INTERPOLATING A VELOCITY FIELD USING MULTILEVEL B-SPLINES

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ABSTRACT

The estimation of regular velocity fields from irregular distributed GPS stations corresponds to the problem of scattered data approximation with free form surfaces. The method of Multilevel B-spline Approximation is a powerful tool to solve this problem. The iterative evaluation of approximation surfaces leads to a best fit approximation of the station velocities. The algorithm for the Multilevel B-spline Approximation has to be extended by the methods of error propagation to evaluate statistically the quality of the interpolated velocity field.
Multilevel B-spline Approximation is applied to generate a velocity field based on roughly 50 GPS stations in Romania. The investigation area includes several CEGRN stations, campaign stations in the framework of the COLLABORATIVE RESEARCH CENTER 461 "STRONG EARTHQUAKES" and a few permanent GPS stations. The application of the law of error propagation provides an opportunity to analyse the accuracy of the velocity field.

1. INTRODUCTION

The aim of the geodetic subproject B1 "Three dimensional plate kinematics" of the COLLABORATIVE RESEARCH CENTER (CRC) 461 "STRONG EARTHQUAKES" is the determination of three dimensional plate movements for Romania as well as strain rates of the tectonic units.

Three dimensional movements of the earth surface can be determined by regional GPS networks. In the framework of the CRC 461 in cooperation with the NETHERLANDS RESEARCH CENTER OF INTEGRATED SOLID EARTH SCIENCES (ISES) a network including roughly 50 stations was established in Romania between 1997 and 2003 (see Fig. 1). Station velocities are estimated using observations of 16 GPS field campaigns between 1995 and 2006.

As usual the GPS stations are located very scattered. For analysing the movement and deformation of the investigation area it is necessary to determine a regular grid or continuous surface using approximation techniques. The methods of freeform surfaces and scattered data interpolation provide both lots of possibilities for the estimation of approximation surfaces. The Multilevel B-spline Approximation unifies both methods

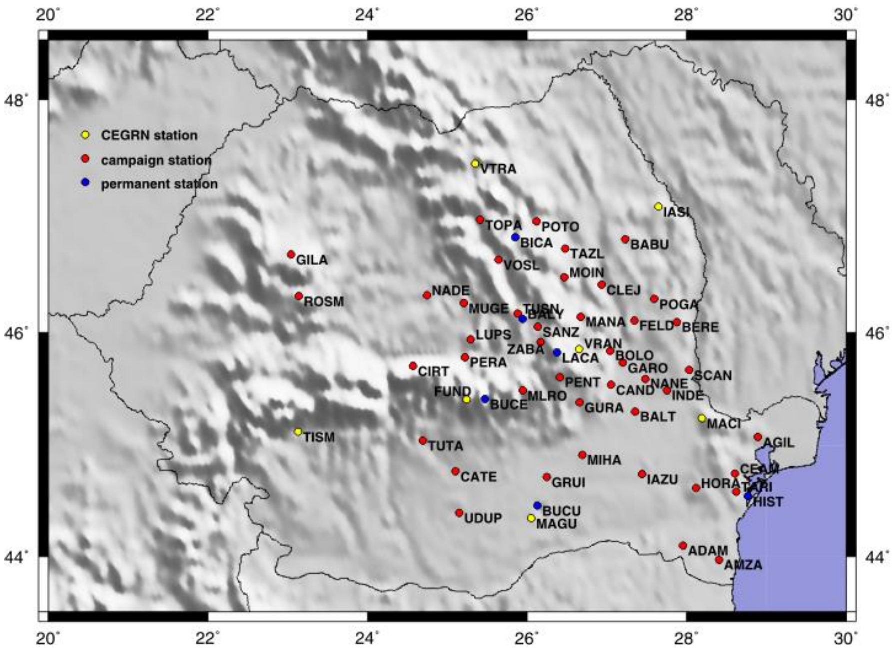


Fig. 1. GPS network used in CRC 461

and provides the possibility to include a computation of strain rates. Due to application of the law of propagation of variances to the approximation algorithm and strain calculation standard deviations can be obtained for the velocity field and the strain rates.

2. DATA APPROXIMATION WITH MULTILEVEL B-SPLINES

Scattered data approximation with Multilevel B-splines was published the first time in 1997 and could be applied and improved successfully for tasks in computer graphics. A very briefly introduction of this approximation technique will be given in this chapter. For studying the details the reader is referred to (Lee et. al., 1997), (Weis and Lewis, 2001) and (Nuckelt, 2006).

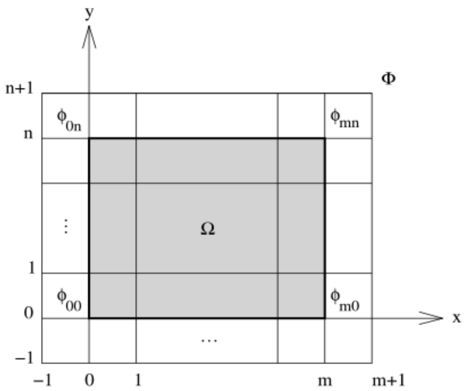


Fig. 2. The configuration of control lattice Φ

2.1. BASIC THEORY OF B-SPLINE APPROXIMATION

The rectangular domain $\Omega = \{(x, y) \mid 0 \le x \le m, 0 \le y \le n\}$ contains a set of scattered points $P = \{(x_c, y_c, z_c)\}$. To approximate the scattered data a approximation function *f*

is formulated as an uniform bicubic B-spline function, which is defined by a control lattice Φ overlaid on the domain Ω (see Fig. 2).

The control lattice can be assumed as a set of $(m+3)\times(n+3)$ points. Let ϕ_{ij} be the value of the *ij*-th control point on lattice Φ located at (i,j) for $i=-1,0,\ldots,m+1$ and $j=-1,0,\ldots,n+1$. The approximation function *f* is defined in terms of these control points by

$$f(x,y)=\sum_{k=0}^{3}\sum_{l=0}^{3}N_k^3(s)N_l^3(t)\phi_{(i+k)(j+l)} \tag{1}$$

where $i=\lfloor x\rfloor-1$, $j=\lfloor y\rfloor-1$, $s=x-\lfloor x\rfloor$ and $t=y-\lfloor y\rfloor$. N_k^3 and N_l^3 are uniform cubic B-spline basis functions defined as

$$\begin{aligned}
N_0^3 &= (1-t)^3/6\\
N_1^3 &= (3t^3-6t^2+4)/6\\
N_2^3 &= (-3t^3+3t^2+3t+1)/6\\
N_3^3 &= t^3/6
\end{aligned}$$

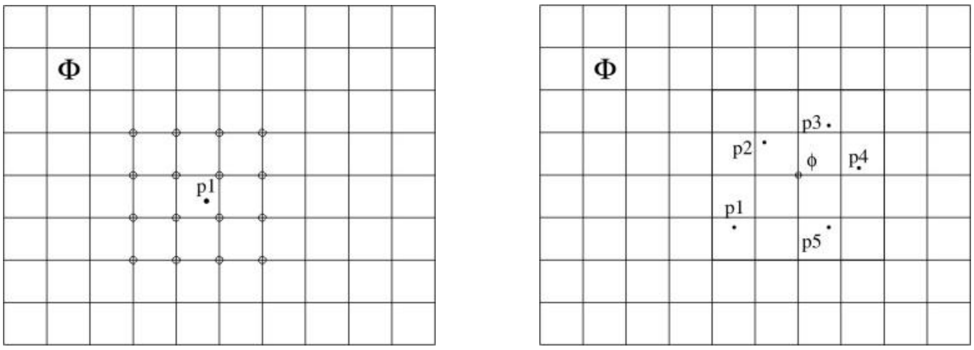
where $0\le t\le 1$. They serve to weigh the contribution of each control point to $f(x,y)$ based on its distance to (x,y). With this formulation the problem of deriving function *f* is reduced to solving for the control points in Φ. To determine the unknown control lattice first only one data point (x_c,y_c,z_c) in *P* is considered. The function value of (x_c,y_c) relates to the sixteen surrounding control points in its neighbourhood, shown in Fig. 3a. For function *f* to take on the value z_c at (x,y) the control points must satisfy

$$z_c=\sum_{k=0}^{3}\sum_{l=0}^{3}N_k^3(s)N_l^3(t)\phi_{kl} \tag{2}$$

where $s=x_c-1$ and $t=y_c-1$. There are many values for the ϕ_{kl}'s that satisfy (2). In the least-squared sense $\sum_{k=0}^{3}\sum_{l=0}^{3}\phi_{kl}^2$ is minimized to set the deviation of *f* zero over the domain Ω. The solution is derived with

$$\phi_{kl}=\frac{N_k^3(s)N_l^3(t)z_c}{\sum_{a=0}^{3}\sum_{b=0}^{3}\left(N_a^3(s)N_b^3(t)\right)^2} \tag{3}$$

Now all data points in *P* are considered. For each point a set of 4×4 control points in its neighbourhood can be determined with (3). These neighbourhoods may overlap for sufficiently close data points. Thus the shared control points obtain different values. The multiple assignments to a control point ϕ can be resolved by considering the data points in its 4×4 neighbourhood (Fig. 3b). Only these points may influence the value of ϕ by (3). Let P_{ij} be the set of these data points of the control point ϕ_{ij}.



(a) neighbourhood of a data point (b) neighbourhood of a control point

Fig. 3. Positional relationship between data points and control points

For each point (x_c, y_c, z_c) in P_{ij} formula (3) gives ϕ_{ij} a different value in ϕ_c:

$$\phi_c = \frac{\omega_c z_c}{\sum_{a=0}^{3}\sum_{b=0}^{3}\omega_{ab}^2} \tag{4}$$

where $\omega_c = \omega_{kl} = N_k^3(s)N_l^3(t)$, $k = (i+1) - \lfloor x_c \rfloor$, $l = (j+1) - \lfloor y_c \rfloor$, $s = x_c - \lfloor x_c \rfloor$ and $t = y_c - \lfloor y_c \rfloor$. To compromise among the values, ϕ_{ij} is chosen to minimize the error $e(\phi_{ij}) = \sum_c (\omega_c\phi_{ij} - \omega_c\phi_c)^2$. The term $(\omega_c\phi_{ij} - \omega_c\phi_c)$ is the difference between real and expected contributions of ϕ_{ij} to function *f* at (x_c, y_c). Differentiating the error $e(\phi_{ij})$ with respect to ϕ_{ij} leads to

$$\phi_{ij} = \frac{\sum_c \omega_c^2 \phi_c}{\sum_c \omega_c^2} \tag{5}$$

Only for the surrounding data points P_{ij} the control point ϕ_{ij} has an influence on function *f*. The approximation function *f* is C^2-continuous because it is a bicubic B-spline surface generated by the control lattice Φ.

2.2 ADAPTED APPROXIMATION ALGORITHM

The basic theory is called B-spline Approximation (BA) algorithm. A tradeoff exists between the shape smoothness and accuracy of the approximation function generated by the BA algorithm. Due to several improvements the algorithm is modified, for details the reader is referred to (Lee et. al., 1997).

MULTILEVEL B-SPLINE APPROXIMATION

A hierarchy of control lattices Φ_0, Φ_1, …, Φ_n is generated. The spacing between control points for Φ_0 is given and the spacing is halved from one lattice to the next. Approximation starts with the coarsest lattice Φ_0. The resulting function f_0 serves a initial approximation and leaves the deviation $\Delta^1 z = z_c - f_0(x_c, y_c)$ for each point

(x_c, y_c, z_c) in P. The next finer control lattice is used to obtain function f_1 that approximates the difference $P_1 = \{(x_c, y_c, \Delta^1 z_c)\}$. The sum $f_0 + f_1$ yields a smaller deviation $\Delta^2 z = z_c - f_0(x_c, y_c) - f_1(x_c, y_c)$ for each point (x_c, y_c, z_c) in P. In general for a level k in the hierarchy a function f_k will be derived using control lattice Φ_k to approximate the data $P_k = \{(x_c, y_c, \Delta^k z_c)\}$, where $\Delta^k z = z_c - \sum_{i=0}^{k-1} f_i(x_c, y_c)$. The final approximation function *f* is defined as the sum of functions f_k: $f = \sum_{k=0}^{h} f_k$.

B-SPLINE REFINEMENT

The evaluation of *f* requires the determination of function f_k from control lattice Φ_k for each level *k* and their addition over the domain Ω. B-spline refinement allows *f* to be represented by only one B-spline function rather than the sum of several B-spline functions. An $(m+3)\times(n+3)$ control lattice Φ is refined to a $(2m+3)\times(2n+3)$ control lattice Φ' whose control point spacing is half as large as that of Φ. Let ϕ_{ij} and ϕ'_{ij} be the *ij*-th control points in Φ and Φ', respectively. Then, the position of control point ϕ_{ij} in Φ coincides with the position of control point $\phi'_{2i,2j}$ in Φ'. The values of the control points in Φ' are obtained from those in Φ, see (Lee et. al., 1997). In each refine level *h* the resulting control lattice Ψ_h is obtained by addition of the refined lattice Ψ'_{h-1} (respectively Φ') and the from $P_h = \{(x_c, y_c, \Delta^h z_c)\}$ generated Φ_h.

Fig. 4 depicts the complete Multilevel B-spline Approximation (MBA) algorithm to generate the approximation function *f*. The algorithm consists of four main operations in each refine level:

I. Compute a control lattice Φ from *P*
II. Compute the deviation $P = P - F(\Phi)$
III. Compute $\Psi = \Psi' + \Phi$
IV. Refine Ψ into Ψ'

2.3 PROPAGATION OF VARIANCES

The law of propagation of variances has to be applied to the four main operations of the MBA algorithm to obtain information about the accuracy of the approximated function *f*, in our case the approximated velocity field.

For the functional relationship $Y = F X$ of the stochastic variables *X* and *Y* the law of propagations of variances is formulated by

$$Q_{YY} = F Q_{XX} F^T \tag{6}$$

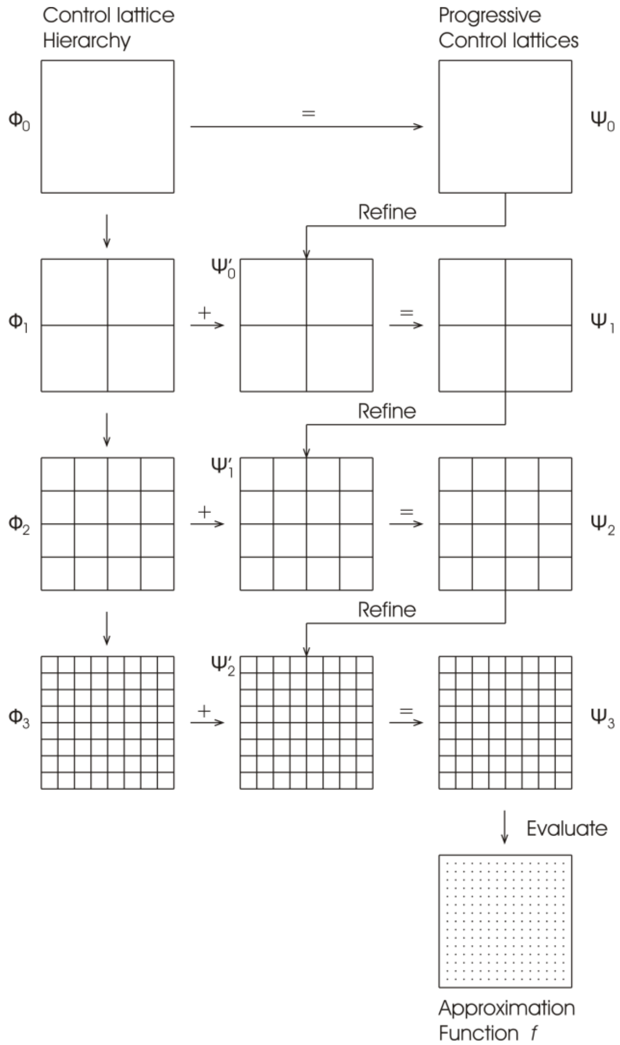


Fig. 4. Approximation function evaluation in the MBA algorithm

The so called *Jacobi* matrix *F* contains the linearised functional relationships between *X* and *Y*. Q_{XX} and Q_{YY} are the covariance matrices of *X* and *Y*. For each step in the approximation algorithm the matrices *F* and Q_{XX} have to be prepared for computing Q_{YY} .

The determination of *F* can be implemented directly into the approximation algorithm. Each operation consists of linear functions, thus differentiations and Taylor expansions are not necessary. A detailed description of the propagation of variances for Multilevel B-spline Approximation is given in (Nuckelt, 2007).

2.4 STRAIN ANALYSIS

Based on the continuous description of an object the theory of continuum mechanics can be applied to perform strain analyses for this object. The necessary continuous description is provided by the velocity field generated by Multilevel B-spline Approximation algorithm. The displacement gradient tensor $Grad\,\bar{u}$ with

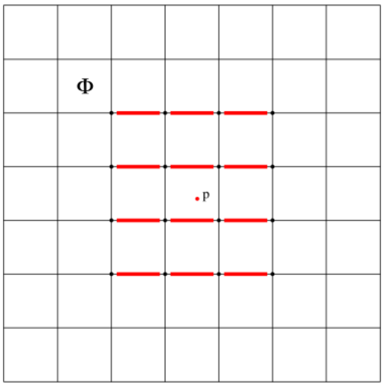
$$Grad\,\bar{u} = \begin{bmatrix} \frac{\partial v_{north}}{\partial X} & \frac{\partial v_{east}}{\partial X} & \frac{\partial v_{up}}{\partial X} \\ \frac{\partial v_{north}}{\partial Y} & \frac{\partial v_{east}}{\partial Y} & \frac{\partial v_{up}}{\partial Y} \\ \frac{\partial v_{north}}{\partial Z} & \frac{\partial v_{east}}{\partial Z} & \frac{\partial v_{up}}{\partial Z} \end{bmatrix} \tag{7}$$

can be obtained directly from the control lattice of the approximation function:

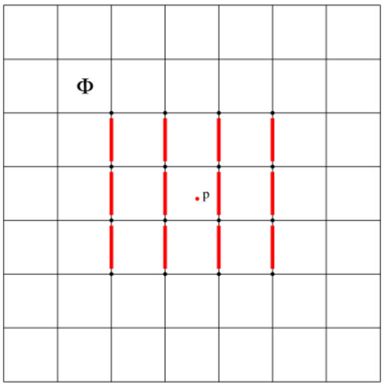
$$\begin{aligned} \frac{\partial v_i}{\partial X} &= \alpha_X \cdot \sum_{k=0}^{2}\sum_{l=0}^{3} N_k^2(s) N_l^3(t)(\phi_{(i+k)(j+l)} - \phi_{(i+k-1)(j+l)}) \\ \frac{\partial v_i}{\partial Y} &= \alpha_Y \cdot \sum_{k=0}^{3}\sum_{l=0}^{2} N_k^3(s) N_l^2(t)(\phi_{(i+k)(j+l)} - \phi_{(i+k)(j+l-1)}) \\ \frac{\partial v_i}{\partial Z} &= 0 \end{aligned} \tag{8}$$

where α_X and α_Y indicate the differentiated mapping function from real coordinates (X,Y) into the coordinate system (x,y) of the control lattice. Fig. 5 illustrates the relation between control points ϕ and the considered point $p(X,Y) = p(x,y)$ in formula (8). The vertical gradient can not be determined, because geodetic observations are provided only for the surface. Thus authentic strain analyses can be performed only for the two dimensional surface.

The infinitesimal deformation tensor $\underline{\varepsilon}$ and the infinitesimal rotation tensor $\underline{\Omega}$ can be derived from $Grad\,\bar{u}$. The principal strains are the results of computing eigenvalues and eigenvectors of $\underline{\varepsilon}$. The shear strains can be derived also from $\underline{\varepsilon}$ and $\underline{\Omega}$. The law of propagation of variances is applied also to each computation of the strain analysis.



(a) x – direction



(b) y – direction

Fig. 5. Determination of gradients in x and y direction from the control lattice

For a complete explanation of the propagation of variances for the determination of $Grad\,\bar{u}$, the tensor calculation and the determination of principal and shear strains is referred to (Nuckelt, 2007).

3. THREE DIMENSIONAL VELOCITY FIELD FOR ROMANIA

The velocity field is generated from the estimated velocities of the GPS stations shown in Fig. 1. The GPS observation data were processed using Bernese GPS Software 5.0 (Hugentobler et. al., 2005). The station velocities were estimated from daily coordinate solutions. These linear velocities were inserted into the Multilevel B-spline approximation to generate the three dimensional velocity field and perform the strain analyses. The obtained velocity field plus standard deviations are shown in Fig. 6 and 7.

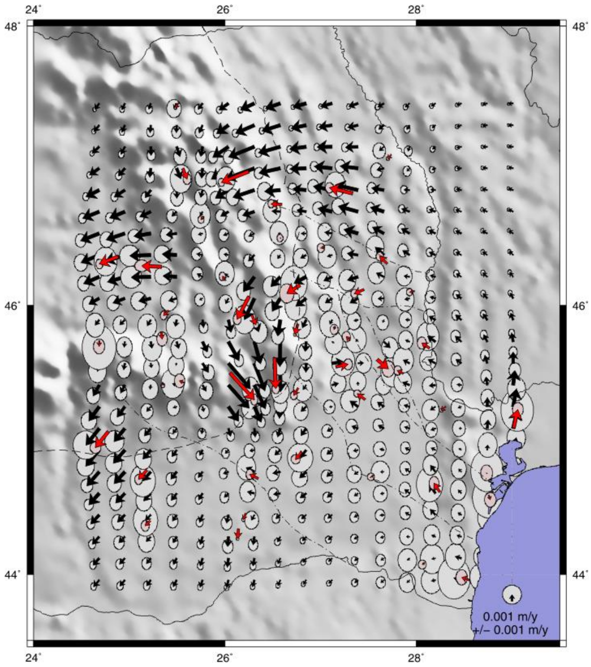


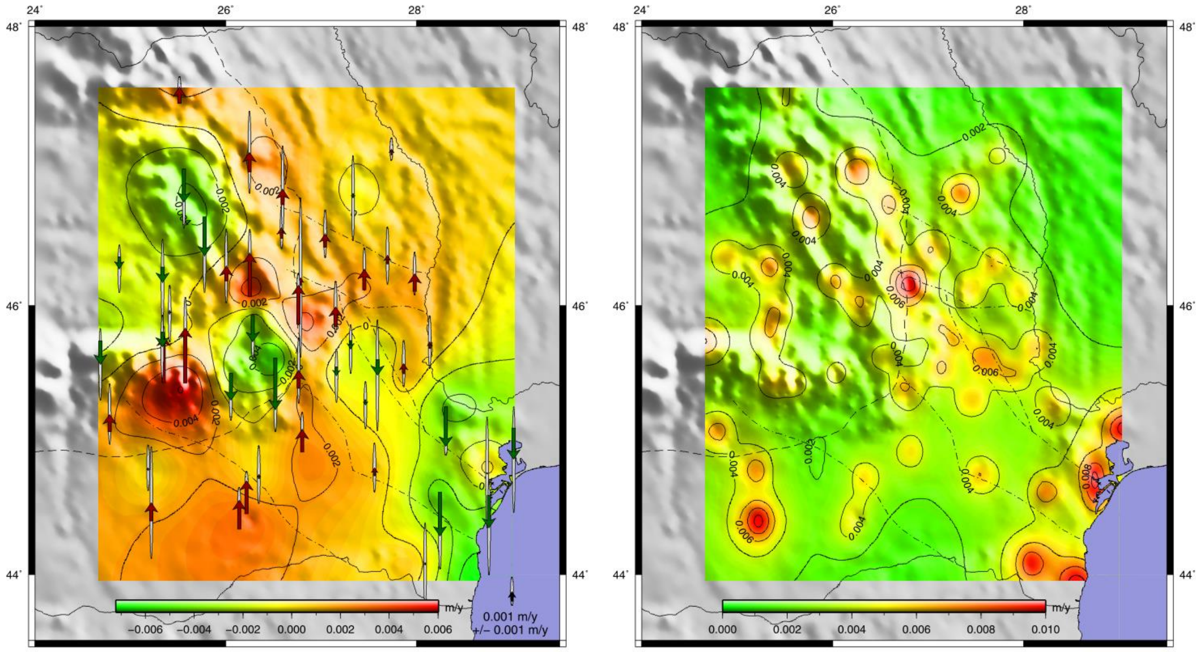
Fig. 6. plane velocity field plus standard deviations

The obtained velocity field fits very good the station velocities. Fig. 6 depicts areas of significant horizontal movements. The Moesian platform (southwest corner in Fig. 6) moves clearly direction southwest. For the part in the north a west movement is shown. The Transylvanian basin inside the Carpathian arc performs a shift to the west. The biggest velocities (up to 5 mm/year) are shown for the south-eastern part of the Carpathian arc. The vertical velocity field in Fig. 7 visualises areas of significant uplift and subsidence. Transylvanian basin, Brasov basin, Focsani basin and the areas close to the Black Sea are evidently subsiding regions. In opposition to this areas the Carpathian arc, Moesian platform and European platform (in the north-eastern part) are uplift areas.

The horizontal as well as the vertical movements match more or less with geological studies (Tarapoanca et. al., 2003). The moderate uplift of Vrancea area (the most south-eastern part of the Carpathian arc) coincides with a geodynamic model developed in the CRC 461. This model proposes the progressive delamination of a soft coupled vertical slab beneath this area (Sperner et. al., 2005).

The approximation with Multilevel B-splines generates smooth best fitting surfaces which represent the velocity field. The accuracies of the GPS stations propagate to the velocity field. Due to the properties of the algorithm areas close to the data points (GPS

stations) obtain bigger standard deviation than other areas, because control points within the 4×4 neighbourhood of the data points are more often included into the approximation process than the other control points. The areas of biggest standard deviations surround the most inaccurate GPS stations. More detailed analyses in terms of accuracy are given in (Nuckelt, 2007)



(a) velocities (b) standard deviations

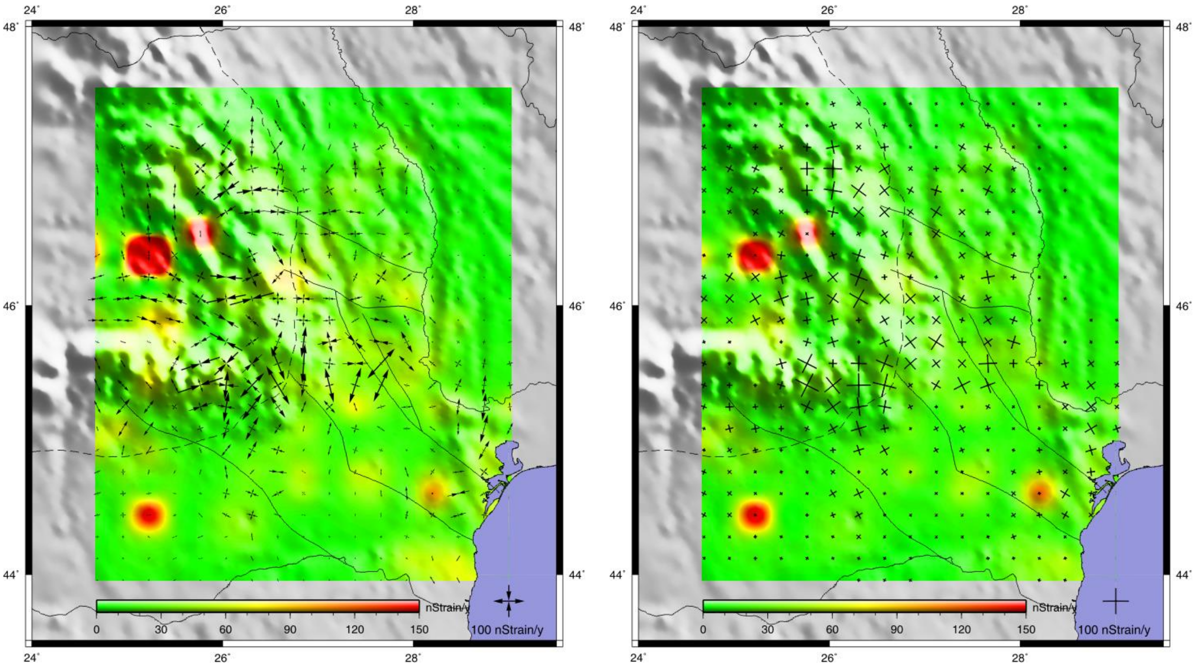
Fig. 7. Vertical velocity field

The computed principal and shear strains are shown in Fig. 8. The coloured surfaces put under the strain crosses represent their formal errors. The pattern of these surfaces is similar to Fig. 7(b). The areas of bigger errors coincide, because the velocity field as well as the strain parameter are based on the same control lattice of the Multilevel B-spline approximation. This lattice itself depends on the GPS stations. That's why you can see the station configuration and their different accuracies also in these figures.

Large principle and shear strain are observable in the centers of both Figures 8(a) and 8(b). Strains are obtained for regions where different movements occur, e. g. the most south-eastern part of the Carpathian arc. Large extensions and shear strains are shown in this zone. In the adjacent area to the east large extensions in different directions are observable. In areas of uniform displacement strains do not occur, e. g. the Moesian platform and the whole southern area, respectively.

4. CONCLUSIONS

Multilevel B-spline approximation is an appropriate tool to generate three dimensional velocity fields based on estimated velocities of scattered GPS stations. The advantage of this algorithm is the simultaneous approximation of several values. Furthermore the law of propagation of variances and algorithms of strain analyses can be implemented easily.



(a) principal strain (b) shear strain

Fig. 8. Horizontal strains plus standard deviations

The accuracy of the obtained velocity field depends on the quality of the input data. The high accuracy as well as the inaccuracy of a GPS station is propagated into the approximated velocity field and strain parameters.

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