# ABOUT HELMERT'S TRANSFORMATION OF PROCESSING FROM ANALOG TO ELECTRONIC MAPS

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## INTRODUCTION

**Problem of accuracy of a transformation has important meaning at the stage of converting analog maps into electronics maps or hybrid and project programs [2, 3]. Scanned maps have a many errors caused by paper deformation and scanner errors. These distortion have linear character and unlinear. It is possible easy to delete linear distortion, by means of turn and scaling map. Non linear distortion can be only eliminated by using advanced transformation. In practice majority of software suggests calibration maps raster by meats of Helmert's transformation or affine transformation [3]. In this work is presented dependence between standard deviation inscribed preliminary document and deformation of processed analog map it can be helpful for fixing ultimate calibration model.**

## 1. FOUNDATION OF GENERAL HELMERT'S TRANSFORMATIONS METHOD

**Coordinates transformation relies on match of the primary system, of scanned analog map on to coordinates of a map graticule (fig. 1). As a result this transformation will be changed following rectangular coordinates parameters:**

- **translation of a primary system about vector $\vec{u}$ is contents between raster point of primary system (*O*) and final point of secondary system ($O^{w}$), which should be placed after ended calibration.**
- **angle of swing $\gamma$**
- **turn rotation about $\gamma$**
- **change of scale about ratio *s*,**

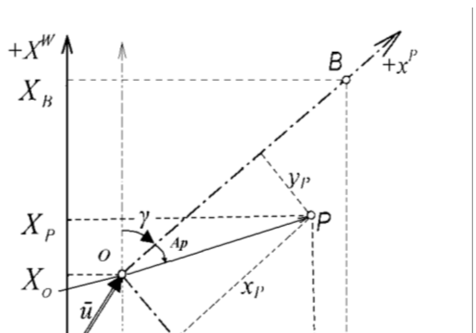


**Fig. 1. Transformation of primary system on secondary system.**

***O*** - **central point of transformation vector of primary system**
$O_w$ - **central point of transformation vector of secondary system**
$X_P$; $Y_P$ – **rectangular coordinate of a point in secondary system**
$x_P$; $y_P$ – **rectangular coordinate of a point in primary system**
$A_P$ – **azimuth of vector transformation in primary system**
$\gamma$ – **angle of swing primary system respect secondary system.**

**If ratio of scale of analog map is equal in all directions, then zero is equal theoretical transformation mean error. Deformed maps about equal deformation coefficient (p=q) should be ideally match in to a map graticule also.**
**From above-mentioned foundation formulas if follows that:**

- **points in secondary system do not undergo angle deformation, they experiencing conformal projection only with translation and change of scale about o constant coefficient *s***
- **form of geometric image has been maintained, after transformation.**

**Deformed maps in unsimilar manner, have different ratio deformation (p≠q) and scale therefore, it is not possible to inscribe their ideally system to secondary by using of Helmert's transformation (fig. 2).**

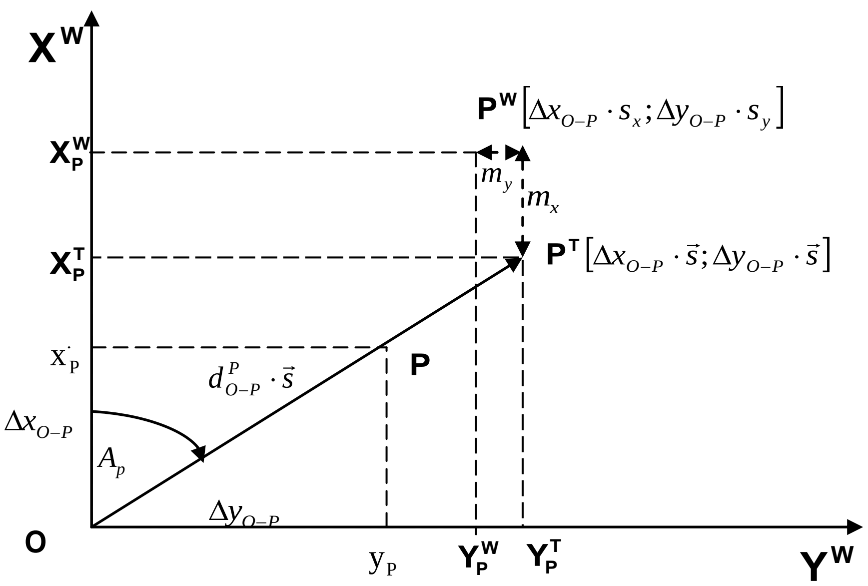


**Fig. 2. Site of point P after transformation.**

$P^T$ **– site of point after transformation (p≠q)**
$P^W$ **– site of point in secondary system**

**We calculate ratio of scale $\vec{s}$ in optional direction of maps system [1]:**

$$s_x = \frac{1}{q} = \frac{\Delta X_n}{\Delta x_n}, s_y = \frac{1}{p} = \frac{\Delta Y_n}{\Delta y_n}$$

$$\vec{s} = s_x \cdot \cos^2 A_p + s_y \cdot \sin^2 A_p$$

$$tgA_p = \frac{\Delta y_{O-P}}{\Delta x_{O-P}}$$

**$s_x$ – ratio of scale along axis X**
**$s_y$ – ratio of scale along axis Y**
**p – ratio of deformation in direction of axis Y**
**q – ratio of deformation in direction of axis X**
**$A_p$ – azimuth vector ($O$ - $P_n$) transformation in optional direction**

**Mean errors of transformed points we calculate from following dependences (fig. 2):**

$$m_x^n = (\vec{s} - s_x) \cdot \Delta x_{O-n} \tag{1}$$

$$m_y^n = (\vec{s} - s_y) \cdot \Delta y_{O-n} \tag{2}$$

$$m_x = \sqrt{\frac{\sum (m_x^n)^2}{n}} \tag{3}$$

$$m_y = \sqrt{\frac{\sum (m_y^n)^2}{n}} \tag{4}$$

$$m_p = \sqrt{(m_x)^2 + (m_y)^2} \tag{5}$$

$\Delta x_{O-n}$ – **increase of coordinate direction of axis X, between central point *O* of primary system and raster point**
$\Delta y_{O-n}$ – **increase of coordinate direction of axis Y, between central point *O* of primary system and raster point**
$m_x^n$ – **transformation error in direction of axis X for point „*n*"**
$m_y^n$ – **transformation error in direction of axis Y for point „*n*"**
$m_p$ – **mean error of points after transformation**

**We can calculate ratio of scale in optional direction of vector of transformation from formulas after changing (1), (2):**

$$\vec{s} = \frac{\Delta X_{O-n} + m_x^n}{\Delta x_{O-n}} \tag{6}$$

**or**

$$\vec{s} = \frac{\Delta Y_{O-n} + m_y^n}{\Delta y_{O-n}} \tag{7}$$

**From above figure and gave formulas it follows, that theoretical mean error of point after transformation is smallest in central area ( $\Delta x$ = 0) and biggest for extreme points of processed raster.**

## 2. ANALYSIS OF ACCURACY OF RASTER IMAGE TRANSFORMATION HELMERT'S METHOD

**Raster images obtained in result of scanning of analog map about different ratio of deformation with using of flat scanner:**

- **Scanner „ Mustec SP" - A3, about resolution 1200x1200 dpi;**
- **Scanner „HP ScanJet 2400"- A4, about resolution 1200x1200 dpi**
- **Scanner „Microtek 4850" - A4, about resolution 4800x2400 dpi**

**Typical cartographic base on which is drawing analog maps are cartographic foil, cartographic board and drawing paper. By that reason compose subject analysis of map on these materials. A minimum resolution of scanning 300 was assumed, 400 dpi which is completely being enough and doesn't influence the quality of the image of the processed analogue map. Analog maps about weak quality contaminated about indefinite „line", but particularly it which ultimately will be processed scan for monochrome form in resolution 400 – 600 dpi. For transformation was used geodetic software „Micromap"- version 4.4 with module *„Raster"* which inserts image of maps and calibrates Helmert's method or affine transformation [4].**

### 2.1 Juxtaposition of gotten result

**Tab. 1. Analysis of accuracy of transformation about ratio of deformation p=q.**

**RASTER IMAGES FROM SCANNER *HP***

| | **Cartographic board** | | **Cartographic foil** | | **Drawing paper** | |
|---|---|---|---|---|---|---|
| **Ratio of def.** | **1,00** | **1,00** | **1,00** | **1,00** | **0,99** | **0,99** |
| **Error** | $\boldsymbol{m_x}$ | $\boldsymbol{m_y}$ | $\boldsymbol{m_x}$ | $\boldsymbol{m_y}$ | $\boldsymbol{m_x}$ | $\boldsymbol{m_y}$ |
| No. | m | | m | | m | |
| 1 | -0,03 | -0,01 | -0,06 | -0,11 | -0,11 | -0,05 |
| 2 | -0,02 | 0,14 | -0,04 | 0,10 | -0,02 | -0,16 |
| 3 | 0,01 | 0,04 | -0,09 | 0,08 | 0,08 | -0,08 |
| 4 | 0,09 | -0,10 | 0,00 | -0,03 | -0,02 | 0,01 |
| 5 | -0,01 | 0,03 | 0,11 | -0,01 | 0,06 | 0,03 |
| 6 | -0,04 | -0,10 | -0,04 | 0,03 | -0,03 | -0,06 |
| 7 | | | 0,05 | 0,01 | 0,07 | 0,16 |
| 8 | | | 0,08 | -0,10 | -0,03 | 0,05 |
| $m_x$ ; $m_y$ | **0,043** | **0,084** | **0,067** | **0,071** | **0,061** | **0,092** |
| $m_p$ [m] | **0,094** | | **0,092** | | **0,110** | |

**Tab. 2. Analysis of accuracy of transformation about ratio of deformation p≠q.**

**RASTER IMAGES FROM SCANNER *"MICROTEK"***

| | **Drawing paper** | | **Cartographic board** | | **Cartographic foil** | |
|---|---|---|---|---|---|---|
| **Ratio of def.** | **0,96** | **0,99** | **0,99** | **0,99** | **1,00** | **1,00** |
| **Error** | $m_x$ | $m_y$ | $m_x$ | $m_y$ | $m_x$ | $m_y$ |
| No. | [m] | | [m] | | [m] | |
| 1 | -3,28 | 2,07 | 0,07 | -0,02 | -0,11 | -0,06 |
| 2 | -0,49 | 2,18 | 0,01 | -0,03 | -0,02 | 0,06 |
| 3 | -0,45 | -0,02 | -0,11 | -0,01 | 0,08 | -0,08 |
| 4 | 2,54 | 2,21 | -0,04 | 0,03 | -0,02 | 0,01 |
| 5 | 2,66 | 0,02 | -0,01 | -0,02 | 0,06 | 0,03 |
| 6 | 2,63 | -2,12 | 0,06 | 0,03 | -0,03 | -0,06 |
| 7 | -0,40 | -2,15 | -0,02 | 0,10 | 0,07 | 0,05 |
| 8 | -3,21 | -2,19 | 0,04 | -0,01 | -0,03 | 0,06 |
| $m_x$ ; $m_y$ | **2,294** | **1,865** | **0,064** | **0,048** | **0,061** | **0,055** |
| $m_p$ [m] | **2,957** | | **0,080** | | **0,082** | |

**Tab. 3. Analysis of accuracy of transformation about ratio of deformation p=q.**

**RASTER IMAGES FROM SCANNER *"MUSTEC"***

| | **Cartographic board** | | **Drawing paper** | | **Cartographic foil** | |
|---|---|---|---|---|---|---|
| **Ratio of def.** | **q=0,99** | **p=0,99** | **q=1,00** | **p=1,00** | **q=1,00** | **p=1,00** |
| **Error** | $m_x$ | $m_y$ | $m_x$ | $m_y$ | $m_x$ | $m_y$ |
| No. | [m] | | [m] | | [m] | |
| 1 | -0,02 | 0,03 | -0,06 | -0,15 | -0,02 | 0,03 |
| 2 | -0,06 | -0,06 | -0,04 | 0,12 | -0,05 | -0,11 |
| 3 | -0,06 | -0,08 | -0,14 | 0,08 | -0,06 | -0,12 |
| 4 | 0,05 | 0,08 | 0,11 | -0,03 | 0,03 | 0,08 |
| 5 | -0,01 | -0,01 | 0,00 | -0,01 | -0,01 | -0,01 |
| 6 | -0,05 | 0,10 | -0,04 | 0,03 | -0,05 | 0,11 |
| 7 | -0,05 | 0,08 | 0,08 | 0,01 | -0,05 | 0,08 |
| 8 | 0,10 | -0,05 | 0,10 | 0,02 | 0,10 | 0,05 |
| 9 | 0,10 | -0,08 | -0,01 | -0,08 | 0,10 | -0,10 |
| $m_x$ ; $m_y$ | **0,063** | **0,069** | **0,065** | **0,071** | **0,033** | **0,071** |
| $m_p$ [m] | **0,093** | | **0,096** | | **0,079** | |

**For maps about similar deformation were calculated mean errors of points after transformation on the basis data of tables 1, 2, 3.**

**$m_p$= ±0,09 m - cartographic foil**

**$m_p$= ±0,09 m - cartographic board**
**$m_p$= ±0,10 m - drawing paper**

**Form of geometric object was checked and scale after transformation. Form of geometric image has been maintained on all processed maps and scale.**
**According to earliest foundation, not deformation maps and maps about homogeneous shrinkage “inscribed” almost ideally in secondary system with minimal mean errors $m_p$ = ± 0,10 m, on which had influence scanning error, identification of points on raster, drawing of cartographic net. For purpose of comparison of independent observation was calibrated strong deformation raster about shrinkage p = 0,90 and q = 0,80. Resultant coefficient of scale $\vec{s} = 1{,}194$ was calculated for all vectors transformation in the basis of points of cartographic nets of raster image according with fig. 3.**

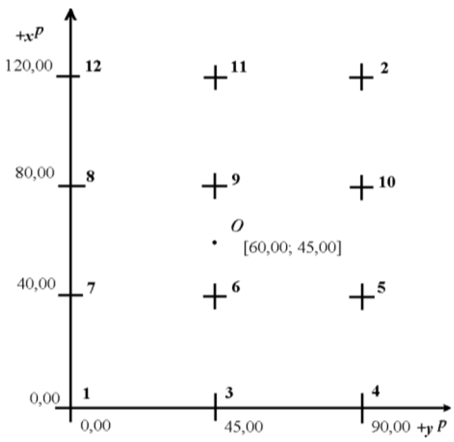


**Fig. 3. Accommodation of points on deformation raster (p≠q).**

**Tab. 4. Analysis of accuracy of transformation about ratio of deformation (p≠q).**

| **Scanner „ Mustec - A3”** | | | **Theoretical accounts** | | | **Difference error trans.** | |
|---|---|---|---|---|---|---|---|
| **Drawing paper** | | | | | | | |
| **Ratio of def.** | **0,80** | **0,90** | Ratio | **0,80** | **0,90** | | |
| **Error** | $m_x^n$ | $m_y^n$ | Error | $m_x^n$-t. | $m_y^n$-t | $\Delta m_x$ | $\Delta m_y$ |
| No. | [m] | | No. | [m] | | [m] | [m] |
| 1 | 3,30 | -3,69 | 1 | 3,36 | -3,73 | 0,06 | -0,04 |
| 2 | -3,31 | 3,65 | 2 | -3,36 | 3,73 | -0,05 | 0,08 |
| 3 | 3,24 | -0,01 | 3 | 3,36 | 0,00 | 0,12 | 0,01 |
| 4 | 3,30 | 3,67 | 4 | 3,36 | 3,73 | 0,06 | 0,06 |
| 5 | 1,10 | 3,60 | 5 | 1,12 | 3,73 | 0,02 | 0,13 |
| 6 | 1,03 | -0,01 | 6 | 1,12 | 0,00 | 0,09 | 0,01 |
| 7 | 1,07 | -3,60 | 7 | 1,12 | -3,73 | 0,05 | -0,13 |
| 8 | -1,07 | -3,63 | 8 | -1,12 | -3,73 | -0,05 | -0,10 |
| 9 | -1,11 | 0,03 | 9 | -1,12 | 0,00 | -0,01 | -0,03 |
| 10 | -1,00 | 3,67 | 10 | -1,12 | 3,73 | -0,12 | 0,06 |
| 11 | -3,30 | -0,02 | 11 | -3,36 | 0,00 | -0,06 | 0,02 |
| 12 | -3,26 | -3,67 | 12 | -3,36 | -3,73 | -0,10 | -0,06 |
| **$m_x$; $m_y$** | **2,021** | **2,574** | **$m_{x-t}$; $m_{y-t}$** | **2,504** | **3,046** | **0,074** | **0,073** |
| **$m_p$ [m]** | **3,273** | | **$m_p$-t** | **3,943** | | **$m_s$ = 0,104** | |

**Theoretical mean errors for individual points of cartographic nets after transformation from formulas (1) and (2).**
**On base of difference $\Delta m_x^n = m_x^n - m_x^n\text{-}t$, $\Delta m_y^n = m_y^n - m_y^n\text{-}t$ error were calculated $m_s = 0{,}104$, on which had influence of scanning error, points identification on raster. It is possible to accept calculated mean error for error of scanning.**

**Tab. 5. Analysis of accuracy of transformation on base of result from publication [ 2].**

| **Theoretical results (1, 2)** | | | **Data from publication [ 2 ]** | | | **Difference error of trans.** | |
|---|---|---|---|---|---|---|---|
| **Ratio of def.** | **0,80** | **0,90** | **Ratio** | **0,80** | **0,90** | | |
| **Error** No. | $m_x^n$-teor. [m] | $m_y^n$-teor. [m] | **Error** No. | $m_x^n$ [m] | $m_y^n$ [m] | $\Delta m_x$ [m] | $\Delta m_y$ [m] |
| 1 | 3,36 | -3,73 | 73 | 3,46 | -3,79 | 0,10 | -0,06 |
| 2 | -3,36 | 3,73 | 24 | -3,41 | 3,87 | -0,05 | 0,14 |
| 3 | 3,36 | 0,00 | 74 | 3,34 | -0,11 | -0,02 | -0,11 |
| 4 | 3,36 | 3,73 | 75 | 3,52 | 3,79 | 0,16 | 0,06 |
| 5 | 1,12 | 3,73 | 58 | 1,18 | 3,88 | 0,06 | 0,15 |
| 6 | 1,12 | 0,00 | 57 | 1,08 | -0,06 | -0,04 | -0,06 |
| 7 | 1,12 | -3,73 | 56 | 1,13 | -3,73 | 0,01 | 0,00 |
| 8 | -1,12 | -3,73 | 39 | -1,20 | -3,76 | -0,08 | -0,03 |
| 9 | -1,12 | 0,00 | 40 | -1,18 | -0,12 | -0,06 | -0,12 |
| 10 | -1,12 | 3,73 | 41 | -1,07 | 3,85 | 0,05 | 0,12 |
| 11 | -3,36 | 0,00 | 23 | -3,47 | -0,07 | -0,11 | -0,07 |
| 12 | -3,36 | -3,73 | 22 | -3,38 | -3,74 | -0,02 | -0,01 |
| $m_x\text{-}t$; $m_y\text{-}t$ | **2,504** | **3,046** | $m_x$; $m_y$ | **2,556** | **3,105** | **0,076** | **0,091** |
| $m_p\text{-}t$ **[m]** | **3,943** | | $m_p$ **[m]** | **4,022** | | $m_s$ **= 0,118** | |

**Raster maps calibrated Helmert's method, which had unequal ratios of deformations, inscribed with enough big error, into cartographic, which value increased proportionally to difference between longitudinal and transverse shrinkage and for distance from center of system to transformed points. Farthest analysis of gotten result, contained in tables 4 and 5 proved that reason big error is caused by non-homogeneous shrinkage maps.**

## 3. **RECAPITULATION**

**At processing of maps and analog drawings for electronic form in software for preliminary calibration of image raster should be suggested method Helmert's, because this method:**

- **behaved assures of form of raster image after transformation**
- **enable inscription of map similarly deformed to secondary system with mean error $m_p = \pm 0{,}10$m**
- **it allows to establish shrinkage maps in optional direction on base of calculated mean errors of points after transformation.**

## 4. CONCLUSIONS

**Raster maps after calibration about mean error ± 0,40 mm of transformed point in scale of map are comparable [5] in respect of accuracy with map manuscript and can be serve with whole certitude for project purposes, analyses of geographic areas and maps update.**

**Raster images about unsimilar shrinkage , before ultimate transformation should have changed scale in optional graphic program based on minimum four fitting points symmetrically located, on raster along axis X and Y.**

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