ELABORATION OF GEODETIC CONTROL NETWORK AT CONSTRUCTION OF BRIDGE OVER VISTULA RIVER IN PULAWY

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ABSTRACT

In this paper the method of geodetic control network elaboration is presented. That network consists of GPS vectors and distances measured classically. The method of reduction of GPS vectors from WGS84 to local coordinate system was described. Common adjustment of GPS vectors and distances in a local coordinate system was performed. The observations set was obtained from measurements of geodetic control network at the construction site of bridge over Vistula River in Pulawy.

1. INTRODUCTION

In engineering surveys local coordinate systems are usually used. The reason for this is the necessity to keep high accuracy level within the survey area (Polish Norm, 1980). There are a lot of methods to define local coordinate system (Cellmer, 2005), (Gajderowicz, 1991). In GPS surveys simple and commonly used approach is to define local topocentric coordinate system with its origin in the centre of measurement area. Interesting subject in GPS observation processing in local coordinate system is determination of linear distortion after reduction.

The analysis of influence of the vector distance from the origin of topocentric coordinate system on linear distortion, is performed in further part of this paper. The methodology of common processing of reduced vectors with linear measurements in a local coordinate system is shown in the next part. Data from realization of network created for construction of bridge over Vistula River in Puławy is processed using above algorithms. The results are shown in the last part of this paper.

2. THE LOCAL COORDINATE SYSTEM REALIZATION

The local coordinate system is defined as topocentric coordinate system with its origin in the middle of the measurement area. In the adjustment problem the coordinates in this local coordinate system were assumed as parameters. After adjustment the coordinates X and Y were shifted through vector t and axes X and Y of local coordinate system and rotated around Z axis by angle φ. The components of vector t were determined as differences between coordinates X and Y of point 1006 in local coordinate system and coordinates of this point in national coordinate system "1965". The angle φ was determined as difference between azimuth $A_{1006\text{-}1008}$ calculated from coordinates in national coordinate system "1965" and azimuth $A_{1006\text{-}1008}$ calculated from coordinates in local coordinate system (after translation). This transformation was

needed to approximately fit coordinate system of bridge's geodetic network to coordinate system of geodetic network established for the purpose of road construction. For small measurement areas the results of distance observations are put to adjustment directly. However before putting GPS-vectors to adjustment they must be reduced from geocentric system to a local system.

3. THE REDUCTION OF THE GPS-VECTORS TO LOCAL COORDINATE SYSTEM

The results of preliminary elaboration of GPS data consist of the following quantities: approximate coordinates B, L, h of the points, 3-dimensional components of GPS-vectors: ΔX, ΔY, ΔZ and their covariance matrices. These data are placed in output file created by commercial software e.g. Aschtech Inc. software: 'GPPS' (Aschtech Inc., 1991).
The reduction algorithm consists of the following stages:

1. **Determination of approximate geodetic coordinates: B_p, L_p, h_p (on 'GRS80' ellipsoid) for origin points of each vector**
2. **Cartesian coordinates X_p, Y_p, Z_p calculation**
3. **End of each vector coordinates X_k, Y_k, Z_k determination:**
   $X_k = X_p + \Delta X$
   $Y_k = Y_p + \Delta Y$
   $Z_k = Z_p + \Delta Z$
4. **All points coordinates X_t, Y_t, Z_t on the plane of topocentric coordinate system calculation (assuming origin of the axes X_t, Y_t, Z_t in the middle of the measurement area)**
5. **Reduced components ΔX_t, ΔY_t, ΔZ_t calculation (coordinates of end of each vector minus coordinates of origin of each vector)**
6. **The covariance matrices of the reduced vectors determination**

In the first stage the presented algorithm is searching for adequate ID points and their approximate coordinates in the data file. Succesive stages require calculations performed on the basis of well-known formulas e.g. (Gajderowicz, 1991) (Czarnecki, 1992). In the fourth stage coordinates of all points are calculated by means of the following formula:

$$X_t = R(X - X_0), \qquad (1)$$

where:

$$X_t = \begin{bmatrix} X_t \\ Y_t \\ Z_t \end{bmatrix}$$ **- point coordinates in topocentric coordinates system**

X – point coordinates in geocentric coordinate system
X_0 – coordinates of the origin of the topocentric system in geocentric system

$$R = \begin{bmatrix} -\sin B\cos L & -\sin B\sin L & \cos B \\ -\sin L & \cos L & 0 \\ \cos B\cos L & \cos B\sin L & \sin B \end{bmatrix}$$ **- rotation matrix**

The orientation of X and Y-axes are in North and East-direction respectively.
In the last stage we use the law of the errors propagation in the following form:

$$C_T = R C_{Grs80} R^T, \qquad (2)$$

where:

C_T – reduced vectors covariance matrix
C_{Grs80} – covariance matrix of vectors before reduction

4. IMPACT OF THE DISTANCE TO THE ORIGIN OF TOPOCENTRIC COORDINATE SYSTEM ON LINEAR DISTORTION OF THE REDUCED GPS VECTOR

To analyze the influence of distance from vector to origin of coordinate system on linear distortion the 1000 m long vector (with known X_0, Y_0, Z_0) in the XY plane was taken. The assumption was made that the beginning of the of vector lies in the origin of local topocentric coordinate system. Three variants of vector layout were processed: meridian, parallel and 45 degrees slope. For each of them the coordinates X, Y, Z of the end of vector were calculated in WGS84 geocentric coordinate system. Next step was to derive the coordinates of the beginning and the end of each vector in topocentric coordinate system with its origin in some distance from the vector. The length of component lying in XY plane of new topocentric coordinate system was evaluated. The linear distortion was derived from:

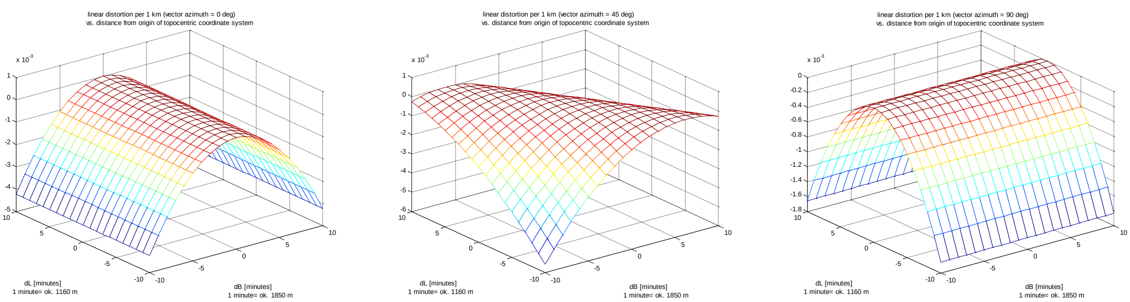
$$\delta d = d_n - d_0, \qquad (3)$$

where:

$d_n = \sqrt{\Delta X^2 + \Delta Y^2}$ - length of component lying in XY plane of new topocentric coordinate system

$d_0 = 1000$ m – length of component lying in XY plane of first system.

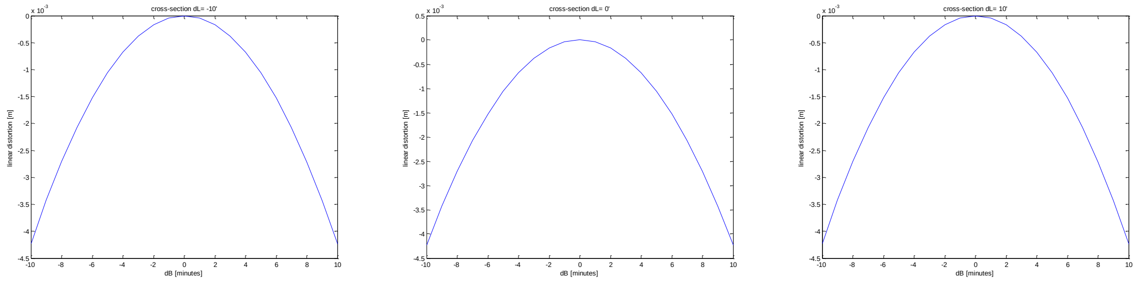
As an origin of new topocentric systems the nodes of grid with dL=1', dB=1' placed around origin point of first coordinate system were taken. The results are shown in Fig. 1.



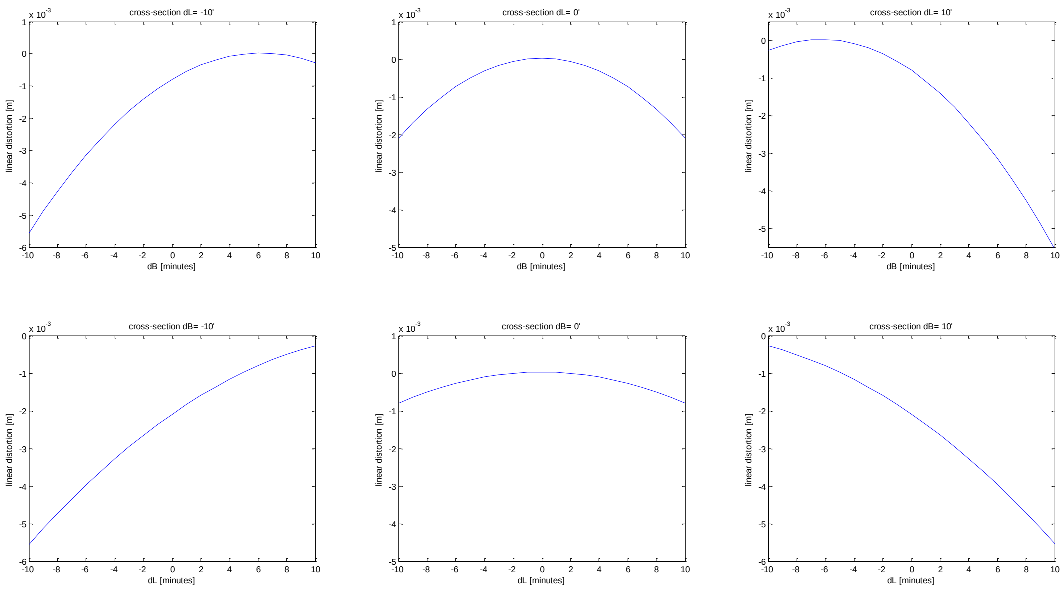
a) b) c)

Fig. 1. Linear distortion of vector after reduction. Azimuth of vector 0^0 (a), 45^0 (b), 90^0 (c).

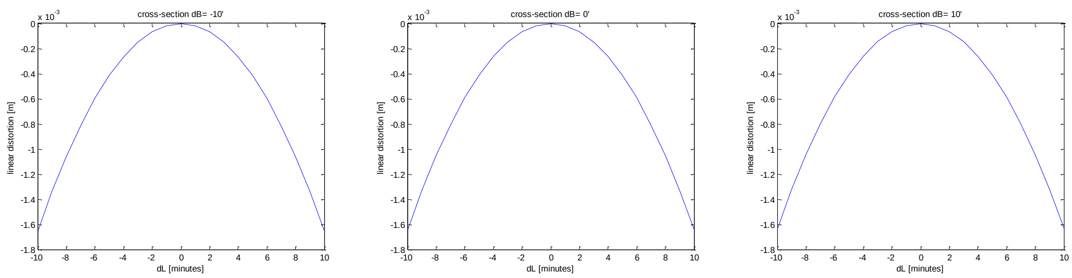
Fig. 2 shows chosen cross – sections of Fig. 1.



a)



b)



c)

Fig. 2. Cross–sections of plot shown in Fig. 1.

It can be seen that for the distance smaller then 7.5 km the linear distortion is smaller then 1 mm. For the longest distance (about 22 km) the linear distortion is about 5 mm.

5. THE STATISTICAL MODEL OF ADJUSTMENT PROBLEM

The statistical model can be presented in the form of the following covariance matrix:

$$C_L=\sigma^2\begin{bmatrix} Q_w & \\ & Q_d \end{bmatrix} \quad (4)$$

where:

- **C_L – observations covariance matrix**
- **σ^2 – variance coefficients**
- **Q_w, Q_d – cofactors matrices for vectors and distances**

Matrix Q_w contains only elements concerning the components Δx, Δy of reduced vector (four elements of matrix C_T from equation (2)).

6. ELABORATION OF THE CONTROL NETWORK AT CONSTRUCTION OF BRIDGE OVER VISTULA RIVER IN PULAWY

In fig. 3 layout of points of control network and measured vectors is shown.

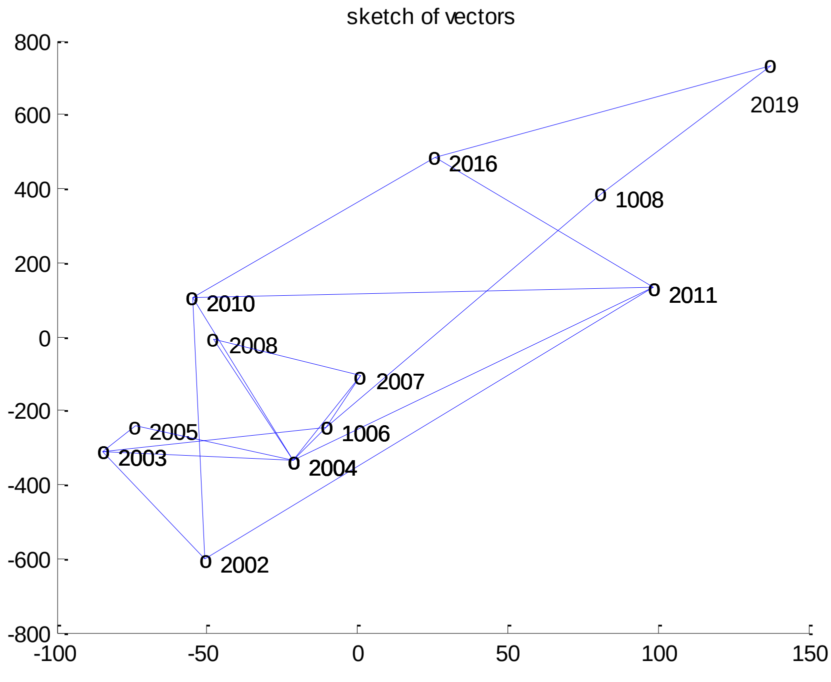


Fig. 3. Layout of points of control network and measured vectors.

In tab. 1, 2, 3 the results of adjustment are presented.

To perform preliminary standardization of observations the mean errors values were assumed as follows: for GPS vectors - values obtained after reduction, for distances - m_d=2mm.

Nr 1	Nr 2	$V_{\Delta X}$ [mm]	$V_{\Delta Y}$ [mm]
1006	1008	-4	5
2002	2003	1	0
2002	2010	2	0
2002	2011	-1	0
2003	2004	0	0
2003	1006	0	2
2004	1006	0	1
2004	2008	4	-4
2004	2011	0	0
2005	2003	0	0
2005	2004	0	0
1006	2007	-1	-2
2007	2004	2	-2
2010	2016	-4	1
2011	2010	4	0
2011	2016	3	-1
2016	2019	0	-1
2019	1008	1	-1

Tab. 1. Vector components residuals.

Nrp	Nrk	d [m]	V_d [mm]
1006	2007	138.368	0.0
2010	2011	155.509	0.2
2011	2008	200.458	0.0
2008	2007	111.421	0.2
2010	2008	112.889	0.0

Tab. 2. Distances residuals.

Nr	mX [mm]	mY [mm]
1008	6	4
2002	3	2
2003	3	2
2004	3	1
2005	4	2
2007	3	0
2008	3	1
2010	3	1
2011	3	1
2016	4	2
2019	6	3

Tab. 3. Mean errors.

7. FINAL REMARKS

For vectors being 7.5 km away from the origin of the topocentric, local coordinate system the value of linear distortion after reduction is lower than 1 mm/km. This value is usually not significant for construction surveys and deformation measurements.

The method of local coordinate system realization is proper for areas smaller than about 6.25 km^2.

The algorithm of space GPS vector reduction makes a common adjustment the GPS-vectors and classical observations in a local coordinate system possible.

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