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PROGNOZOWANIE STREFY RAŻENIA GŁOWICY PRĘTOWEJ

Streszczenie: W pracy przedstawiono metodę przybliżonego oszacowania kształtu statycznej strefy rażenia prętowej głowicy przeciwlotniczego pocisku rakietowego. Metoda oparta jest na wzorach Gurneya dla oceny prędkości miotania powłoki pocisku produktami detonacji materiału wybuchowego oraz na założonej postaci zależności prędkości elementu powłoki od czasu. Numeryczne całkowanie równań trajektorii poszczególnych elementów prętów pozwala określić położenie prętów w kolejnych momentach czasu. Przedstawiono wyniki przykładowych obliczeń.

PREDICTION OF FIELD OF FIRE OF A BAR WARHEAD

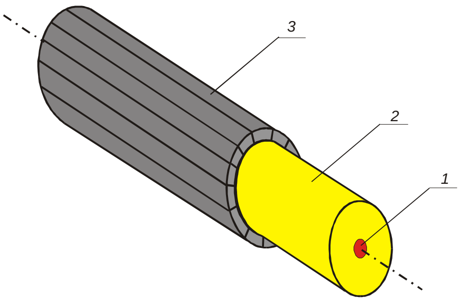
Abstract: In the paper, a method of prediction of the shape of a field of fire of a bar warhead of an antiaircraft missile has been presented. The method is based on Gurney formula for predicting velocity of a shell launched by detonation products of an explosive and an assumed form of function describing time dependence of the velocity of shell elements. Numerical integration of equations of trajectory of individual elements of bars provides bar positions in consecutive moments of time. Exemplary results of calculations have been presented.

1. Introduction

Bar warheads, although somewhat “out-of-date” [1], are still widely used in antiaircraft missiles. For designing fuses, used in these warheads, it is essential to predict the shape of the field of fire. For this reason, motion of bars, launched by the pressure of detonation products of an explosive charge, should be modelled. In the paper, an approach to this problem is proposed. It is based on the classical Gurney formula [2], with some modifications proposed in [3].

Basic assumptions of proposed model and used formulae are presented in the section 2. The algorithm of constructing solution is described in the section 3. The model is analysed on the basis of exemplary results of calculations in the section 4. Basic conclusions are given in the section 5.

2. Description of the model



Construction of a considered warhead is shown schematically in Fig.1. The igniter ignites the explosive charge. A detonation wave moves from the point of ignition. Pressure of detonation products launches the bars.

Fig.1. Considered warhead: 1 – igniter, 2 – explosive, 3 – metal bars

The proposed model is based on the following assumptions:

1. In the point of the axis, where the igniter is situated, a spherical detonation wave is initiated. Its front approaches individual points of bars at various values of the angle α – Fig.2.

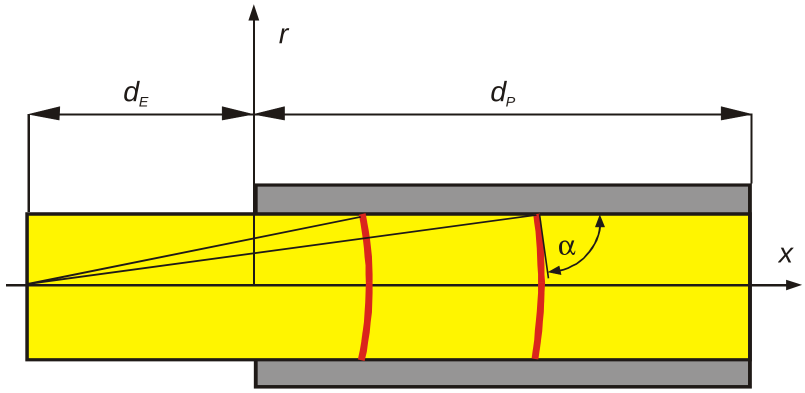


Fig.2. Consecutive positions of detonation wave front

2. The maximum velocity of individual parts of bars is assessed on the basis of the Gurney formula for a cylindrical shell [2]:

$$u_{m0} = \frac{u_G}{\sqrt{\mu + 0,5}}, \quad \mu = \left[\left(\frac{r_B}{r_E}\right)^2 - 1\right]\frac{\rho_B}{\rho_E} \tag{1}$$

where: u_G –Gurney velocity for grazing detonation, r_B – external radius of bars, r_E – internal radius of bars, ρ_B – density of bars material, ρ_E – density of explosive.

3. The fact of incline impact of the detonation front on the bars is taken into account by proper modification of the value of Gurney velocity. The following formula is used ([3]):

$$u_G(\alpha) = \begin{cases} \sqrt{4/3}u_G \quad dla \quad \sin\alpha \le 0,7 \\ u_G\sqrt{1 + (1 - \sin\alpha)/0,9} \quad dla \quad \sin\alpha > 0,7 \end{cases} \tag{2}$$

4. The following formula describing dependence of the velocity of bar elements on time is used ([3]):

$$u = u_m\sqrt{1 - \frac{1}{1 + [(t - T)/\tau]^2}}, \quad \tau = \frac{m_0 u_m}{p_H}, \quad m_0 = \frac{\rho_B\left(r_B^2 - r_E^2\right)}{2r_E} \tag{3}$$

where: p_H - detonation pressure, T – time from the ignition of the charge to the moment of detonation front impact, u_m – maximum velocity of a given bar element.

5. The gas escape through the opening spaces between bars is not taken into account. This assumption is based on the fact, that the acceleration time τ is very short and the Poisson's effect causes that the bars move for a time without loosing their contact.
6. We do not take into account aerodynamic drag (this assumption will be discussed further on).
7. The rigidity of the bars material is not taken into account. Initial assessments proved that internal forces are much lower than the forces exerted by detonation products and the inertial forces.

3. Method of constructing solution

The bars are divided into n elements. For each element i coordinates of center of gravity x_{ci}, r_{ci}, time of the impact of detonation wave front T_i, maximum velocity u_{mi} and time constants τ_i are calculated:

$$x_{ci} = d_E + d_P(i - \tfrac{1}{2})/n \quad i = 1, n \tag{5}$$

$$r_{ci} = \left(r_E + r_B\right)/2 \tag{6}$$

$$T_i = \frac{\sqrt{x_{ci}^2 + r_E^2}}{D} \tag{7}$$

$$sa = \frac{x_{ci}}{\sqrt{x_{ci}^2 + r_E^2}} \tag{8}$$

$$u_{mi} = \begin{cases} \sqrt{4/3}u_{m0} \quad dla \quad sa \le 0,7 \\ u_{m0}\sqrt{1+(1-sa)/0,9} \quad dla \quad sa > 0,7 \end{cases} \tag{9}$$

$$\tau_i = \frac{m_0 u_{mi}}{p_H}, \quad m_0 = \frac{\rho_B\left(r_B^2 - r_E^2\right)}{2r_E} \tag{10}$$

where: d_E – distance along the axis form the ignition point to the beginning of bars, d_P – bar length, D – detonation velocity, u_{m0} – maximum velocity given by (1).

Equations of trajectories of individual elements of bars are integrated numerically:

$$x_{ci}(t+\Delta t) = x_{ci}(t) + [u_{xi}(t) + u_{xi}(t+\Delta t)] \quad \Delta t/2 \tag{11}$$

$$r_{ci}(t+\Delta t) = r_{ci}(t) + [u_{ri}(t) + u_{ri}(t+\Delta t)] \quad \Delta t/2 \tag{12}$$

Axial and radial components of velocity v_{xi} and v_r for $t + \Delta t$ are determined making use of the fact that the acceleration vector is perpendicular to the line linking centers of gravity of bar elements.

$$u_{xi}(t+\Delta t) = u_{xi}(t) + [u_i(t+\Delta t) - u_i(t)] \quad \sin \quad \beta \tag{18}$$

$$u_{ri}(t+\Delta t) = u_{ri}(t) - [u_i(t+\Delta t) - u_i(t)] \quad \cos \quad \beta \tag{19}$$

$$\beta = \mathrm{arctg}\{[r_{i-1}(t) - r_{i+1}(t)]/[x_{i+1}(t) - x_{i-1}(t)]\} \tag{20}$$

For $i = 1$and $i = n$, angle β is calculated by the use of (20), taking $i = 2$ and $i = n-1$.

The value of velocity $u_i(t+\Delta t)$ is calculated by the fomula:

$$u_i(t+\Delta t) = u_{mi}\sqrt{1 - \frac{1}{1+[(t+\Delta t - T_i)/\tau_i]^2}} \quad -a_s\Delta t \tag{21}$$

Time increment Δt is set equal to the time of passing distance Δx by detonation wave.

4. Analysis of the model

Fig. 3 shows exemplary results of calculations of positions of a bar in consecutive moments of time. The bar is generally straight, although the near end is somewhat curved. It is a consequence of the higher values of velocity of this part of the bar. Velocity distributions are shown in Fig.4. Higher value of velocity stems from the fact, that detonation wave impacts this part of the bar at lower values of angle α, so the larger part of detonation products energy is used for launching the bar.

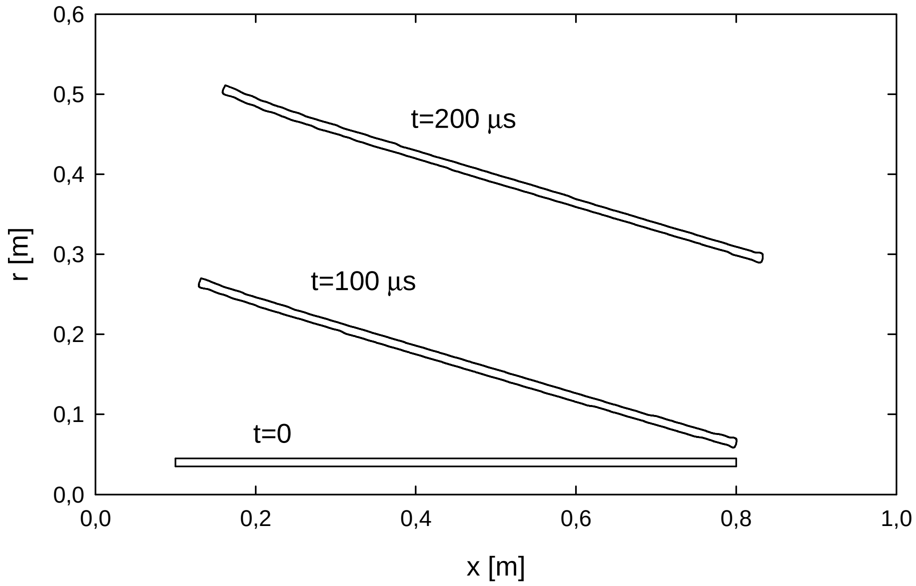


Fig.3. Positions of a bar in three moments of time (time is measured from the moment of ignition)

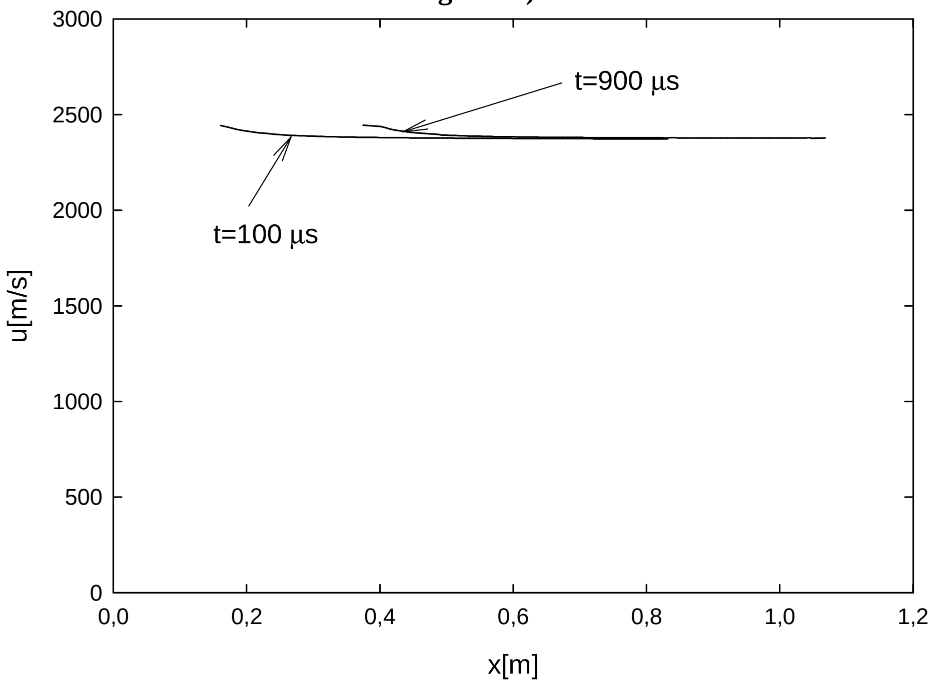


Fig.4. Velocity distributions along the bar

The effect of distorting the bars depends on the distance d_E. Fig. 5 illustrates this influence.

The angle between the straight part of the bar and the axis – 18,5^o is slightly higher than Taylor angle γ– 16,8^o:

$$\gamma = 2\arcsin\left(\frac{u_{\max 0}}{2D}\right) \qquad (22)$$

This difference is caused by the fact, that the model takes into account curvature of the detonation front and finite time of launching.

Results of calculations enable us to predict shape of a static field of fire of the warhead (in the reference frame moving with the rocket) with the positions of bars in consecutive moments of time – Fig. 6. Basing on the results of calculations it is possible to predict the zone of action and time of action of proximity fuse. For the warhead with 24 bars (approximately isometric cross section) distances between centers of gravity of bars are shown in Fig.7.

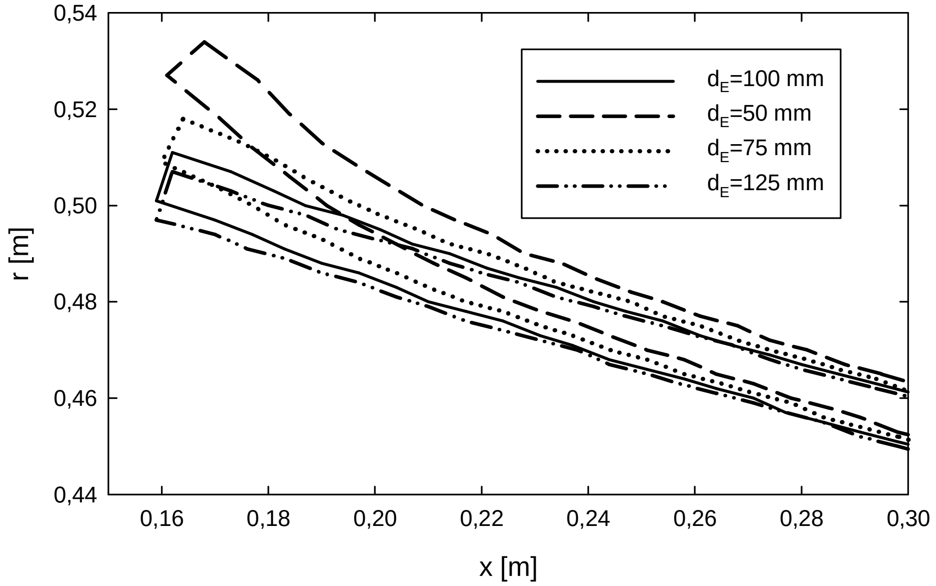


Fig.5. Distortion of the near end of a bar for various distances from the point of ignition to bars

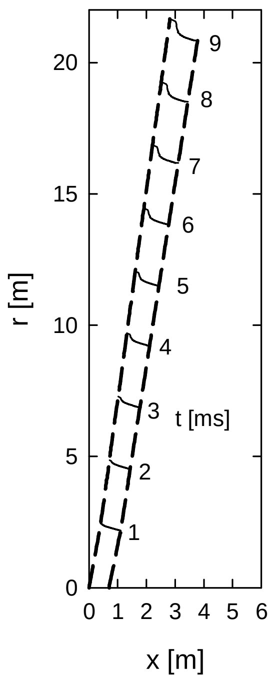


Fig.6. The shape of the field of fire

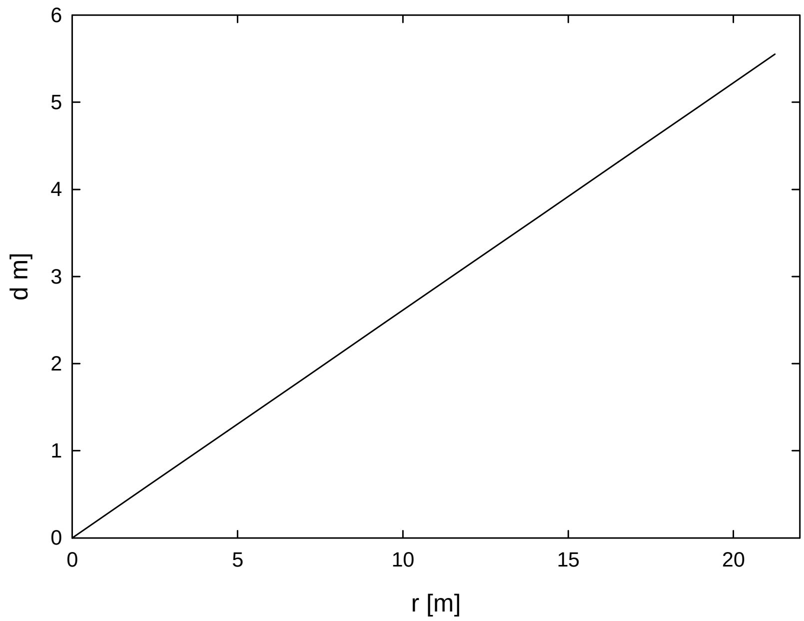


Fig.7. Distances between bars as a function of radius

Positions of bars shown in Fig. 6 are overestimated, because the aerodynamic drag is not taken into account. In reality they reach given radius *r* later than it is calculated by the model. It is difficult to assess the aerodynamic drag, because there is no data concerning the value of the drag coefficient in the range of predicted velocities. For a rough estimation a limiting value of the drag coefficient for shell fragments C_D = 2 [3] is used. The force of the aerodynamic drag is calculated by the formula:

$$F = -C_D \cdot S \cdot \frac{\rho_p \cdot u^2}{2} \tag{23}$$

where: S – area of the external surface of a bar, ρ_p – density of air, u – velocity of a bar.

The equation of inertial motion of bars has the form:

$$m\frac{du}{dt} = -C_D \cdot S \cdot \frac{\rho_p \cdot u^2}{2} \tag{24}$$

which can be transformed to:

$$\frac{1}{u}\frac{du}{dt} = -\frac{1}{L}u, \quad L = \frac{2m}{C_D S \rho_p} \tag{25}$$

Taking into account, that the bars move mainly in the radial direction, we can use the following approximation:

$$\frac{1}{u}\frac{du}{dt} = -\frac{1}{L}\frac{dr}{dt} \tag{26}$$

Hence:

$$\int_{u_0}^{u} \frac{1}{\mu} d\mu = -\frac{1}{L}\int_0^r d\xi \tag{27}$$

After integration we obtain:

$$\ln\left(\frac{u}{u_0}\right) = -\frac{r}{L} \tag{28}$$

Hence:

$$u = u_0 \exp\left(-\frac{r}{L}\right) \tag{29}$$

and further:

$$\frac{dr}{dt} = u_0 \exp\left(-\frac{r}{L}\right) \tag{30}$$

$$\int_0^r \exp\left(\frac{r}{L}\right) dr = \int_0^t u_0 d\tau \tag{31}$$

After integration we obtain:

$$L\left[\exp\left(\frac{r}{L}\right) - 1\right] = u_0 t = r_0 \tag{32}$$

where r_0 is a radial position of a bar for no drag.

Finally:

$$r = L \ln\left(\frac{r}{r_0} + 1\right) \tag{33}$$

The value of L can be estimated by the formula:

$$L = b\frac{\rho_b}{\rho_p} \tag{34}$$

Fig.8 shows calculated difference between the position estimated on the basis of the model and position calculated by (33). From the diagram one can find the value of $r_0 - r$ for given r. Then the value of r_0 can be calculated. Making use of this value one can determine value of t corresponding to given distance r.

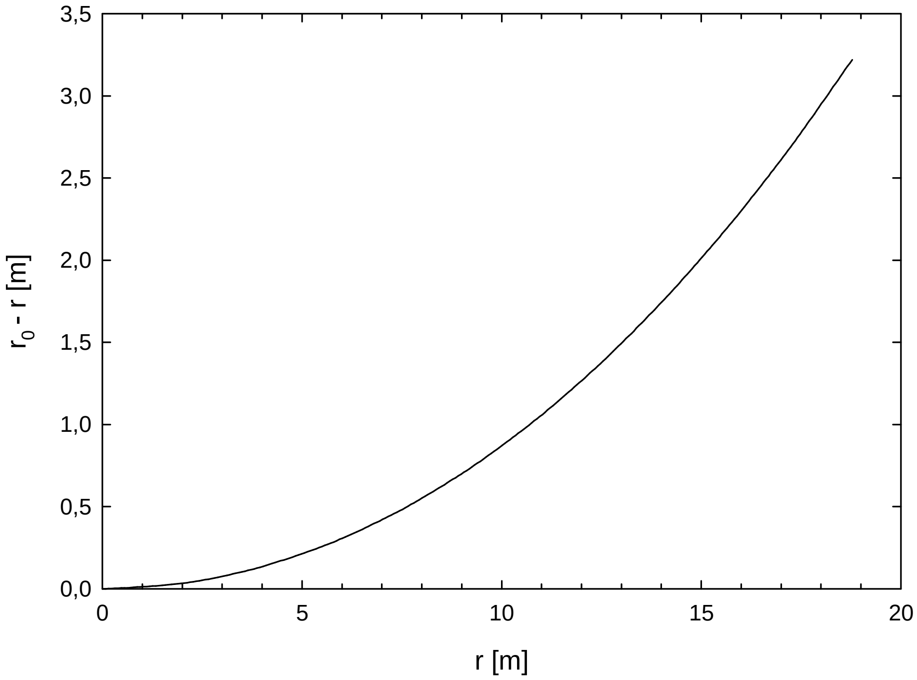


Fig.8. Difference between radial position of a bar calculated by the model and radial position estimated with taking into account limiting value of aerodynamic drag

5. Conclusions

Analysis presented in the Sec. 4 reveals that:

1. The model predicts the angle of driving bars about 10% higher than the Taylor angle.
2. The model predicts deformation of the near ends of bars. The scale of deformation depends on the distance between the ignition point and bars.
3. The model provides results, that can be used to predict the shape of the field of fire of a bar warhead.
4. The effect of aerodynamic drag is not high in the vicinity of the warhead. However it rises fast with increasing radial distance.

References

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