WGS-84 ELLIPSOID AS VERTICAL REFERENCE SYSTEM FOR HYDROGRAPHIC SURVEYS

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ABSTRACT

Depth measurement, as an element of hydrographic surveys, is realised in local reference system connected with a hydrographic vessel. Taking echosounder transducer's draught into consideration enables determination of the distance between water surface and the bottom. Sea level is changeable, especially in tide areas, even during hydrographic surveys. This fact created the necessity to work out many vertical reference systems. In tideless areas MSL – Mean Sea Level is generally used.
Alternative for many vertical reference systems can be determination of the depth on reference ellipsoid using geopotential model of the Earth. This solution enable use of positioning systems for depth measurements and reduction of heaving.

INTRODUCTION

During hydrographic surveys, three different reference surfaces (figures) are used:

- **natural physic surface of the Earth,**
- **reference ellipsoid,**
- **equipotential (level) surface of the Earth's gravity field.**

Because of complicated and compound shape of the Earth, there are many reference systems for description the Earth's shape. In hydrography (in tideless areas), the level of seas and oceans with full balance of water masses fulfills Mean Sea Level.
For description the shape of sea bottom in navigational charts, there are used two reference systems: one of them (horizontal) is connected with the reference ellipsoid, second one (vertical) is connected with the sea's level. During GPS measurements there is possible to compare horizontal component of position (λ, φ) obtained using GPS receiver with vertical component read from navigational chart. For vertical component of the position H obtained using GPS receiver and h obtained from the chart, there is not possible to compare them because of different vertical reference systems. The solution of this problem can be establishing the model of the geoid for calculating separation between the geoid and the reference ellipsoid.
The geoid is that equipotential (level) surface of the Earth's gravity field which, on average, coincides with mean sea level in the open undisturbed ocean. In practical terms, the geoid is the imaginary surface where the oceans would seek mean sea level if allowed to continue into all land areas so as to encircle the Earth. The geoid

undulates up and down with local variations in the mass and density of the Earth. The local direction of gravity is always perpendicular to the geoid.
The orthometric height is the height of a point above the geoid as measured along the plumbline between the geoid and a point on the Earth's surface, taken positive upward from the geoid. It is defined as H in the equation:

$$h = H + N \tag{1}$$

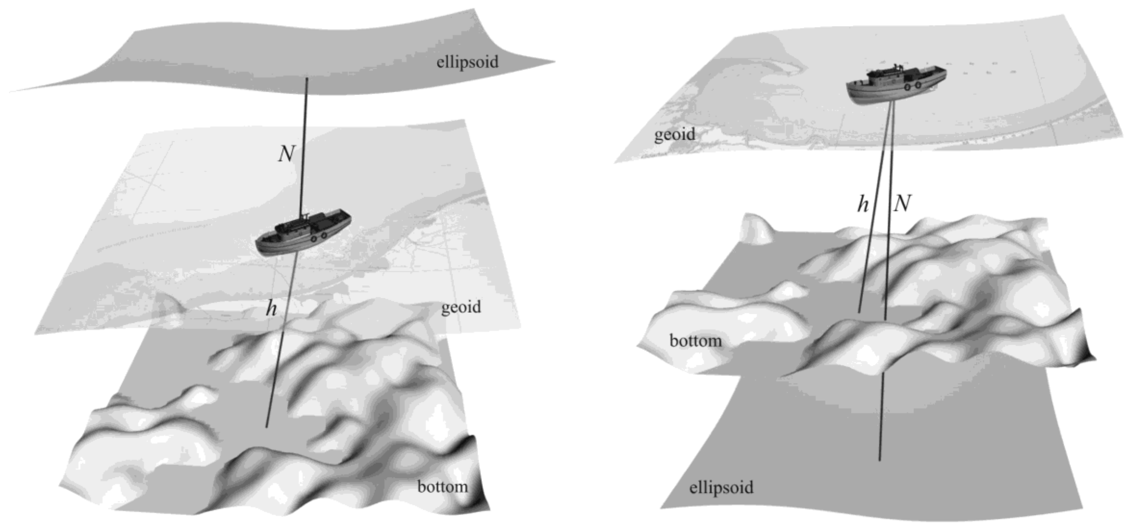

Fig. 1. Location of geoid, ellipsoid and Earth's surface

COORDINATE CONVERSION – RELATIONSHIP BETWEEN ELLIPSOIDAL AND CARTESIAN COORDINATES

To determine the position of a point in space, a three-dimensional coordinate reference system is necessary. Each reference system can be mapped into infinitely many curvilinear coordinate systems. Coordinate systems establish the ordered relation between physical points in space and real numbers (coordinates).
In modern three-dimensional geodesy, the three-dimensional Cartesian coordinate system is applied for global tasks. It is defined by three orthogonal coordinate axes which form a right-handed system. The X, Y, Z coordinate axes intersect each other at the origin of the coordinate system.
The coordinate lines of the ellipsoidal coordinate system are curvilinear lines on the surface of the ellipsoid. They are called parallels at constant latitude (φ) and meridians for constant longitude (λ).
When the ellipsoid is related to the shape of the Earth, the ellipsoidal coordinates are named geodetic coordinates. Traditionally an alternative to geodetic coordinates have been astronomical latitude and astronomical longitude. Geographic coordinates is the generic term including astronomical latitude and astronomical longitude as well as geodetic latitude and geodetic longitude. Since astronomical observations have been taken over by satellite based methods, in most cases today the term geographic coordinates implies geodetic coordinates.

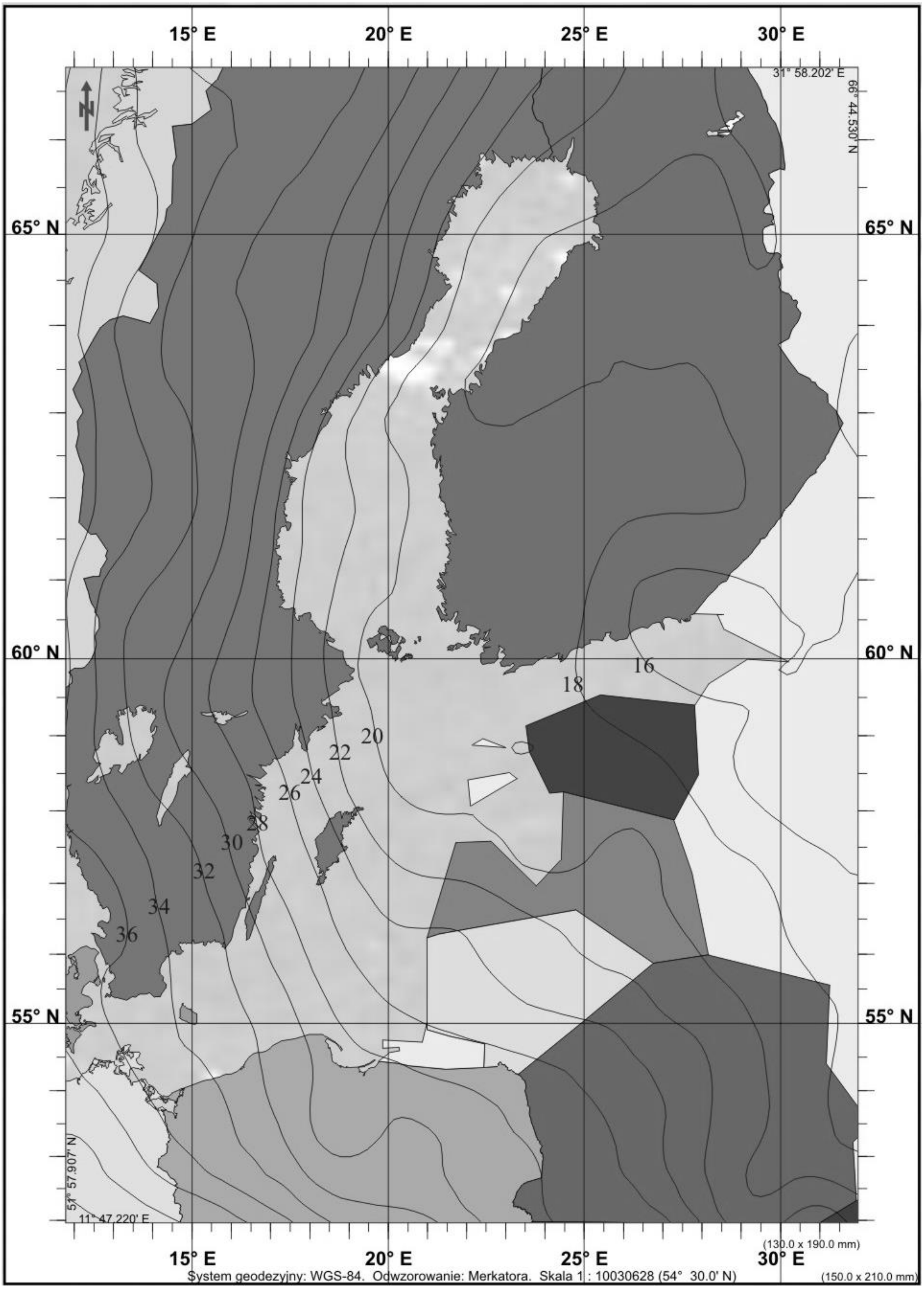

Fig. 2. Distribution of distance between geoid and elipsoid at Balic Sea

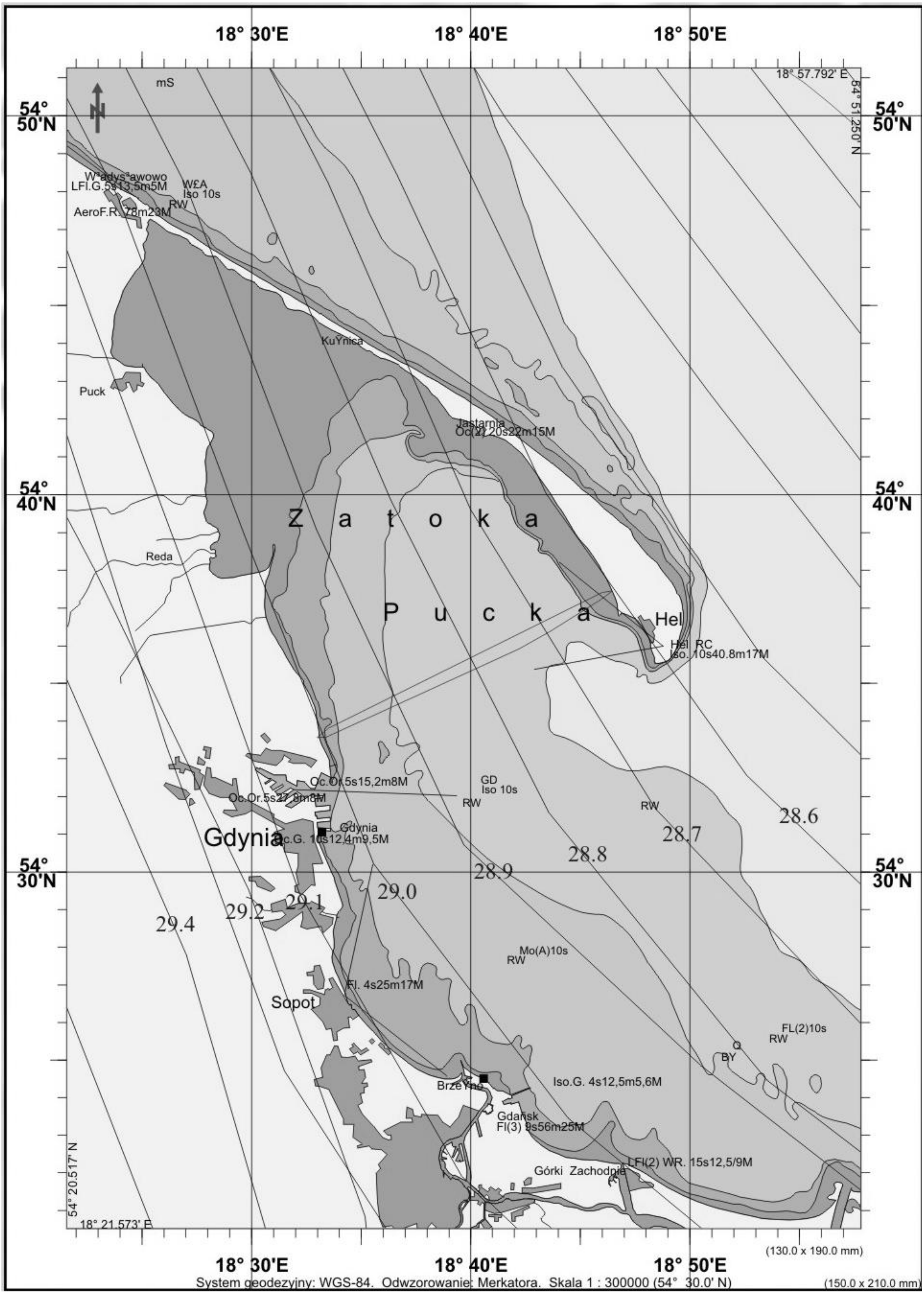

Fig. 3. Distribution of distance between geoid and elipsoid at Gulf of Gdansk

If the origin of a right-handed Cartesian coordinate system coincides with the centre of the ellipsoid, the Cartesian Z-axis coincides with the axis of rotation of the ellipsoid and the positive X-axis passes through the point φ = 0, λ = 0, then the following formula converts ellipsoidal coordinates to geocentric Cartesian coordinates (ISO, 2002):

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} [N+h]\cos\varphi\cos\lambda \\ [N+h]\cos\varphi\sin\lambda \\ [N(1-e^2)+h]\sin\varphi \end{bmatrix} \qquad (2)$$

with the radius of curvature in the prime vertical (perpendicular to the meridian)

$$N = \frac{a}{\sqrt{\left(1-e^2\sin^2\varphi_k\right)}}, \qquad (3)$$

and the first numerical eccentricity of the ellipsoid

$$e = \sqrt{2f - f^2}, \qquad (4)$$

where $f = \frac{a-b}{b}$ **is flattening of the ellipsoid.**

EARTH GRAVITY MODEL EGM96 IN HYDROGRAPHIC SURVEYS DURING MOVEMENT DISRUPTIONS

In special situation, when the transducer of the echosounder is mounted below the antenna of the positioning system's receiver, location of the measurement point $\mathbf{P_d}$**, where the acoustic wave reflects can be determined on the basis of the position of the receiver's antenna:**

$$\overrightarrow{\mathbf{P_a P_d}} = \begin{bmatrix} (l+d)\cdot\cos\beta_x \\ (l+d)\cdot\cos\beta_y \\ (l+d)\cdot\cos\beta_z \end{bmatrix}. \qquad (5)$$

where

$$P_a = \begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix}, \; P_d = \begin{bmatrix} X_d \\ Y_d \\ Z_d \end{bmatrix}, \qquad (6)$$

$\cos\beta_x, \cos\beta_y, \cos\beta_z$ **are direction cosines of angles between axes of the reference system connected with the hydrographic vessel and the line antenna – transducer,** l **is the distance between the antenna and the transducer and** d **is the depth measured by the echosounder (distance between the transducer and the reflection point** $\mathbf{P_d}$ **on the bottom).**

For transforming ortocartesian co-ordinates X^d, Y^d, Z^d of the measurement point P_d to ellipsoidal co-ordinates $\varphi^d, \lambda^d, H^d$ can be used many calculation methods. The iteration method uses following equations (Makar, Naus, 2005):

$$\lambda^d = arc\ tg \frac{Y^d}{X^d} \tag{7}$$

$$\varphi_i^d = arc\ tg \frac{Z^d + \dfrac{a \cdot e^2 \cdot \sin^2(\varphi_{i-1}^d)}{\sqrt{1 - e^2 \cdot \sin^2 \varphi_{i-1}^d}}}{X^d \cdot \cos\lambda^d + Y^d \cdot \sin\lambda^d} \tag{8}$$

$$H^d = \frac{X^d}{\cos\varphi^d \cdot \cos\lambda^d} - \frac{a}{\sqrt{1 - e^2 \cdot \sin^2 \varphi^d}} \tag{9}$$

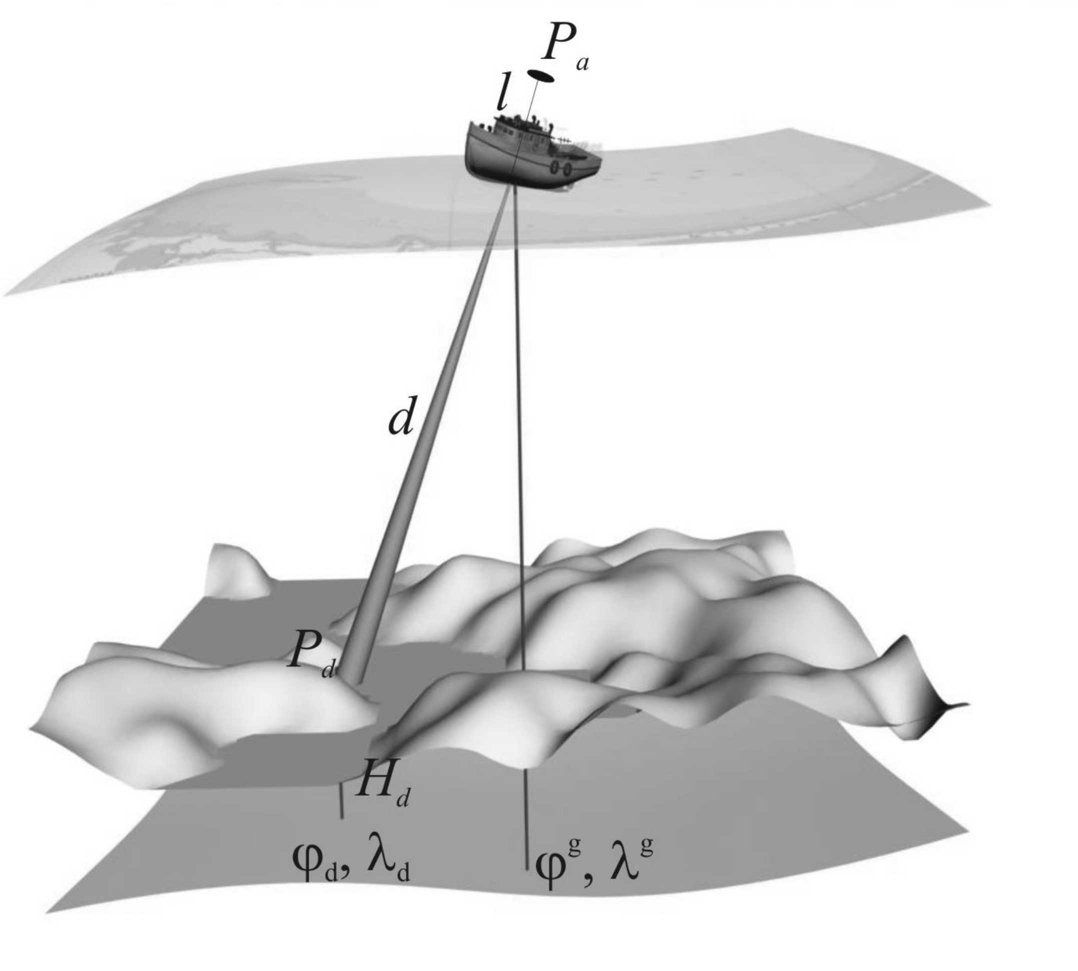

Fig. 4. Depth measurement of the reflecting point

bottom

The point P_g will be determined, when there are no pitching, rolling and heaving, in the point of crossing the water surface and the pole, where the echosouner's transducer is mounted.

CONCLUSIONS

Presented method allows to increase the accuracy of determination of position and the depth the point on the bottom, where the acoustic wave is reflected during hydrographic surveys. Using this method allows eliminate movement disruptions of the sounding vessel, which are results of pitching, rolling and heaving and eliminate the error of depth measurements as a result of wavy motion. Usage of one vertical reference system enable to avoid errors of visualization in Electronic Navigational Charts, especially spatial one.

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