrix,**

$\mathbf{P}_V$ **– speed vector covariance matrix.**

A general form of these matrices is as follows:

- **dead reckoned position covariance matrix**

$$\mathbf{P}_z=\begin{bmatrix}\sigma_\varphi^2 & \sigma_{\varphi\lambda}\\ \sigma_{\varphi\lambda} & \sigma_\lambda^2\end{bmatrix}, \tag{24}$$

where:

σ_φ^2 **– latitude variance,**

σ_λ^2 **– longitude variance,**

$\sigma_{\varphi\lambda}$ **- covariance between latitude and longitude.**

At the zero step (t_0) we take the values of the position covariance matrix elements from the initial position covariance matrix.

- **Speed vector covariance matrix:**

$$\mathbf{P}_V=\begin{bmatrix}\sigma_{V_\varphi}^2 & \sigma_{V_\varphi V_\lambda}\\ \sigma_{V_\varphi V_\lambda} & \sigma_{V_{\lambda E}}^2\end{bmatrix}, \tag{25}$$

where:

$\sigma_{V_\varphi}^2$ **- variance of speed component along the meridian,**

$\sigma_{V_\lambda}^2$ **- variance of speed component along the parallel,**

$\sigma_{V_\varphi V_\lambda}$ **- covariance between the speed components.**

Particular quantities are calculated from these formulas:

$$\sigma_{V_\varphi}^2 = k_\varphi^2\left[\left(\sigma_{V_d}\cos KDd\right)^2 + \left(V_d\sigma_{KDd}\sin KDd\right)^2\right], \tag{26}$$

$$\sigma_{V_\lambda}^2 = k_\lambda^2\left[\left(\sigma_{V_d}\sin KDd\right)^2 + \left(V_d\sigma_{KDd}\cos KDd\right)^2\right], \tag{27}$$

$$\sigma_{V_\varphi V_\lambda} = \frac{1}{2}\left(k_\varphi^2\sigma_{V_d}^2 - k_\lambda^2 V_d^2\sigma_{KDd}^2\right)\sin 2KDd, \tag{28}$$

where:

σ_{V_d} **- mean error of determining speed over ground;**

σ_{KDd} **- mean error of determining course made good; we use here the following formula:**

$$\sigma_{KDd} = \sqrt{\sigma_{K\dot{Z}}^2 + \sigma_{cp_{\dot{Z}}}^2 + \sigma_z^2}\,. \tag{29}$$

The mean error of position (*M*) is calculated by this relation

$$M = \sqrt{\operatorname{tr}\mathbf{P_z}} = \sqrt{\left(\frac{\sigma_\varphi}{k_\varphi}\right)^2 + \left(\frac{\sigma_\lambda}{k_\lambda}\right)^2}\,. \tag{30}$$

According to IMO recommendations, the mean circle of error 95% should be used in navigation. From the practical point of view, navigation can make use of the double mean error 2*drms*), the probability of which ranges from 95.5-98.2%. We will therefore assess position accuracy using the double mean error calculated from this formula

$$2M = 2\sqrt{\left(\frac{\sigma_\varphi}{k_\varphi}\right)^2 + \left(\frac{\sigma_\lambda}{k_\lambda}\right)^2}\,. \tag{31}$$

Figure 2 shows differences between the deterministic (a) and stochastic (b) method of determining ship position coordinates in dead reckoning.

CONCLUSION

From the viewpoint of the mathematical model (formula (22)) the method of dead reckoning can be considered as a linear stochastic dynamic system, in which we determine position coordinates at any moment of ship's motion on the basis of speed vector measurements. Compared to classical methods, the presented above dead reckoning navigation method has a number of advantages. Firstly, it is assumed that measurements are random and their certain accuracy (measurement errors of all navigational parameters are allowed for). Secondly, it can describe a dynamic system of any small time interval between measurements, as an analogue system in particular. This permits for a reproduction of an actual curvilinear trajectory of a ship. In practice it is most often assumed that ship's movement goes along sufficiently long section of rhumb lines, which leads to significant systematic errors (error of the method). Thirdly,

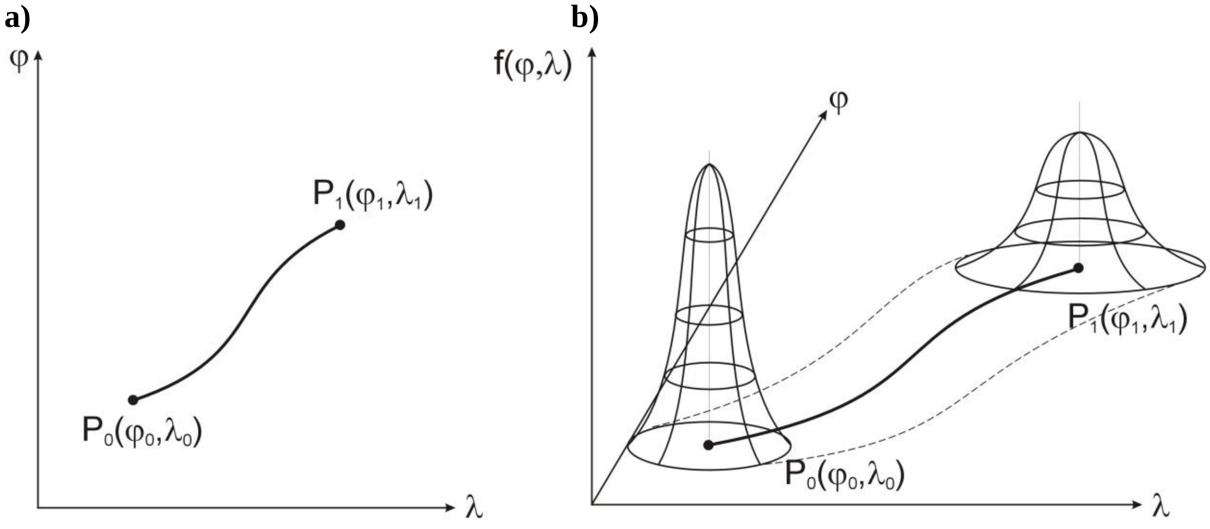

Fig. 2. Comparison of dead reckoning positions: a) deterministic, b) estimated.

it can be successfully used in integrated navigational systems [2], [3], [5]. Fourthly, calculations are made directly on the surface of a relevant reference ellipsoid.

If ship speed vector measurements refer to speed through the water (relative to the water), then to calculate position coordinates and its accuracy we will use the vector of speed over ground (relative to the bottom) calculated by the formulas (11), (12). In this connection, in the formulas (26) – (29) we have to allow for the fact that the vector of speed over ground is a sum of two vectors – speed through the water and leeway.

Elements of the mean error ellipsis can be calculated directly from the covariance matrix of position coordinates reckoned at each step of the estimation [4].

REFERENCES

[1] Banachowicz A., 1985, *Teoretyczne podstawy modeli nawigacji zliczeniowej*, Rozprawa doktorska. Wydział Nawigacji i Uzbrojenia Okrętowego Wyższej Szkoły Marynarki Wojennej, Gdynia 1985.

[2] Banachowicz A., Urbański J., 1986, *Nawigacja zliczeniowa*, Zeszyty Naukowe, WSMW Nr 4, Gdynia 1986.

[3] Banachowicz A., 1988, *Ocena dokładności pozycji w nawigacji zliczeniowej*, Zeszyty Naukowe AMW Nr 2, Gdynia 1988.

[4] Banachowicz A., 1991, *Wykorzystanie macierzy kowariancji pozycji do obliczania elementów elipsy lub elipsoidy błędów*, Zeszyty Naukowe AMW Nr 2, Gdynia 1991.

[5] Banachowicz A., 2001, *Variants of Structural and Measurement Models of an Integrated Navigational System*. Polish Academy of Sciences, Annual of Navigation. No 3. 2001.

[6] Banachowicz A., 2006, *Uogólnione prawo przenoszenia błędów losowych*, Prace Wydziału Nawigacyjnego Akademii Morskiej w Gdyni Nr 18. AM, Gdynia 2006.

[7] Banachowicz A., 2006, *Elementy geometryczne elipsoidy ziemskiej*, Prace Wydziału Nawigacyjnego Akademii Morskiej w Gdyni Nr 18. AM, Gdynia 2006.