THE SUITABILITY OF ANGLE MEASUREMENTS MADE WITH STEEP TARGETS FOR DETERMINING SPATIAL POSITIONS OF POINTS

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1. INTRODUCTION

The variety of measuring tasks performed with theodolites or electronic tachimeters is the reason why in some cases the working conditions of an instrument are quite different from those prevailing in typical geodesic measurements.

Sometimes, for instance, it may be necessary to measure near zenithal directions in relation to the points above the theodolite position. A situation like this may occur, e.g. during the measurement of roof structure deformations. With such directions the influence of both various instrumental errors and the main axis deflection of a theodolite increases considerably so it would be reasonable to analyze this influence and find out if such targets can be used for the tasks involving the positioning of points inaccessible to direct measurements.

2. ANALYSIS OF INSTRUMENTAL ERRORS INFLUENCE ON ANGLE MEASUREMENTS WITH A THEODOLITE

The basic construction of a theodolite is generally well-known and has been described in numerous textbooks, that is why I will confine myself to the elements which are essential for the present discussion.

2.1. Deflection of the main theodolite axis

As everybody knows, the main axis of a theodolite is brought to its vertical position following the indications of an alidade level. The accuracy of this level is usually several dozen ^{cc} (e.g. for a seconds theodolite it is c. 60^{cc}). As a result, the main axis of a theodolite will be deflected from plumb, which, in return, will influence both the measurement of the horizontal and the vertical angle.

In order to determine the influence of a theodolite vertical axis deflection on the circle readout, the total deflection angle ϑ of the theodolite main axis is divided into two components. One of the components lies on the collimation plane (ϑ_l) and the other (ϑ_c) is perpendicular in relation to the former. The influence of the main axis deflection on theodolite circle readout is expressed by the following equations:

$$\Delta Hz = \vartheta_c \operatorname{ctg} V + \frac{1}{2}\vartheta_l \vartheta_c \left(1 + 2\operatorname{ctg}^2 V\right) \tag{1}$$

$$\Delta V = \vartheta_l - \frac{1}{2}\vartheta_c^2 \operatorname{ctg} V \tag{2}$$

As the above equations indicate, the horizontal circle readout is mainly influenced by the component transverse to the observed direction ϑ_c, whereas the vertical circle readout is, first of all, influenced by the lengthwise component ϑ_l. If we assume that the angle ϑ is a small one, then it is possible to simplify somewhat the above relationships.

$$\Delta Hz = \vartheta_c \operatorname{ctg} V \tag{3}$$

$$\Delta V = \vartheta_l \tag{4}$$

Equations (3) and (4) are generally available in the literature on the subject. As they are approximations, their application is connected with certain errors resulting from their simplification. Approximation errors are functions of deflection constitutive values as well as the zenithal angle. On the basis of equation (1) and (2), the values of these errors can expressed as follows;

$$\delta Hz = -\frac{1}{2}\vartheta_l \vartheta_c \left(1 + 2\operatorname{ctg}^2 V\right) \tag{5}$$

$$\delta V = \frac{1}{2}\vartheta_c^2 \operatorname{ctg} V \tag{6}$$

Permissible deflection values of the main axis of a theodolite dependent on the zenithal angle and permissible correction errors of the Hz readout are shown in the table below.

Table 1. Maximum deflection values for permissible Hz circle correction erros.

δHz\V	50	25	10	5	2	1	0.50	0.20	0.10
2^{cc}	9^c21^{cc}	4^c49^{cc}	1^c78^{cc}	89^{cc}	35^{cc}	18^{cc}	9^{cc}	3.5^{cc}	1.8^{cc}
1^{cc}	6^c51^{cc}	3^c17^{cc}	1^c25^{cc}	63^{cc}	25^{cc}	12^{cc}	6.3^{cc}	2.5^{cc}	1.3^{cc}
0.5^{cc}	4^c61^{cc}	2^c24^{cc}	89^{cc}	44^{cc}	18^{cc}	9^{cc}	4.4^{cc}	1.8^{cc}	0.9^{cc}
0.1^{cc}	2^c06^{cc}	1^c00^{cc}	40^{cc}	20^{cc}	8^{cc}	4.0^{cc}	2.0^{cc}	0.8^{cc}	0.4^{cc}

As for zenithal angle corrections, the appropriate values are shown in the following table:

Table 2. Maximum deflection values for permissible V circle correction erros.

δV\V	50	25	10	5	2	1	0.50	0.20	0.10
2^{cc}	15^c96^{cc}	10^c27^{cc}	6^c35^{cc}	4^c48^{cc}	2^c83^{cc}	2^c00^{cc}	1^c41^{cc}	89^{cc}	63^{cc}
1^{cc}	11^c28^{cc}	7^c26^{cc}	4^c49^{cc}	3^c17^{cc}	2^c00^{cc}	1^c41^{cc}	1^c00^{cc}	63^{cc}	45^{cc}
0.5^{cc}	7^c98^{cc}	5^c13^{cc}	3^c17^{cc}	2^c24^{cc}	1^c41^{cc}	1^c00^{cc}	71^{cc}	45^{cc}	32^{cc}
0.1^{cc}	3^c57^{cc}	2^c30^{cc}	1^c42^{cc}	1^c00^{cc}	63^{cc}	45^{cc}	32^{cc}	20^{cc}	14^{cc}

It can be easily observed that approximation affects, to a large extent, the size of ΔHz obtained from equation (3).

In the tables above an area was shaded, for which the assumed accuracy of deflection influence could be determined with an alidade accuracy level of 60^{cc}. For smaller zenithal angles (the unshaded area) strict equations should be used.

In modern digital theodolites used in industrial measurement systems to compensate for the influence of the main axis deflection on the graded circle readout, automatic systems are applied. Their main part are two-way electronic levels defining both constitutive deflections. On the basis of their readout, the controlling theodolite processor introduces appropriate amendments not only to the V vertical component but also to the Hz horizontal. As a result, it was possible to perform precise observations also with steep targets.

Current compensation of the theodolite main axis is especially important in the case of position changes of a theodolite connected with , e.g. the tripod subsiding.

2.2. Other instrumental errors affecting the measurement of angles with a theodolite

Apart from the main axis deflection, the readout of a theodolite is also affected by the so-called instrumental errors resulting from the difference between the geometric definition of individual theodolite elements and their real mechanical-optical realization.

In the context of the present discussion, the closest attention should be devoted to those instrumental errors, the influence of which depends on the zenithal angle. They are mainly of the collimation and inclination kind.

The influence of collimation error on a horizontal error measurement is expressed by the equation:

$$\sin \Delta Hz_k = \frac{\sin k}{\sin V} \approx \frac{k}{\sin V} \tag{7}$$

Where k is the angle value of collimation error.

As a matter of fact, the average of two measurements of a given direction in two telescope positions is free from collimation error, but there are situations when the measurement in two telescope positions is not possible.

The influence of inclination error on the readout of a horizontal circle is described by the following relationship:

$$\sin Hz_i = \mathrm{tg}\, i \cdot \mathrm{ctg} V \tag{8}$$

where i is the angular value of inclination error.

In the case of modern theodolites, this error is practically non-existent, however with steep targets its influence can be noticeable.

Collimation error is overlapped with a change in the position of the target axis coinciding with a focusing change . It results from the non-rectolinear movement of the inner focusing lens. According to Płatek 1992, for a T3000 theodolite, with a focusing change of the 0.6 -∞ range , a change in the position of a target axis does not exceed 1.5^{cc} (0.5”).

In the construction of digital theodolites rigorous adherence to geometrical conditions ceased to be essential as most of instrumental errors can be numerically compensated. It also concerns such errors as collimation, inclination or that of vertical circle index. Determining the values of errors alone is carried out as a standard measurement procedure. The possibility of determining the above mentioned errors and accounting for their influence on the graded circle readout is quite essential, especially as mentioned earlier, sometimes it is not possible to measure the direction to the observed points in two telescope positions.

2.3. The influence of instrumental errors on the measurement of a horizontal direction

For near zenithal directions the horizontal components of spatial directions will be, to a large extent, influenced by collimation, inclination errors, as well as those of vertical theodolite axis deflection from plumb.

With precise electronic theodolites it is possible to correct digitally the influence of collimation and inclination, although it is essential to have angular values of each of these errors determined by means of a special measurement procedure. This determination, as every other measurement, is burdened with a certain error, as a result of which we cannot ignore their residual values. Also the influence compensation system for the theodolite main axis deflection from plumb has its high, however limited, accuracy.

For near zenithal directions the value of coefficients in the equations determining the influence of collimation, inclination and deflection from plumb errors on the Hz readout grow endlessly, as a result of which this influence even for small values of the aforementioned errors will be quite essential. To illustrate the issue, let us analyze the influence of the above mentioned errors on the Hz readout for various V values. Let us assume the following values of instrumental errors:

- residual collimation error 3^{cc},
- residual inclination error 3^{cc},
- error of determining the component of a theodolite main axis deflection 1^{cc}.

The results of the analysis are presented in the table below. Subsequent lines of this table represent; ΔHz_k collimation influence, that of ΔHz_i, standing axis deflection ΔHz_ϑ, a well as the combined, average influence of the three above mentioned errors specified on the basis of the following equation:

$$\Delta Hz = \sqrt{\Delta Hz_k^2 + \Delta Hz_i^2 + \Delta Hz_\vartheta^2} \tag{9}$$

Table 3. Influence of instrumental errors on Hz circle readout.

V [^{g}] Wpływ [cc]	100	50	20	10	5	2	1	0.50	0.20	0.10	0.05	0.01
ΔHz_k	3.0	4.2	9.7	19.2	38.2	95.5	1 91.0	3 82.0	9 54.9	19 09.9	38 19.7	1 91 01.5
ΔHz_i	0.0	3.0	9.2	18.9	38.1	95.5	1 91.0	3 82.0	9 54.9	19 09.9	38 19.7	1 91 01.5
ΔHz_ϑ	0.0	1.0	3.1	6.3	12.7	31.9	64.1	1 28.9	3 28.4	6 77.1	14 35.4	1 04 19.0
ΔHz	3.0	5.3	13.7	27.7	55.5	1 38.8	2 77.6	5 55.4	13 89.8	27 84.5	55 89.4	2 89 53.2

As the above table shows, despite low values of collimation and inclination errors, with a precise compensation system for transverse deflection influence, the influence of these factors on the Hz readout for near zenithal directions is significant, and for $V<2.5^c$ it exceeds 1^g. It is worth noting that the combined influence of instrumental errors on the Hz component is, approximately, adversely proportional to the zenithal angle V.

As the Hz component for near zenithal directions is charged with a significant influence of instrumental errors, it is not used in many measuring methods, e.g. in the area of astronomical geodesy.

2.4. Influence of instrumental errors on the measurement of a vertical angle

Among instrumental errors affecting the measurement of a vertical angle one can name:

- index error of a vertical circle,
- position shift of a target axis with a focusing change,
- influence of a theodolite deflection from plumb,
- unstable vertical axis.

Similarly as in the case of the Hz component, there is no need to analyze the last two influences separately. Thus the vertical angle will be affected by index error of a vertical circle and deflection from plumb error, but in the case of the vertical error this influence is constant and does not depend on the value of this angle. The combined, average influence of instrumental errors on the vertical angle V is thus expressed by the following equation:

$$\Delta V = \sqrt{\Delta V_0^2 + \Delta V_\vartheta^2} \tag{10}$$

Where ΔV_0 is the influence of a vertical circle index.

2.5. Influence of instrumental errors on accurate location of points

In the case of measurements defining the spatial position of observed points, a simple accuracy characteristics expressed by the mean error of an Hz component does not provide an answer to the question whether accuracy characteristics for a given theodolite involving angle measurements meets accuracy requirements of a performed task. To this aim it would be more appropriate to analyze the figure formed on the surface of a certain conventional sphere of the centre Q of a theodolite (the intersection of the main axis with the horizontal standing axis of a telescope) by the vacillation band, corresponding to the influence of analyzed instrumental errors on measured components of a spatial direction. The vacillation band for the horizontal direction will be the strip contained between two "meridians" corresponding to the values Hz_0-ΔHz and Hz_0+ΔHz . Its width near point P (Hz_0 , V_0) will amount:

$$a = 2 * r * \Delta Hz \sin V_0 \tag{11}$$

where r is the sphere radius.

The band for the vertical angle will assume the shape of a ring defined by two "parallels" corresponding to the values of Vz_0-ΔVz and Vz_0 +ΔVz . The band width will be expressed by the equation:

$$b = 2 * r * \Delta V \tag{12}$$

It will be constant as the influence of instrumental errors charging the vertical angle does nor depend on this circle. The intersection of two bands forms an error figure in the shape of a curve-sided trapezium (Fig.1), the two sides of which, corresponding to Hz_0-ΔHz and Hz_0+ΔHz, are big circles, whereas the side corresponding to Vz_0-ΔVz and Vz_0+ΔVz are small circles.

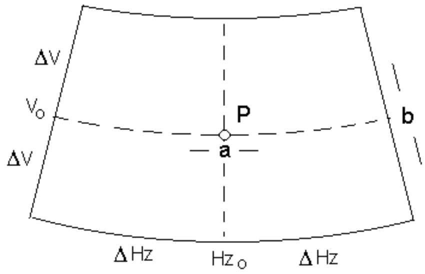

Fig. 1.

To carry out an appropriate digital analysis, let us assume, additionally, the following numerical values :

- residual index error of vertical circle error 3^{cc},
- sphere radius r = 100m.

On the assumptions that that the value ΔV is constant and equals 3.3^{cc}, the corresponding value set from the relationship (12) b = 0.993 mm. The combination of values obtained from the dependence (11) is represented by the table and the ensuing diagram below.

Table 4. Horizintal dimensio of curve-sided trapezium

V [g]	100	50	20	10	5	2	1	0.50	0.20	0.10	0.05	0.01
a [mm]	0.942	1.175	1.335	1.361	1.367	1.369	1.369	1.369	1.369	1.369	1.370	1.370

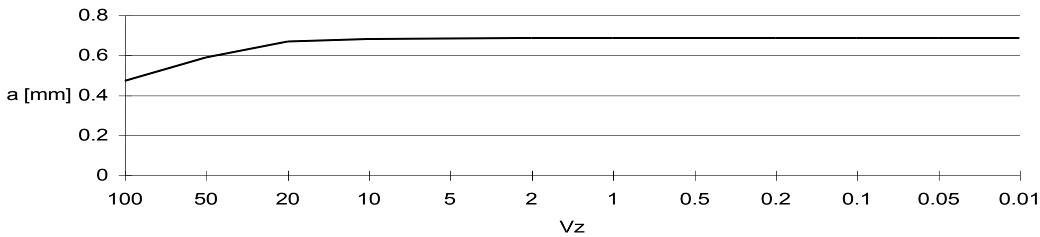

Fig. 2.

As the above analysis shows, a rapidly growing Hz component in the zenithal area does not influence significantly the defining accuracy of a spatial direction represented by trapezium error. There is a slight change in the value of the error figure, evenly spread throughout the entire range of vertical angles analyzed. This conclusion is also right for the value V_0 = 0, despite the fact that the Hz component is indeterminate then. The error trapezium degenerates then into a circle, the centre of which is the zenithal point and the radius - b/2.

Thus the conducted analysis justifies the conclusion that there is no question of accuracy loss for near zenithal directions so in order to define the spatial position of points one can use a full range of telescope positions, at which observations are possible. Moreover, it should be noted that at the level of instrumental influence, the components of spatial directions for various points are mutually dependent values (correlated). It stems from the fact that individual directions are affected by the same instrumental errors.

3. TEST MEASUREMENTS

The measurements conducted by A.Kaleciński and S.Wykowski (2005) were aimed, among others, at verifying the accuracy of the results of the analysis presented above. These measurements were carried out with a two-theodolite measuring system using the angle intersection method. The system consisted of two digital theodolites Wild T2002 and a computer (laptop) with a piece of software enabling to collect measurement results and supervise individual measuring activities. Due to target limitations of up to 79^g it was impossible to check the proper functioning of the system for the steepest targets. As a result of model section measurements, no noticeable accuracy decrease was observed on a precise leveling staff placed above the instruments, while measuring the length of a section with an increase in the target axes inclination. In all the tests this accuracy was very high and oscillated at the 0.04 ÷0.08 mm level. (deviation for the standard length).

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Platek A.1992: ***Geodezyjne dalmierze elektromagnetyczne i tachnimetry elektroniczne*** **cz. I i II.PPWK (*Geodesic elektro-mangetic range-finders and tacheometers* , Part I and II).**

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