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A TESTING METHOD OF PROPULSION SYSTEMS ON SIMILE MICRO MODELS

In this paper is presented a testing method of propulsion systems on simile micro models. In order to calculate and design a micro engine simile with a real rocket engine, physical and geometrical relations where deduced. On the base of experimental data measured after testing of simile micro engine, can be establish the ballistic parameters of real rocket engine.

1. Introduction

Thermogasdynamic and energetic parameters of a rocket engine are determinated by used propellant, structure and geometry of the engine. If two or more engines use the same propellant, have the same structure and geometry, then they will realize the same burning rate of propellant, the same pressure distribution in combustion chamber, the same burning temperature, the same composition of gases, the same gases flux velocity, the traction parameters of these engines being in strong simile relations.

In work [1] are presented a few methods for determination of solid propellant rocket-engine traction parameters, which are based on experimental results obtained during the firing with similar micro engine. The purpose of these methods is to establish the difference between calculated values and experimental values of various traction parameters (traction force coefficient, specific impulse, etc.) of engine. These differences appear because of calculus model imperfections, which consist of neglecting or incomplete consideration of losses caused by two phase character of gases, turbulent character of flowing, frictions in limit layer, heat transfer, etc. Although authors of this work affirm that simile micro engine with real engine was designed and built from condition of realizing of same distribution of pressure in combustion chamber (with +/-10% deviation), are not presented enough detailed geometrical and physical similitude relations.

Also, is very known and is used on a large scale the method for determination of burning rate and specific impulse of a solid propellant, by firing on testing bench with self-styled STM (Standard Testing Motor), these engines are not similar to a certain specified engine. Values of burning rate and specific impulse which are determined in this way will be used only comparing of different propellants, in standard conditions, they suffering modifications in case of a certain engine, in function of concrete geometry of propellant and engine in general.

2. Formulation of problem

In work [2] it was tried to deduce relations between geometrical dimensions of two or more solid propellant engines, which assure physical similitude of these, starting from condition that all engines realize the same distribution of pressure in combustion chamber, being loaded whit the same type of propellant (from the same charge), in order to elaborate a method for establishing of real values of traction parameters for a certain engine. For that, was analyzed all magnitudes which intervene in pressure distribution law in combustion chamber of a solid propellant rocket-engine (1). From relation (1),it is observed that for the same variation in time of pressure p , in case of two engines loaded with propellant from the same charge, is necessary that geometrical sizes A_a, A_{cr} and V_ℓ to be correlated in manner which will be presented.

It is necessary to remark that relation (1) which is the start in developing of this method doesn't take into account consideration some complex processes like flowing of biphasic medium, loss of energy in limit layer, unstable end erosive combustion, dissociation and redissociation of gases, etc., this relation being deduced by application of theorems of conserving mass and momentum, using hypotheses which are presented in [3]. In the case of this similitude theory are fulfilled the general similitude criteria: $\mathrm{Re}, M, Nu, \mathrm{Pr}\, Bi, Fr, Ar, Eu$.

Above notations represent: $\mathrm{Re} = {u\ell}/{\nu}$, Reynolds criterion, ratio between inertia forces and viscous forces of the fluid; $M = {u}/{a}$, Mach criterion, ratio between fluid velocity and local velocity of sound; $Nu = {\alpha\ell}/{\lambda}$, Nusselt criterion, ratio between thermal transferred flux by convection and thermal transferred flux by conduction, through a section with thickness ℓ; $\mathrm{Pr} = {\nu}/{a_t}$, Prandtl criterion, ratio between intensity of heat transfer and mass; $Bi = {\alpha\ell}/{\lambda_s}$; Biot criterion, ratio between temperature gradient in solid body and difference between fluid and solid body; $Fr = {u^2}/{g\ell}$, Froude criterion, ratio between inertia forces and gravitational forces of fluid; $Ar = \left({g\ell^3}/{\nu^2}\right)\left({\Delta\rho}/{\rho}\right)$, Archimedes criterion, ratio between upward forces and inertia forces of fluid; $Eu = {\Delta\rho}/{\rho u^2}$, Euler criterion, ratio between pressure forces and inertia forces of fluid; u - medium velocity of fluid, [m/s], on characteristic distance ℓ, [m]; ν - cinematic viscosity of fluid, [m^2/s]; a - local velocity of sound, [m/s]; α - convection coefficient of fluid, [W/m^{2}K]; λ - thermal conductivity of fluid, [W/m^{2}K]; a_t - diffusivity of fluid, [m^2/s]; λ_s - thermal conductivity of solid body [W/m^{2}K]; g -gravitational acceleration, [m/s^2]; p - fluid pressure, [N/m^2]; ρ - fluid density, [Kg/m^3]; *i*=1,2,…,n, - engine order number in its similitude class.

Because in enounced similitude condition all simile engines realize the same composition of combustion products, the same distribution of temperature, pressure and velocity, result that physical magnitudes which used in similitude enounced criteria was taken equal values in homologue points, therefore realize identical temporal and spatial distribution:
w_l=const./share; α_i=const./share; v_i=const./share; λ_i=const./share;a_i=const./share;

a_{ti}=const./share;Re_i=const./share;Pr_i=const./share;M_l=const./share;Bi_l=const./share; Nu_l=const./share;Fr_i=const./share;λ_s=const./share;ρ_i=const./share;p_i=const./share;Ar_i =const./share;Eu_i=const./share.

3. Similitude general relations

The pressure equation in combustion chamber of a rocket-engine with solid propellant is

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \rho_p u_a \frac{R_c T_c}{V_\ell} A_a - C_D p \frac{R_c T_c A_a}{V_\ell}, \qquad (1)$$

where:

- p - pressure in combustion chamber, [N/m^2];
- t - time, [s];
- ρ_p - solid propellant density [kg/m^3];
- u_a - burning rate of propellant [m/s];
- A_a - propellant burning area [m^2];
- V_ℓ - free volume of combustion chamber [m^3];
- A_{cr} - critical area of nozzle [m^2];
- R_c - gases constant [Nm/kgK];
- T_c - temperature in combustion chamber [K];
- C_D - debit coefficient;
- k - division of specific heat of gases (depends on gases composition),

From this relation it is we observed that the pressure varies in function of geometrical sizes V_ℓ, A_a and A_{cr} which are proper for each engine. In order to realize the same distribution of pressure for all engines, it is enough to make constant V_ℓ, A_a and A_{cr}, in relation (1). For that is necessary to divide respectively variable magnitudes to other dimensional magnitude proper for each engine. This dimensional magnitude, d can be any geometrical sizes of engine, for example high of propellant stellar channel or combustion chamber diameter. In this way, sizes V_ℓ, A_a and A_{cr} become nondimensionaly and they will be constant for all class of similar engines:

$$\begin{aligned} \overline{V_\ell} &= V_\ell / d^3 = \text{const/lot}; \\ \overline{A}_a &= A_a / d^2 = \text{const/lot}; \\ \overline{A}_{cr} &= A_{cr} / d^2 = \text{const/lot}; \end{aligned} \qquad (2)$$

Pressure equation (1) will be multiplied with d. Physical meaning of equation does not change. Time of combustion become ${dt}/{d} = d\left({t}/{d}\right)$. Final version of pressure equation for a similar engine class is

$$\frac{\mathrm{d}p}{\mathrm{d}(\bar{t})} = \rho_p u_a \frac{R_c T_c}{\bar{V}_\ell} \bar{A}_a - C_D p \frac{R_c T_c}{\bar{V}_\ell} \bar{A}_{cr} \; . \tag{3}$$

4. Aplication for stellar geometry of propellant

Because free volume and combustion surface are global parameters must be determinated the relations of dependence with final similitude relations. These relations will be characteristic of different solid propellant engines, depending on internal geometry of engine, propellant, etc. So, it is used in this paper an engine with propellant founded in chamber, with internal stellar channel (frontal sides are covered). This type of propellant has 4 phases of combustion (fig. 1).

Area of combustion has relation

$$A_a = PL \, , \tag{4}$$

where:

P - combustion perimeter in a transversal section of combustible [m^2];

L - length of propellant [m].

Free volume is

$$V_\ell = A_t L \, , \tag{5}$$

in which A_t is free surface.

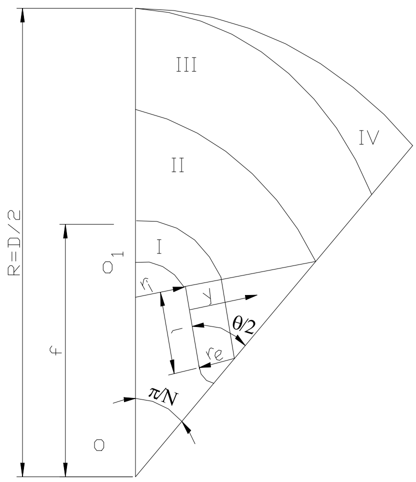

Fig. 1. Scheme of combustion for a piece of solid propellant

Calculus relations are followings:

PHASE I: $y \in [0, r_e]$

$$P_I = \frac{\pi}{n} y + r_i \left(\frac{\pi}{2} + \frac{\pi}{n} - \frac{\theta}{2} \right) + \ell + r_e \left(\frac{\pi}{2} + \frac{\theta}{n} \right); \tag{6}$$

$$A_{T_I} = \frac{1}{2}\left[f + r_i\left(\frac{1}{\sin\frac{\theta}{2} - \frac{\pi}{2}} - 1\right)\right]^2 \frac{\sin\left(\frac{\theta}{2} - \frac{\pi}{2}\right)\sin\frac{\pi}{n}}{\sin\frac{\theta}{2}} - \frac{r_i^2}{2}\left[tg\left(\frac{\theta}{2} - \frac{\pi}{n}\right) - \beta\right] + \tag{7}$$

$$+ \frac{r_e^2}{2}\left(ctg\frac{\theta}{2} - \frac{\pi}{n} - \frac{\theta}{2}\right) + \frac{\beta}{2}\left(2r_i y + y^2\right) + y\ell + \left(\frac{\pi}{4} - \frac{\theta}{4}\right)\left(2r_e y + y^2\right),$$

where:

$$\ell = \left[r_i\left(\frac{1}{\sin\left(\frac{\theta}{2} - \frac{\pi}{n}\right)} - 1\right) + f\right]\frac{\sin\frac{\pi}{n}}{\sin\frac{\theta}{2}} - r_e ctg\frac{\theta}{2} r_i ctg\left(\frac{\theta}{2} - \frac{\pi}{n}\right); \tag{8}$$

$$\beta = \frac{\pi}{n} - \frac{\pi}{2} - \frac{\theta}{2}. \tag{9}$$

PHASE II: $y \in \left[r_e, r_e + \ell tg\theta/2\right]$

$$P_{II} = y\left[\frac{\pi}{2} - \frac{\theta}{2} + \frac{\pi}{n} - ctg\frac{\theta}{2}\right] + r_i\left[\frac{\pi}{2} - \frac{\theta}{2} + \frac{\pi}{n}\right] + r_e ctg\frac{\theta}{2} + 1; \tag{10}$$

$$A_{T_{II}} = A_{T_I}(r_e) + \frac{(r_i + y)^2}{2}\beta - \frac{(r_i + r_e)^2}{2}\beta + y\ell - \frac{y(y - r_e)ctg\frac{\theta}{2}}{2}. \tag{11}$$

PHASE III: $y \in \left[r_e + \ell tg\theta/2, R - t\right]$

$$P_{III} = (y + r_i)\left[\frac{\pi}{n} + \arcsin\left(\frac{f - r_i}{y + r_i}\sin\frac{\pi}{n}\right)\right]; \tag{12}$$

$$A_{T_{III}} = A_{T_{II}}\left(r_e + ltg\frac{\theta}{2}\right) + \frac{(y + r_i)^2\left[\frac{\pi}{n} + \arcsin\left(\frac{f - r_i}{y + r_i}\sin\frac{\pi}{n}\right)\right]}{2} - \left(r_e + \ell tg\frac{\theta}{2} + r_i\right)^2\beta \tag{13}$$

All relations depend by geometrical dimensions of star: f, r_1, r_e, R, L. Universal character is realized dividing these variables at size f. In this way it is obtain constant size for all class of similar engines:

$\bar{f} = \frac{f}{f} = 1$; $\bar{r}_i = \frac{r_i}{f} = \text{const/lot}$; $\bar{r}_e = \frac{r_e}{f} = \text{const/lot}$; $\bar{R} = \frac{R}{f} = \text{const/lot}$; $\bar{L} = \frac{L}{f} = const/lot$;

Calculus relations for combustion area and free volume become:

PHASE I: $\bar{y} \in [0, \bar{r}_e]$

$$\bar{A}_a(\bar{y}) = \frac{A_a}{f^2} = \frac{P_I L 2n}{f^2} = \bar{P}_I \bar{L} 2n = \left[\frac{\pi}{n}\bar{y} + \bar{r}_i\left(\frac{\pi}{2} + \frac{\pi}{n} - \frac{\theta}{2}\right) + \bar{\ell} + \bar{r}_e\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\right]\bar{L} 2n \tag{14}$$

$$\bar{V}_\ell\left(\bar{y}\right)=\frac{V_\ell}{f^3}=\frac{A_{T_l}L2n}{f^3}=\bar{A}_{T_l}\,\bar{L}2n=$$

$$=\left\{\begin{array}{l}\frac{1}{2}\left[1+\bar{r}_i\left(\frac{1}{\sin\frac{\theta}{2}-\frac{\pi}{2}}-1\right)\right]^2\frac{\sin\left(\frac{\theta}{2}-\frac{\pi}{n}\right)\sin\frac{\pi}{n}}{\sin\frac{\theta}{2}}-\frac{\bar{r}_i^{\,2}}{2}\left[tg\left(\frac{\theta}{2}-\frac{\pi}{n}\right)-\beta\right]+\frac{\bar{r}_e^{\,2}}{2}\left[ctg\frac{\theta}{2}-\frac{\pi}{n}+\frac{\theta}{2}\right]+\\ +\frac{\beta}{2}\left(2\bar{r}_i\,\bar{y}+\bar{y}^2\right)+\bar{y}\,\bar{\ell}+\left(\frac{\pi}{4}-\frac{\theta}{4}\right)\left(2\bar{r}_e\,\bar{y}+\bar{y}^2\right)\end{array}\right\}\bar{L}2n \tag{15}$$

where :

$$\bar{\ell}=\left[\bar{r}_i\left(\frac{1}{\sin\left(\frac{\theta}{2}-\frac{\pi}{n}\right)}-1\right)+1\right]\frac{\sin\frac{\pi}{n}}{\sin\frac{\theta}{2}}-\bar{r}_e\,ctg\frac{\theta}{2}\bar{r}_i\,ctg\left(\frac{\theta}{2}-\frac{\pi}{n}\right); \tag{16}$$

$$\beta=\frac{\pi}{n}-\frac{\pi}{2}-\frac{\theta}{2}. \tag{17}$$

PHASE II: $\bar{y}\in\left[\bar{r}_e,\bar{r}_e+\bar{\ell}tg\frac{\theta}{2}\right]$

$$\bar{A}_a\left(\bar{y}\right)=\frac{A_a}{f^2}=\bar{P}_{II}\,\bar{L}2n=\left\{\bar{y}\left[\frac{\pi}{2}-\frac{\theta}{2}+\frac{\pi}{n}-ctg\frac{\theta}{2}\right]+\bar{r}_i\left[\frac{\pi}{2}-\frac{\theta}{2}+\frac{\pi}{n}\right]+\bar{r}_e\,ctg\frac{\theta}{2}+\bar{\ell}\right\}\bar{L}2n \tag{18}$$

$$\bar{V}_\ell\left(\bar{y}\right)=\frac{V_\ell}{f^3}=\bar{L}2n\left[A_{T_I}\left(\bar{r}_e\right)+\frac{\left(\bar{r}_i+\bar{y}\right)^2}{2}\beta-\frac{\left(\bar{r}_e+\bar{r}_i\right)^2}{2}\beta+\bar{y}\,\bar{\ell}-\frac{\bar{y}\left(\bar{y}-\bar{r}_e\right)ctg\frac{\theta}{2}}{2}\right] \tag{19}$$

PHASE III: $\bar{y}\in\left[\bar{r}_e+\bar{\ell}tg\frac{\theta}{2},\bar{R}-\bar{f}\right]$

$$\bar{A}_a\left(\bar{y}\right)=\frac{A_a}{f^2}=\bar{P}_{III}\,\bar{L}2n=\left\{\left(\bar{y}+\bar{r}_i\right)\left[\frac{\pi}{n}+\arcsin\left(\frac{1-\bar{r}_i}{\bar{y}+\bar{r}_i}\sin\frac{\pi}{n}\right)\right]\right\}\bar{L}2n\,; \tag{20}$$

$$\bar{V}_\ell\left(\bar{y}\right)=\frac{V_\ell}{f^3}=\frac{A_{T_{III}}}{f^2}\bar{L}2n=\left\{\bar{A}_{T_{II}}\left(\bar{r}_e+\bar{\ell}\,tg\frac{\theta}{2}\right)+\frac{\left(\bar{y}+\bar{r}_i\right)^2\left[\frac{\pi}{n}+\arcsin\left(\frac{1-\bar{r}_i}{\bar{y}-\bar{r}_i}\sin\frac{\pi}{n}\right)\right]}{2}-\frac{\left(\bar{r}_e+\bar{\ell}\,tg\frac{\theta}{2}+\bar{r}_i\right)^2\beta}{2}\right\}\bar{L}2n \tag{21}$$

In this way for each phase of combustion were established calculus relations for area of combustion and free volume, which intervene in pressure equation (3).

It is better to write the pressure equation function of combustion distance because it is variable in sizes, which intervene in study of combustion geometry:

$$\frac{\mathrm{d}p}{\mathrm{d}\bar{t}}=\frac{\mathrm{d}p}{\mathrm{d}\bar{y}}\frac{\mathrm{d}\bar{y}}{\mathrm{d}\bar{t}}=\frac{\mathrm{d}p}{\mathrm{d}\bar{y}}u_a\,, \tag{22}$$

where , for burning rate, it is used correspondent law, function of utilized propellant nature. If this law is $u_a=cp^{\upsilon}$, where c and υ are constants of burning rate, introducing (22) in (3) it is obtained:

$$\frac{\mathrm{d}p}{\mathrm{d}\bar{t}}=\rho_p\frac{R_cT_c\bar{A}_a(\bar{y})}{\bar{V}(\bar{y})}-\frac{C_D}{c}p^{1-\upsilon}\frac{R_cT_c\bar{A}_{cr}}{\bar{V}(\bar{y})}\,. \tag{23}$$

Time of combustion results from relation

$$\frac{\mathrm{d}\bar{t}}{\mathrm{d}\bar{y}}=\frac{1}{u_a}\,. \tag{24}$$

So, pressure variation in combustion chamber of the engine from similitude classes described by differential equation (23), having next initial conditions:

$$\bar{t}=0\Rightarrow\bar{y}=0;\,p=p_0\,, \tag{25}$$

where p_0 is initial pressure in combustion chamber [N/m^2].

Solving equation (23), with initial conditions (25), it's obtained variation of pressure in combustion chamber function of covered distance by front of flame, curve valid for all engines from correspondent simile class of engines which respect simile geometrical condition found above. In these conditions similar engines will realize the same temperature of outcomes in combustion chamber, ca composition of outcomes, the same parameters of outcomes at emergence from nozzle (pressure, temperature, density, speed) and other energetic magnitudes (total impulse, traction force, debit) will be in physical similitude relations, which can be determined also by dividing at chosen dimensional magnitude, so:

- For traction force P:

$$P=C_pA_{cr}p\,,$$

where C_p is traction force coefficient, which is function of propellant nature, distribution of pressure and engine geometry; $\overline{P}=\frac{P}{f^2}C_p\frac{A_{cr}}{f^2}p=C_p\overline{A_{cr}}p$, $p=\mathrm{f}(\overline{y})$.

Therefore,

$$\overline{P}(\overline{y}) = C_p \overline{A_{cr}} p(\overline{y}) . \tag{26}$$

- For total impulse:

$$I_\Sigma = \int_0^t P(t)\mathrm{d}t \Rightarrow \frac{\mathrm{d}I_\Sigma}{\mathrm{d}t} = P(t) \Rightarrow \frac{\mathrm{d}\left(\frac{I_\Sigma}{f^3}\right)}{\mathrm{d}\left(\frac{t}{f}\right)} = \frac{P(t)}{f^2} \Rightarrow \frac{\mathrm{d}\overline{I_\Sigma}}{\mathrm{d}\overline{t}} = \overline{P}(\overline{t}) ,$$

where $\overline{I_\Sigma} = \frac{I_\Sigma}{f^3} = \mathrm{const/lot}$.

Making the changing of variable t in y, it is obtained

$$\frac{\mathrm{d}\overline{I_\Sigma}}{\mathrm{d}\overline{t}} = \frac{\mathrm{d}\overline{I_\Sigma}}{\mathrm{d}\overline{y}} \frac{\mathrm{d}\overline{y}}{\mathrm{d}\overline{t}} = \frac{\mathrm{d}\overline{I_\Sigma}}{\mathrm{d}\overline{y}} u_a \Rightarrow \frac{\mathrm{d}\overline{I_\Sigma}}{\mathrm{d}\overline{y}} = \frac{\overline{P}(\overline{y})}{u_a} . \tag{27}$$

- For debit of combustion produces:

$$G = C_D A_{cr} p ,$$

where $C_D = f(R_c, T_c, k)$, debit coefficient depending by pressure and temperature in combustion chamber. Dividing by f, it is obtained

$$\overline{G} = \frac{G}{f^2} = C_D \overline{A_{cr}} p . \tag{28}$$

In conclusion, obtained similitude relations are:

a. Geometrical similitude (for engines with solid combustible molded in chamber, with internal star channel):

$$\overline{r_e} = \frac{r_e}{f} = \mathrm{const/lot} \; ; \; \overline{r_i} = \frac{r_i}{f} = \mathrm{const/lot} \; ;$$
$$\overline{R} = \frac{R}{f} = \mathrm{const/lot} \; ; \; \overline{L} = \frac{L}{f} = \mathrm{const/lot} \; ; \tag{29}$$
$$\overline{A_{cr}} = \frac{A_{cr}}{f^2} = \mathrm{const/lot} \; ; \; \frac{L}{D} = \mathrm{const/lot} \; ;$$
$$\frac{A_e}{A_{cr}} = \mathrm{const/lot} ,$$

where were added last two relations which resulted from the first ones, with notations:

D - diameter of propellant or internal diameter of chamber;
L - length of propellant or internal length of chamber;
A_e - area of emergence section of nozzle;
A_{cr} - area of critical section of nozzle.

b. Physical similitude:

$$\bar{t} = \frac{t}{f} = \text{const/lot}\,;\ \ p = \text{const/lot}\,;$$

$$T_c, R_c, k, \rho_c = \text{const/lot}\,;\ T_e, \rho_e, \omega_e = \text{const/lot}\,;$$

$$\overline{G} = \frac{G_I}{f^2} = C_D \frac{A_{cr}}{f^2} p = \text{const/lot}\,; \qquad (30)$$

$$\overline{P} = \frac{P}{f^2} = \text{const/lot}\,;\ \overline{I_\Sigma} = \frac{I_\Sigma}{f^3} = \text{const/lot}\,.$$

5. Conclusions

Two or more engines, made so that geometry of combustion chamber, propellant and nozzle, respect similitude geometrical relations (29), are loaded in the same way and with the same propellant type, realize physical similitude described by relations (30);

Similitude relations were deduced only from condition to realize the same distributions of pressure for all similar engines. Therefore, this similitude theory does not take into consideration other aspect like unstable combustion, but this phenomenon is manifest identically for all engines from respectively similitude class. Measured values of traction parameters during the firing on bench for one of simile engine can be extrapolated for each engine from its simile class;

This simile criterion can be used in following situations:

- the determination of performances of an engine for which the testing is very expensive; in this case it is necessary to construct a micro engine which must respect similitude conditions with real engine; the experimental results can be extrapolated at real engine;
- the data obtained during testing of an engine can be used to design new engines, simile with tested engine.

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