ON INTEGRATION OF MULTIPLE-SOURCE DATA IN GEODETIC MONITORING OF STRUCTURAL DISPLACEMENTS

Witold Prószyński

Warsaw University of Technology, Institute of Applied Geodesy, Section of Engineering Surveying, phone: (22) 234 7287, mobile: 606 616 420, wpr@gik.pw.edu.pl

1. INTRODUCTION

A variety of data on the behaviour of a controlled structure can be available, as multiple are also the methods of gathering such data. Reference can be made here to such techniques as the geodetic, geotechnical and photogrammetric measurements, remote sensing, simulations of structural performance, as well as expert reviews. Such data provide potential inputs for evaluation of the safety of a structure. Figure 1 contains a flow diagram showing an idealised concept of the process, which involves complete processing of all available data.

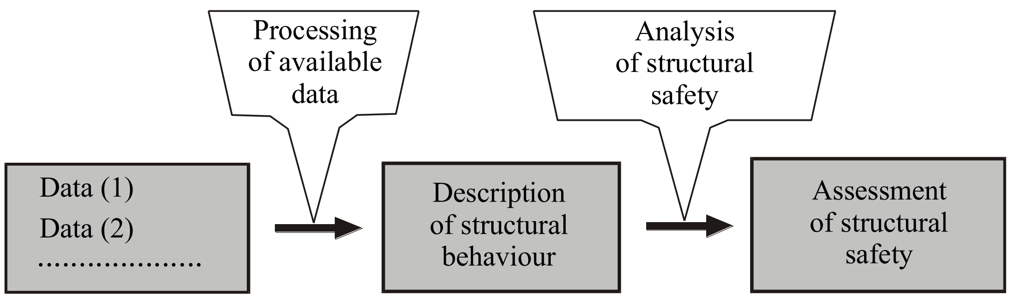

Fig. 1. Flow diagram for evaluation of structural safety.

Such an opportunity could be offered by a global system for structural safety assessment.

Taking for granted that the integrity of the data obtained from each source is ensured, the key advantages that can be seen in the integration of information resources seem to be as follows:

- **possibility of verifying the consistency of multiple-source data and hence, better reliability of data processing;**
- **extended description of structural behaviour;**
- **more reliable and accurate assessment of structural safety.**

Certain rules and methods for integration of measurement data should obviously be developed not only for global (specialised) systems of structural safety control, but also for more or less interlinked geodetic systems for monitoring of displacements, that would incorporate outputs from many instruments and measuring devices. To surveyors, it is important what kind of information would be processed by such systems and how they would be linked to the said specialised systems. There seems to be no general answer to this question. Critical

elements here are the type of structure, its current condition, as well as the scope and the broadly defined requirements of specialised monitoring (including the accuracy and frequency of data updates). Considered as an exemplary solution for safety control of water reservoirs and dams, because supported by success stories worldwide, can be the deployment of geodetic systems for monitoring of displacements, that are part of specialised systems for safety control of such structures (Duffy et al, 2001). Geodetic measurements are integrated, in those systems, with data obtained by means of other measurement methods.

To resume the discussion started by Chrzanowski at al (1985 and 1986), Chen and Yang (1994) and Prószyński (1999), we will concentrate our attention on data integration within geodetic monitoring systems, confining ourselves to quantitative information of discrete type.

2. PRERECQUISITES FOR DATA INTEGRATION

The data denoted as $l_1, l_2, ..., l_n$, will be considered as continuous random variables, each with a normal distribution, a zero expectation value, and an estimable dispersion parameter.

By referring here to data integration, we will mean the integration of data for clearly defined purposes, e.g. in order to determine the coordinates of points, the displacements of points, the displacements of and/or deformations in the body of a structure, etc. Accordingly, we will deal with a data integration method that should be recognised as fundamental in geodetic monitoring of displacements, that is, based on an explicitly defined parametric model. There is no question that with some other data integration methods being also available for structural safety assessment the scope of methodological considerations has to be narrowed.

We will say that data $l_1, l_2, ..., l_n$ can be integrated with each other if:

a) **each data can be represented as a (at least C^1 class) function of parameters $X_1, X_2, ..., X_u$ which we are interested in for a strictly defined purpose**

$$l_i = f_i(X_1, X_2, ..., X_u) \tag{1}$$

or in linearised form, i.e. after expansion at point $X_1^o, X_2^o, ..., X_u^o$

$$l_i - l_i^o = a_{1,i}x_1 + a_{2,i}x_2 + ... + a_{u,i}x_u \tag{2}$$

In addition to pseudo-observations (i.e. $l_i = X_i$), notation (1) includes a specific type of data representing a certain relationship which is formed, to our understanding, by parameters $X_1, X_2, ..., X_u$, i.e. $f_i(X_1, X_2, ..., X_u) = 0$ (e.g. an equation corresponding to our knowledge of the point's motion parameters); then, as the value of random variable l_i in notation (1), we put zero;

b) **that of data $l_1, l_2, ..., l_n$ which due to its nature depends on the reference system must be expressed in a single common reference system;**

c) **for each data we are able to estimate the accuracy as standard deviation $\sigma_{l,i}$ or (less often) limit error $\Delta_{l,i}$.**

Such estimation allows one to include the randomness of available data and use the following well-known observation equation:

$$a_{1,i}x_1 + a_{2,i}x_2 + \ldots + a_{u,i}x_u = l_i^{obs} - l_i^{o} - \varepsilon_i \quad ; \quad \sigma_{l,i} \tag{3}$$

This also applies for data representing knowledge of a structure's behaviour.

In this case $l_i^{obs} = 0$, and ε_i is the true error of such data.

If we know only limit error $\Delta_{l,i}$ of any data, we can determine standard deviation $\sigma_{l,i}$ based on the assumption that the distribution of variable l_1 is normal.

Hence, the fulfilment of the prerequisites for data $l_1, l_2, \ldots, l_n$, as described above, implies that a mathematical (stochastic) model is available to us which cross-relates individual sets of such data to each other and allows, provided that certain algebraic requirements are also met, to include all such data in the determination of its parameters and estimate the accuracy of such determinations.

As it can be seen from the approach presented here, creating models and integrating data for a specified purpose are strictly interrelated. The concept of such a relationship is demonstrated in the following notations, i.e.

$$l_1, l_2, \ldots, l_n \rightarrow \{w_M\} \qquad\qquad M \rightarrow l_1, l_2, \ldots, l_k \tag{4}$$

Meant to be integrated, data $l_1, l_2, \ldots, l_n$ define a certain set of properties $\{w_M\}$ that should be inherent in their integration model, without providing a clear definition of what form such a model might take. With the specific model M being available, only certain types of data can be integrated, i.e. $l_1, l_2, \ldots, l_k$.

The integrability of data $l_1, l_2, \ldots, l_n$ through model M with parameters $x_1, x_2, \ldots, x_u$ can be expressed as follows:

$$l_1 \wedge l_2 \wedge \ldots \wedge l_n \,|\, M(x_1, x_2, \ldots, x_u) \tag{5}$$

If any of the obtained data does not meet the requirements for integrability with a group of the remaining data which are integrable with each other through an available model, they cannot be used to estimate the parameters of such a model. When describing the behaviour of a structure, they could be used as quantitative or only qualitative controls, however, that is, they can be supportive of structural safety assessment. With non-integrable data being identified as l_r, l_s the foregoing can be expressed as follows:

$$l_1 \wedge l_2 \wedge \ldots \wedge l_n \,|\, M(x_1, x_2, \ldots, x_u); \quad l_r, l_s \quad \rightarrow \quad \hat{x}_1, \hat{x}_2, \ldots, \hat{x}_u; \quad l_r, l_s \tag{6}$$

Data l_r, l_s will be complementary to the estimated parameters of a model and help interpret the behaviour of a structure. It is clear that full output information will also contain the estimated accuracy of parameters and complementary data.

The integrability of data through a specific model does not ensure the suitability of such a model for particular data values. The lack of suitability may suggest the need to use another model.

Finally, we will see that the integration of the obtained data l_1, l_2, ..., l_n can be interpreted as their transformation into a set of parameters $x_1, x_2, ..., x_u$, which we are interested in due to some reasons, i.e. {l} → {x}, which does not include any complementary data.

The target integration model for measurement data l can also be obtained by combining two models. By defining the input functional model as

$$\mathbf{Ax} = \mathbf{l} \tag{7}$$

and assuming that x can be input information to the model

$$\mathbf{Bc} = \mathbf{x} \tag{8}$$

where B – coefficient matrix, c – parameter vector of a new model,

we will finally obtain the target functional model for integration of data l, which will be as follows:

$$\mathbf{Hc} = \mathbf{l}, \qquad \text{where } \mathbf{H} = \mathbf{AB} \tag{9}$$

For practical reasons, it will certainly be used as a stochastic model, i.e. with equations of type (3).
By using some examples of geodetic systems for monitoring of displacements, it can be demonstrated (Chrzanowski et al 1986, Chen and Yang 1994) that target model (9) can allow for integration of additional measurement data, which are non-integrable in input model (7).

With the correctness of the model itself, which is assumed in the considerations above, only the inaccuracy of data entered into it is taken into account.

3. EXAMPLES OF DATA INTEGRATION THROUGH A PARAMETRIC MODEL

The simplest form of data integration is a model for multiple measures of a single quantity, i.e. a one-parameter model. We will call it an elementary model and identify it as M_e. We deal with it when preparing data for each of the following examples of integration methods:

a) integration through a control network geometry model (geometric model - M_g)

Such is a static model of the control network relating to a single epoch (parameters – coordinates of the network points) or a pair of epochs (parameters – components of displacement vectors of the network points).

Every measurable quantity, which can be expressed as a function of such parameters, is assigned an observation equation and can be incorporated into a model together with its measurement result and estimated accuracy. Thus, inclinometer or pendulum measurement data can also be included. These are the models which can be used to determine the displacements of selected points of a structure if the motion of such points during measure-

ment can be considered negligible in terms of measurement accuracy. They are exact models in which only the inaccuracy of measurement data inputs is considered.
The interpretation value of the displacements identified in a geometric model is conditioned by the need to record certain complementary quantities, which, in model M_g, are not integrable with the measurement results of the network elements (e.g. time of day, building temperature, wind direction and strength, progress of construction works), that is, information that could be used to provide a more reliable assessment of the behaviour of a structure.

b) integration through the point motion model of a single-epoch control network (kinematic model – M_k)

The parameters here are the coordinates of the network points at a certain moment of time, as well as the velocities and acceleration rates, if any, of such points during network measurement. This model, which serves here only as an example of how measurement data can be integrated, is used to create multi-epoch kinematic models.

These are the models which can be used to determine the displacements of selected points of a structure if motion of such points, even during a single network measurement, should be considered significant in terms of the measurement accuracy of the elements of such a network. Then a properly selected model of motion should be applied to enable integration of measurement data obtained at various moments of time. In general, these models are not exact models. As with the geometric model, additional physical data should be collected to raise the interpretation value of determined parameters.

c) integration through the geometric model of structural behaviour (deformation model – M_o)

This model incorporates only geometric effects (deformations, rotations, mutual displacements of structural elements) of various factors affecting the structure under review. In addition to measurement data integration as in model M_g, it is possible, when using an approximation model (e.g. Chrzanowski et al, 1986), to integrate the measurement data of linear deformations and measure any inclination changes at short sections set out from the certain point of a structure. Models M_o are, in general, approximate models which are relations defined in space domain or time-space domain. As with models M_g and M_k, additional physical data should be collected to raise the interpretation value of determined parameters. We will see here that, given the material integrity and non-deformability of the body of a structure, instead of model M_o we can use the geometric (M_g) or kinematic (M_k) model as the basis for integration of measurement data.

d) integration through the physical model of structural behaviour (strength model - M_w)

These are the so-called cause-effect models which describe relations between the broadly defined loads (e.g. dead weight, useful load, temperature variations, wind pressure) that work on a structure (*causes*) and the resultant displacements and deformations of such a

structure (*effects*). They can also be relations representing the transformation of a structure from one state to another as a result of such effects.
In addition to measurement data integration as in model M_0, it is possible to integrate here, among other data, the results of stress measurements that are conducted at short sections set out from the certain point of a structure.

In general, the relations making up the model of structural behaviour are relationships showing a certain degree of approximation to the reality, and it is necessary to verify their correctness by confronting them with collected information.

Theoretically, there are many possibilities of creating cause-effect models. However, the complexity of how a structure may actually respond to the loads applied to it makes such models too complicated and, at the same time, too little exact when confronted with the accuracy levels of measurement results. Therefore, in practical (non-research) applications they are superseded by simplified analysis based not only on the relevant theoretical know-how of construction engineers, but also on their expertise and technical intuition. Noticeably, measurement data (including that obtained with surveying methods) are increasingly used to upgrade (calibrate) the models of structural behaviour, which are primarily based on theoretical solutions. The cause-effect models seem to become an important tool in the future to process the behavioural data and assess the security levels of structures. They will create a convenient platform for closer collaboration between structural mechanical engineers, geotechnicians and engineering surveyors, that is, interdisciplinary collaboration.

4. PRACTICAL EXAMPLE

Figure 2 provides a schematic view of the results of monitoring the displacements of selected points of a building in time interval $\Delta t_{1,2} = t_2 - t_1$ (t_1 - initial measurement, t_2 - current measurement), which include:

- **vectors of vertical displacements (p_V) relative to the reference system that is external to the structure (measurement technique: precise levelling network);**
- **vectors of horizontal displacements (p_H) relative to the reference system that is external to the structure (measurement technique: trigonometric network);**
- **inclination changes (α_V) of short survey bases in selected places of the structure (measurement technique: electronic level);**
- **change in temperature outside the building (ΔT).**

So, we have the following set of data:

$$p_{V,1}, p_{V,2}, \ldots, p_{H,1}, p_{H,2}, \ldots, \alpha_{V,1}, \alpha_{V,2}, \ldots, \Delta T$$

The integration of all such data may pose a serious problem. The body of a building may be heterogeneous, e.g. dilated segments of the structure, self-supporting wall facing that loosely interacts with the main structure. Due to various types of impacts (including temperature variations), the building is subject to deformations and displacements. The model that could integrate all collected information would be very complex, and considering that

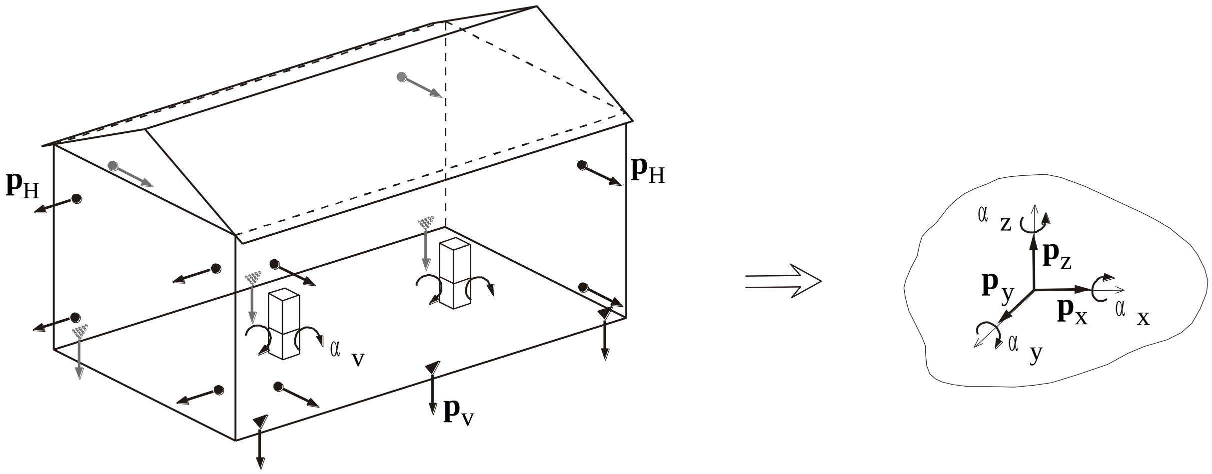

Fig. 2. Displacements of selected points of the building and displacement components for the body of the building.

the outer wall facing becomes deformed on its own, creating such a model would be hardly practicable. Another problem in integrating the said data may be that it is difficult to ensure the integrity which is required in respect of the reference system for positional data, as the reference bases are not identical and the measurements cannot be conducted at the same time.

If omitting possible deformations and mutual displacements of structural elements and considering the temperature readings as complementary information, geometric model M_g with the parameters as shown in figure 2 can be used, i.e.

$$p_{V,1} \wedge p_{V,2} \wedge \ldots \wedge p_{H,1} \wedge p_{H,2} \wedge \ldots \wedge \alpha_{V,1} \wedge \alpha_{V,2} \wedge \ldots \big| M_g(p_x, p_y, p_z, \alpha_x, \alpha_y, \alpha_z); \quad \Delta T$$
$$\rightarrow \quad \hat{p}_x, \hat{p}_y, \hat{p}_z, \hat{\alpha}_x, \hat{\alpha}_y, \hat{\alpha}_z; \quad \Delta T$$

If only vertical displacements and temperature readings were available, we would write as follows:

$$p_{V,1} \wedge p_{V,2} \wedge \ldots \big| M_g(p_z, \alpha_x, \alpha_y); \quad \Delta T \qquad \rightarrow \quad \hat{p}_z, \hat{\alpha}_x, \hat{\alpha}_y; \quad \Delta T$$

5. CERTAIN RELIABILITY FEATURES OF MEASUREMENT DATA INTEGRATION MODELS

From the generally known dependence pertaining to internal network reliability, i.e.

$$\mathbf{R} = \mathbf{I} - \mathbf{Q} \tag{10}$$

where: R – reliability matrix, Q – covariance matrix of adjusted observations in a standardised system, I – identity matrix,

it follows immediately that

- **higher gain in the accuracy of each observation (that is, lower ratio $u_i = \sigma_{\hat{l},i} / \sigma_{l,i}$) due to its involvement in the adjustment of all observations corresponds to a higher degree of its controllability by the remaining observations**

$$R_{ii} = 1 - Q_{ii} = 1 - \left(\frac{\sigma_{\hat{l},i}}{\sigma_{l,i}} \right)^2 = 1 - u_i^2 \qquad i = 1,2, ...,n \tag{11}$$

- **if the *a priori* declared standard deviation of an observation is changed, the dependence is maintained**

$$\frac{\partial R_{ii}}{\partial \sigma_{l,i}} = - \frac{\partial Q_{ii}}{\partial \sigma_{l,i}} \tag{12}$$

where, in a system of more complex structure, the detailed form of these derivatives could be difficult to derive because $\sigma_{\hat{l},i} = f(\sigma_{l,1}, \sigma_{l,2}, ..., \sigma_{l,n})$.

Given that the pre-estimated accuracy of the observations is correct, we will trace the effect of accuracy disproportions in an adjustment model, assuming elementary model M_e for convenience.

We will consider a system with two (non-correlated) observations,

$$\begin{aligned} x &= l_1^{obs} + v_1 \qquad \sigma_1 = \sigma \\ x &= l_2^{obs} + v_2 \qquad \sigma_2 = k \cdot \sigma \ ; \qquad k \geq 1 \end{aligned} \tag{13}$$

Matrix R and resultant reliability coefficients $\sigma_{\hat{\mathbf{v}},1}$, $\sigma_{\hat{\mathbf{v}},2}$ for l_1^{obs} and l_2^{obs} respectively, will be as follows:

$$\mathbf{R} = \mathbf{I} - \mathbf{Q} = \mathbf{I} - \frac{1}{1+k^2} \begin{bmatrix} k^2 & k \\ k & 1 \end{bmatrix} = \frac{1}{1+k^2} \begin{bmatrix} 1 & -k \\ -k & k^2 \end{bmatrix} \qquad \sigma_{\hat{\mathbf{v}},1} = \frac{1}{\sqrt{1+k^2}} ; \ \ \sigma_{\hat{\mathbf{v}},2} = \frac{k}{\sqrt{1+k^2}} \tag{14}$$

Let us look at the reliability levels of the observations when coefficient k increases as compared to when k = 1 (see Fig. 3):

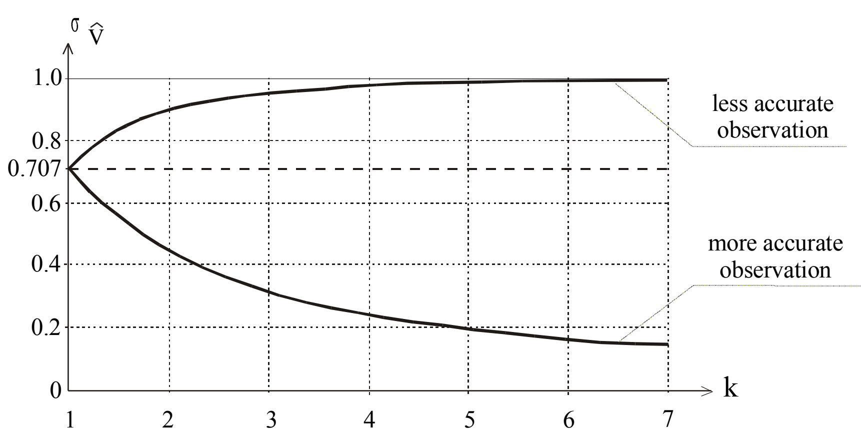

Fig. 3. Differences in reliability levels due to disproportion in the accuracy of observations.

- **an input disproportion in the standard deviations of the observations results in an identical disproportion of the reliability levels;** $\frac{\sigma_{\hat{v},2}}{\sigma_{\hat{v},1}} = \frac{\sigma_2}{\sigma_1} = k$
- **a less accurate observation gains in accuracy more than does a more accurate observation, and it also becomes more reliable at the expense of the reliability level of a more accurate observation (as we have** $\sigma^2_{\hat{v},1} + \sigma^2_{\hat{v},2} = 1$**).**

It can also be demonstrated that $\lim_{k \to \infty} \sigma_{\hat{v},1} = 0$ **and** $\lim_{k \to \infty} \sigma_{\hat{v},2} = 1$**, which means that an observation with unlimitedly high inaccuracy can be fully controlled with an error-free observation. In fact, it is a system that does not meet the internal reliability requirement (Prószyński, Kwaśniak, 2002), but it provides a certain idea of what (negative) effects the observations of considerably varied accuracy levels can bring about when combined.**

Now let us consider a system of type (13), but with n observations, using observation weights to facilitate the notation of the final expressions, i.e.

$$x = l_i + v_i \qquad \sigma_i = \frac{\sigma_o}{\sqrt{p_i}} \qquad i = 1, 2, ..., n \tag{15}$$

Now we will specify the form of the i-th diagonal term for matrices Q and R

$$Q_{ii} = \frac{p_i}{\sum\limits_1^n p_s} \qquad R_{ii} = \frac{\sum\limits_{s \neq i} p_s}{\sum\limits_1^n p_s} \tag{16}$$

For such observations l_i, l_j **that** $p_i < p_j$, **we will have** $Q_{ii} < Q_{jj}$ **and** $R_{ii} > R_{jj}$**. We will see that the reliability level of an observation depends on the relative sum of the weights of the remaining observations.**

The impact of different observation accuracies in system (15) (for n = 3) upon the

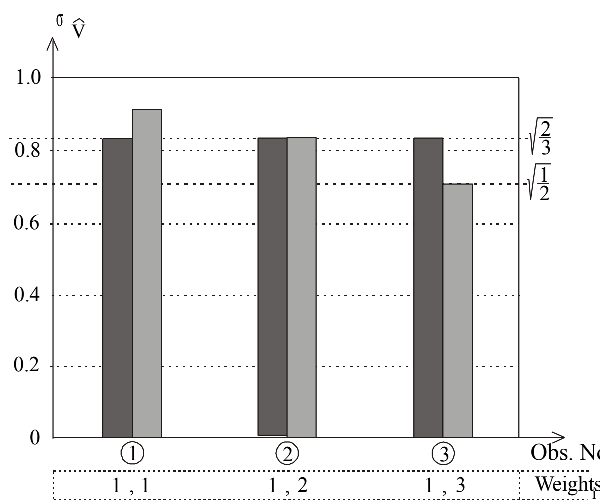

Fig. 4. Changes in reliability levels due to the differentiation of observation weights (the system with three observations).

distribution of internal reliability coefficients is illustrated in figure 4. If the accuracy levels of observations are identical, i.e. $p_1 = p_2 = p_3 = 1$, we have $\sigma_{\hat{V},i} = \sqrt{\frac{2}{3}}$ (i =1, 2, 3), so such a system meets the internal reliability requirement, even with a certain margin. If the accuracy levels differ, however, e.g. $p_1 = 1$, $p_2 = 2$, $p_3 = 3$, we will have $\sigma_{\hat{V},1} = \sqrt{\frac{5}{6}}$, $\sigma_{\hat{V},2} = \sqrt{\frac{2}{3}}$, $\sigma_{\hat{V},3} = \sqrt{\frac{1}{2}}$, which means that the most accurate observation (i.e. l_3) does not meet that requirement any more.

From (16) we will derive

$$\frac{\partial Q_{ii}}{\partial p_i} = \frac{R_{ii}}{\sum_1^n p_s} \qquad\qquad \frac{\partial R_{ii}}{\partial p_i} = -\frac{R_{ii}}{\sum_1^n p_s} \tag{17}$$

which means that the degree of gain in accuracy and the reliability coefficient of ith observation respond to any change in the weight of that observation in proportion to its initial reliability level, and any weight increase results in a higher accuracy, that is, a lower reliability level. The opposition of these effects arises directly from general dependence (10).

In systems that are structurally more complex, which are the subject of displacement monitoring, the features described above occur as well, but not that transparently.

Now let us consider how the correctness of pre-estimated accuracy is significant to non-measurement information when the parameters of a model are to be estimated. Estimating the accuracy of such information satisfactorily may present much difficulty. It is clear that of paramount importance to the correctness of the estimated model parameters are the realistic estimations of the *a priori* accuracy (σ_i) for each data incorporated into such a model.

They should be such that the values of relation $\frac{\varepsilon_i}{\sigma_i}$, where ε_i – unknown true error of the ith data – can be a sample taken from distribution N(0.1). Since we can encounter a high uncertainty when estimating σ for non-measurement information, let us consider the effects of understating and overstating this value:

- $\hat{\sigma} \ll \sigma$ - **an overstatement of the pre-estimated accuracy may cause an effect of gross error in this data, and consequently, as discussed above, a weakening of its controllability by the remaining information;**
- $\hat{\sigma} \gg \sigma$ - **an understatement of the pre-estimated accuracy may weaken the impact of such data upon the solution of a system, and at the same time increase its controllability by the remaining information.**

This implies that if non-measurement information is introduced into a system, it is safer to use, at least at the beginning, an understatement of the pre-estimated accuracy.

6. FINAL CONCLUSIONS

The following general conclusions can be drawn from the research and considerations presented in this study:

- analysis of the reliability features of the models used to integrate measurement data with varied accuracy levels confirms the suitability of the two-step measurement data processing concept (Papo, Perelmuter 1993) for physical models M_W. Pursuant to this concept, measurement results in geometric model M_g should be processed first, as the model is highly reliable and allows for correct estimates of the actual accuracy of the information it integrates;
- with data integrated in a kinematic model, where the point motion model (the motion model is either preset or determined) is employed, the geodetic system for monitoring of displacements can remain autonomous. It is legitimate except that a competent specialist must determine the way of discretising the structure under review, an observation timetable, and the accuracy requirements.
- data integration which requires knowledge of the deformation model of a structure causes that the geodetic system for monitoring of displacements cannot be autonomous, but it can only remain a constituent element (module) of a specialised system of structural safety control.

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