THE HOPFIELD NEURAL NETWORK IN THE ASPECT OF THE STABILIZATION OF THE DISPLACEMENT PHENOMENON

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1. INTRODUCTION

The Hopfield neural network also called the Hopfield model is a dynamic system dissipative in charakter i.e. the vector moves towards the local minimum of an energy function. The network is used for matching, classifying and reproducing images, for solving optimization problems and a number of other purposes. With the development of electronic technologies, interesting characteristics of these networks have been used in practice to build integrated circuits physically realizing theoretical models. The article suggest using the Hopfield neural network for assessing the stability of points in a measurement-control geodetic network.

2. ASSOCIATIVE MEMORY

The Hopfield neural network has been built on the basis of two stage-neurons of the McCulloch-Pitts type (Hertz A., Krogh R., Palmer R., 1991). Hopfield suggested an effective algorithm of the application of a neural network for creating the so-called associative memory. The task of the associative memory is to remember a particular set of learning standards so that while presenting an unknown standard x the system can generate the answer in the form of one of the previously remembered standards. In the process of learning by a network, which consists in the right choice of weights W_{ij} of particular neurons, attraction areas corresponding to learning standards are created.
Fig. 1 presents a diagram of the Hopfield recurrent associative memory, which operates in the mode of asynchronous status updating i.e. only the status of one neuron is updated in one clock cycle. It results from fig. 1 that input signals are simultaneously network input signals, i.e. $y_i = x_i$. For this reason balance equations of the system can be written as (let us omit the polarization value)

$$x_i^{k+1} = \operatorname{sgn}\left(\sum_{j=1}^{n} W_{ij} x_j^k\right). \quad (1)$$

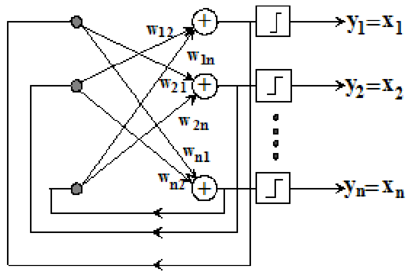

Fig. 1. Diagram of the Hopfield associative memory.

Condition (1) is satisfied when weights are chosen according to the Hebb principle (Hebb D.O., 1949)

$$W_{ij} = \frac{1}{n}\sum_{k=1}^{p} x_i^{(k)} x_j^{(k)} . \qquad (2)$$

One of the most important parameters of the associative memory is its capacity, i. e. ability to effectively remember a particular number of images. The notion of the capacity of the associative memory is connected to the parameter

$$c_i^{(l)} = -x_i^{(l)} \frac{1}{n} \sum_{j=1}^{n} \sum_{\substack{k=1 \\ k \neq l}}^{p} x_i^{(k)} x_j^{(k)} x_j^{(l)} , \qquad (3)$$

called crosstalk. If for the l^{th} learning standard the parametr $c_i^{(l)} < 1$, then in spite of a certain inconsistence of bits the component $x_i^{(l)}$ is stable. Instability appears when the maximum memory capacity is exceeded. Then in the operation of a neuron there is a change of status to the opposite to $x_i^{(l)}$.

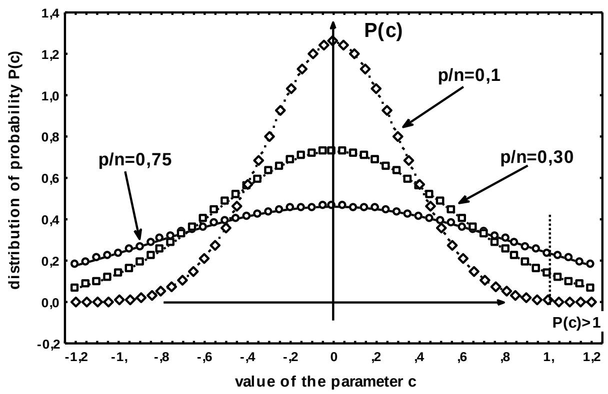

Fig. 2. Distribution of probability of the assumption of the value *c*.

The distribution of the parameter $c_i^{(l)}$ is a binomial distribution, which for large values np approaches the normal distribution

$$P(c) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{p/n}} \exp\left(-\frac{c^2}{2p/n}\right) \qquad (4)$$

with the average value zero and the ariance $\sigma^2 = p/n$ (fig. 2). The value of the probability $\omega = P(c_i^{(l)} > 1)$ increases with the increase of the remembered standards p and the dimension n of the vector x (cf. formula (3)).

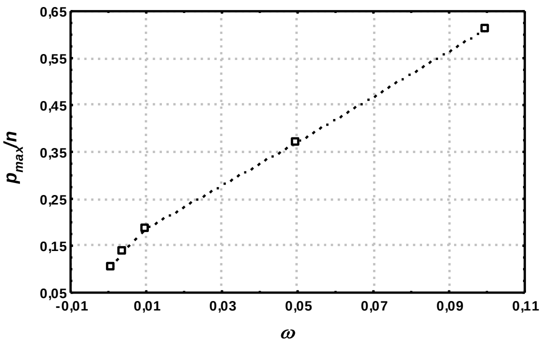

Fig. 3. Diagram of the dependence of the error $\delta = \delta_{\max}$ the number of remembered standards.

Fig. 3 presents the relation $p_{\max}$ on the assumption that $\omega = \omega_{\max}$. It can be seen in the figure that for the error $\omega_{\max} = 10\%$ (10% of bits in the wrong status) the maximum associative memory capacity is about 60% of the number of neurons, from which the associative memory has been created.

3. ASSOCIATIVE MEMORY ENERGY FUNCTION

Hopfield identified the energy function of a two-stage network in the form (Hopfield J., Tank D., 1986)

$$E = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij} y_i y_j \; . \qquad \textbf{(5)}$$

The process of minimization of the energy function (state function) is closely connected to the process of standard recognition. During this process there is a decrease in the value of the energy function $(E_{k-1} < E_k$; k – learning step) called the Lapunov function (Peretto P., 1992), which on the assumption of a symmetrical weight matrix reaches the local minimum at the moment of assigning a standard to one of the attractors. The continuation of the minimization process does not then cause a change in the state of neurons, which means that the energy of the system does not change. The discussion has so for concerned vectors with coordinates corresponding to the position of the vertices of an n-dimensional hypercube $[-1, 1]^n$. In practice an analogue network is more important, in which input signals assume values of a bipolar or unipolar activation function ($f(x) = tgh(\alpha x)$ or $f(x) = 1/(1+\exp(-\alpha x))$). If we designate analogue signals as $y_i = x_i = v_i$, then we will obtain (Gil J., 1995):

$$v_i = f(u_i) = f(\sum_{j=1}^{n} W_{ij} v_j) . \qquad \textbf{(6)}$$

In a specified state the network equation has the form

$$\tau_i \frac{du_i}{dt} = -u_i + \sum_{j=1}^{n} W_{ij} f(u_j), \tag{7}$$

where τ_i denotes the time constant of the adaptation process. In a specified state changes u_i and v_i are zero and the network is balanced. In this case Hopfield defined an energy function in the form

$$E = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij} v_i v_j + \sum_{i=1}^{n}\int_0^{v_i} g^{-1}(v)dv. \tag{8}$$

The energy function during time evolution decreases or remains constant, similarly to internal energy in a magnetic system (Kosiński R. A., 2002, 2004). In the process of adaptation of the input vector the value of the energy function approaches an energetic minimum, in which the network reaches a point attractor. While approaching the point attractor the network can reach a local minimum (a parasite attractor), whose elimination is a difficult task.

4. NETWORK DYNAMISM IN THE VICINITY OF AN ATTRACTOR

In order to solve a system in the time t in the vicinity of an attractor we will adopt the denotation u_i^* - attractor, u_i - actual point of the system operation (working point). Therefore, we will write:

$$u_i = u_i^* + \delta_i, \tag{9}$$

where δ_i - difference close to zero between the value of the working point of the system and the value of the attractor. Thus

$$\frac{du_i}{dt} = \frac{d\delta_i}{dt} \tag{10}$$

and

$$f(u_i^* + \delta_i) = f(u_i^*) + f'(u_i^*)\delta_i. \tag{11}$$

Next, considering the equation (7) we will have (Osowski S., 1996)

$$\tau_i \frac{d\delta_i}{dt} = -\delta_i + \sum_{j=1}^{n} W_{ij} f'(u_j^*)\delta_j + [-u_i^* + \sum_{j=1}^{n} W_{ij} f(u_j^*)]. \tag{12}$$

Bearing in mind that in a specified state the network equation will have the form

$$-u_i + \sum_{j=1}^{n} W_{ij} v_j = 0, \tag{13}$$

the dynamic equation of a linearised network is expressed by the dependence

$$\tau_i \frac{d\delta_i}{dt} = -\delta_i + \sum_{j=1}^{n} W_{ij} f'(u_j^*)\delta_j \, . \qquad \textbf{(14)}$$

The matrix form of the system of equations (14) is expressed as follows:

$$\frac{d\delta}{dt} = -\mathbf{T}^{-1}[\mathbf{1} - \mathbf{GW}]\delta \, , \qquad \textbf{(15)}$$

where: T=*diag*$[\tau_1, \tau_2, ..., \tau_n]$, G=*diag*$[f'(u_1), f'(u_2), ..., f'(u_n)]$, $\delta = [\delta_1, \delta_2, ..., \delta_n]^T$, and

$$\mathbf{W} = \begin{bmatrix} W_{11} & W_{12} & ... & W_{1n} \\ W_{21} & W_{22} & ... & W_{2n} \\ ... & ... & ... & ... \\ W_{n1} & W_{n2} & ... & W_{nn} \end{bmatrix}$$

It results from the form of the equation (15) that the equation of a linearised system is a linear equation. The number of steps n of the time evolution to the solution approaching infinity can be reduced by limiting the precision of changes of the vector δ with respect to the precision of the specified coordinates of the displacement vector.

5. NUMERICAL EXAMPLE

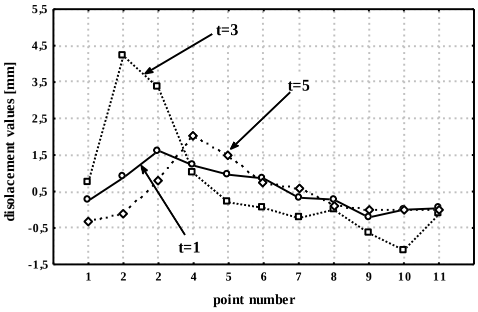

Fig. 4. Displacement values.

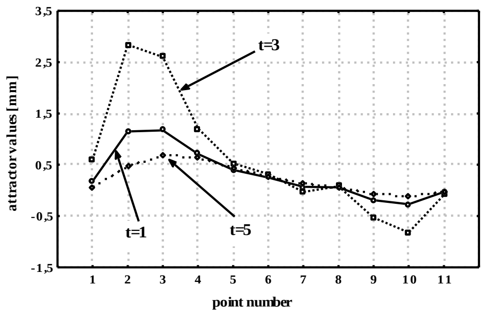

Fig. 5. Attractor values.

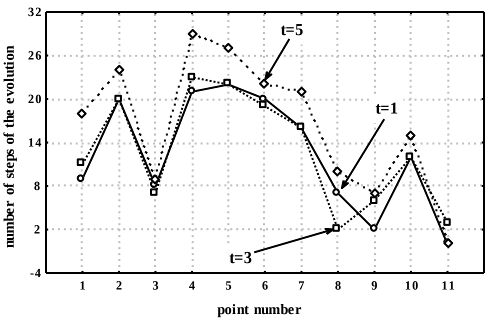

Fig. 6. Number of steps *n* of the time evolution.

The equation of a linearised system will be used for the assessment of the stability of the geodetic matrix point of the measurement-control network, stabilised on a building located on expansive soil. The displacement of points have been determined with regard to the initial measurement, on the basis of three periodical measurements carried out in

two months' time intervals. The state of displacements corresponding to a particular measurement time has been determined on the basis of the minimization of the sum of absolute deviations (Gil J., 1995). The values of attractors and the number of steps of time evolutions as indicators for the assessment of the stability of the phenomenon of motion of particular measurement points have also been determined for a particular measurement time. The results of the numerical simulations have been presented in figures 4, 5 and 6. It results from them that point 11 shows a high level of stability at the time of the research.

CONCLUSIONS

The problem of associative memory and its energy function can be alternative in the approach to the assessment of stability of point of the height matrix on the basis of arbitrarily formulated stability functions (Wolski B., 2006). In general, a measurement-control network set on buildings located on expansive soils is particularly vulnerable to load changes, which cause changes in the direction of the trajectory of the movement of points. The results of the numerical experiments described in the paper with regard to the whole research period confirm the proposition that the displacement of a particular point equal to zero does not prove the stability of the phenomenon of the movement of points. A typical example is point 9, whose displacement in the time t=5 equalled zero with an assigned number of step of time evolution equal to 7. A specified number of steps of time evolutions of points of the measurement-control geodetic network, which represents general information on the stabilization time of the phenomenon of movement of points should be associated with a forecast for displacement on condition that the foundation-soil system is stabilized.

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