FORECASTING OF VERTICAL DISPLACEMENTS BASED ON A TIME SERIES

Jan Gadomski, Maria Gadomska, Maciej K. Kumor
Faculty of Civil and Environmental Engineering
University of Technology and Life Sciences in Bydgoszcz

1. INTRODUCTION

Forecasting of vertical displacements is one of the main purposes of monitoring of a geotechnical structure. The forecasting is an integral part of a complex metrological monitoring. It is because it answers the question about a behavior of the given structure [9]. The problem of a statistical estimation of a displacement (settlement) in a period of time, forecasting or possibility of forecasting basing on several models, are presented in various papers [3, 5, 7, 8, 9, 10], but the forecasts are mainly based on regressive models. Aczel A. D. draws attention to the fact that models of regression have several applications. One of them is to show the relation between two variables. They are also applied to forecast, i.e. to estimate a value of a dependent variable basing on a regression equation. Aczel stresses it is important that the provided values do not exceed limits of values used in the procedure of estimation of the regression equation [1]. There are no such restrictions considering a time series, where the aim of the analysis is to forecast some future values of a time series' variable.

The time series, called also a chronological or dynamic series, is a set of successive observations. In case of a time series it is assumed that analyses should be done on the strength of at least 50 – 100 observations. If it is impossible then any experience or information about the past of the examined effect should be used to obtain a preliminary model. Next, the model should be corrected in accordance with new data [2]. There exist several opinions about the length of a time series.

Methods of time series analysis and forecast use the past values of the forecasted variable. In those models, time and variable's past values represent all influential factors. First of all the following methods are used to diagnose: a movable average value and exponential smoothing, analytic and adaptation models of a growth trend, models of a periodic component, auto regression models and Markov chains. With these methods it is possible to obtain forecasts based on detected past regularities without indicating causes of rise of the regularities [4]. It is well-founded to use the methods if a detected regularity is constant in time (timeless), i.e. if the effect is characterized by high inertia.

The subject literature gives many examples that models of a growth trend are a proper basis to forecast an effect's level in future. There are many advantages of using models of a growth trend [11]:

- **only empirical data related to the forecasted variable is necessary to build a model,**
- **the forecast problem is reduced to a simple extrapolation of the trend function with putting an appropriate value of a time variable in a forecasted period of time,**

- **it is easy to estimate accuracy of the forecast using a forecast accuracy measure ex ante.**

It should be noticed that there are several evaluations of applicability of respective models of a growth trend for forecast purposes. Linear and polynomial trends have a restricted applicability because the functions are bounded, i.e. they do not have horizontal asymptotes.

Linear and polynomial functions give a good compliance of theoretical values of a forecasted variable with its empirical values, but behind that variability the compliance is usually worse [11]. Because of that in the paper linear and polynomial trends are omitted after preliminary analyses.

2. AIM OF THE RESEARCH

The objects of the research are displacements of points marked on structures located in a neighborhood of an earthwork done to build a big trade center.

The realized structure, with the measure of 60x180 m, is founded end to end with a neighboring building, on the depth of 9,0 m and 6,5 m below the foundation of the existing building. Tertiary and quaternary laminations to the depth of at least 20 m are the subsoil. The tertiary clays complex is not homogeneous, as it is laminated with the sand and silt lenses and irrigated (watered).

Monitoring of the geotechnical structure was carried out to establish an impact of the earthwork and foundation works on neighboring structures. Five structures were observed. The range of the monitoring contained:

- **observation of the technical state of buildings (crack, scratch of walls),**
- **surveying of the building construction vibration,**
- **ground water level measurement (piezometer),**
- **geodetic surveying of vertical displacements and walls deviation from a plumb-line.**

Geodetic surveys included 12 points located on a four-floor apartment block (8) and garage-apartment building (4) (picture 1).

In the first period surveying of all points was systematic, made in equal time intervals. During construction of the building some of the points were built over, and others were temporarily inaccessible. Finally, complete systematic observation included only five points.
On the basis of the results obtained through the research, five time series with the length *n*=58 were created. After preliminary analysis of the time series it was claimed that the effect progress in time was polyphasic.

In the paper we present forecasts with the example of two points (1 and 2).

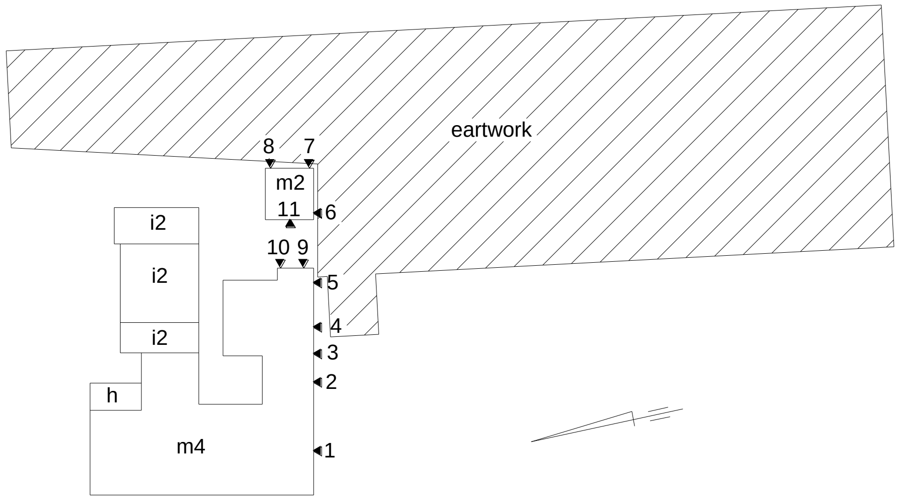

Fig. 1. Draft of measuring points collocation.

3. DETERMINATION OF MODELS OF TREND

Empirical and deductive circumstances should be taken into account while choosing the form of a trend function. If the analytical form of the function is more complicated the deductive circumstances should be superior, with less complicated form of the function – empirical circumstances should prevail. The analysis of an empirical statistic material should confirm relevance of selection of the function form or suggest such a selection [4].

A logistics function *f (t)* was applied to describe displacements [6]:

$$f(t) = y_t = \frac{a}{1+e^{b-ct}} + d$$

where:

- y_t - displacements in moments t,
- t - moments of time ($t = 1, \ldots, n, n+1, \ldots$),
- a, b, c - parameters ($a, b, c \neq 0$),
- d - translation ($d \in < 0, \min_t(y_t) >$)

The function has a horizontal asymptote (in this case it is equal to the sum of parameters *a* and *d*), which determines the so called saturation level. The function has one inflexion point, which separates a period of increasing pace of the effect escalation from a period of decreasing pace until stabilization of the effect is on a stable level.

Many nature effects are characterized by a logistic course, especially those effects whose development is limited by a bounded area.

The logistic function causes many difficulties when it comes to parameters estimation because non-linear relation between variables and the parameters makes it impossible to use the classical least square method. Most of known methods of logistic function's parameters estimation give approximate results and are based on some simplifications.

Deduction based on classical trend models involves a risk of preparing the forecast on the strength of a generally good model, but not current for the last known

observations. It usually causes greater forecast errors. Classical models of a growth trend are constructed with the assumption of constancy of development mechanism of the effect. This assumption was omitted in adaptive models. High flexibility of adaptive models, enabling to include irregular changes in components of time series, permits to build short-term forecasts [4]. One of such models is a model of a creeping trend. The construction of the model is based on time series smoothing for arbitrary fixed constant value of smoothing $k<n$. Parameters of linear trend functions are calculated on the strength of successive parts of the series. For each t, each empirical value y_t is assigned a smoothed (theoretical) value determined with only those functions $f_i(t)$, which were constructed for sequences containing the given value y_t [6]. Average values of all such smoothing constitute the final smoothing.

The main problem of the method is correct determination of a number k of terms of a time series, for which the movable mean is calculated [12]. The number of terms of the movable mean, called a smoothing constant and dependent on the length of the time series, is determined by a researcher. To calculate the optimum constant k an average error ex post of extinct forecasts is used. One of varied initially taken values is accepted as the ultimate. It is the value for which the average error ex post of extinct forecasts is the least. The best results for considered series were received for $k = 7 – 11$.

Errors of extinct forecasts for logistic functions were calculated to compare methods. Errors of extinct forecasts are greater for logistic functions than for a creeping trend (table 1).

**For analyses of time series the computer application PROGNOZY was used [6].
This program doesn't cover the uncertainty of the measurements' results.**

Table 1. Errors of forecasts and errors of trend.

Pt	LOGISTIC FUNCTION			CREEPING TREND		
	Errors of extinct forecasts	Error of trend	Error of forecast	Errors of extinct forecasts	Error of trend	Error of forecast
1	1,14	1,61	1,68	0,47	0,80	0,84
2	0,67	1,42	1,49	0,14	0,97	1,02

**Picture 2 shows time series for points 1 and 2 and creeping and logistic trends.
Forecasts are shown for horizon h = 1, 2, 3, 4, 5, 6, 7, but an error ex ante in the table regards only the horizon h = 7. Errors of extinct (apparent) forecasts were calculated for 7 time periods.**

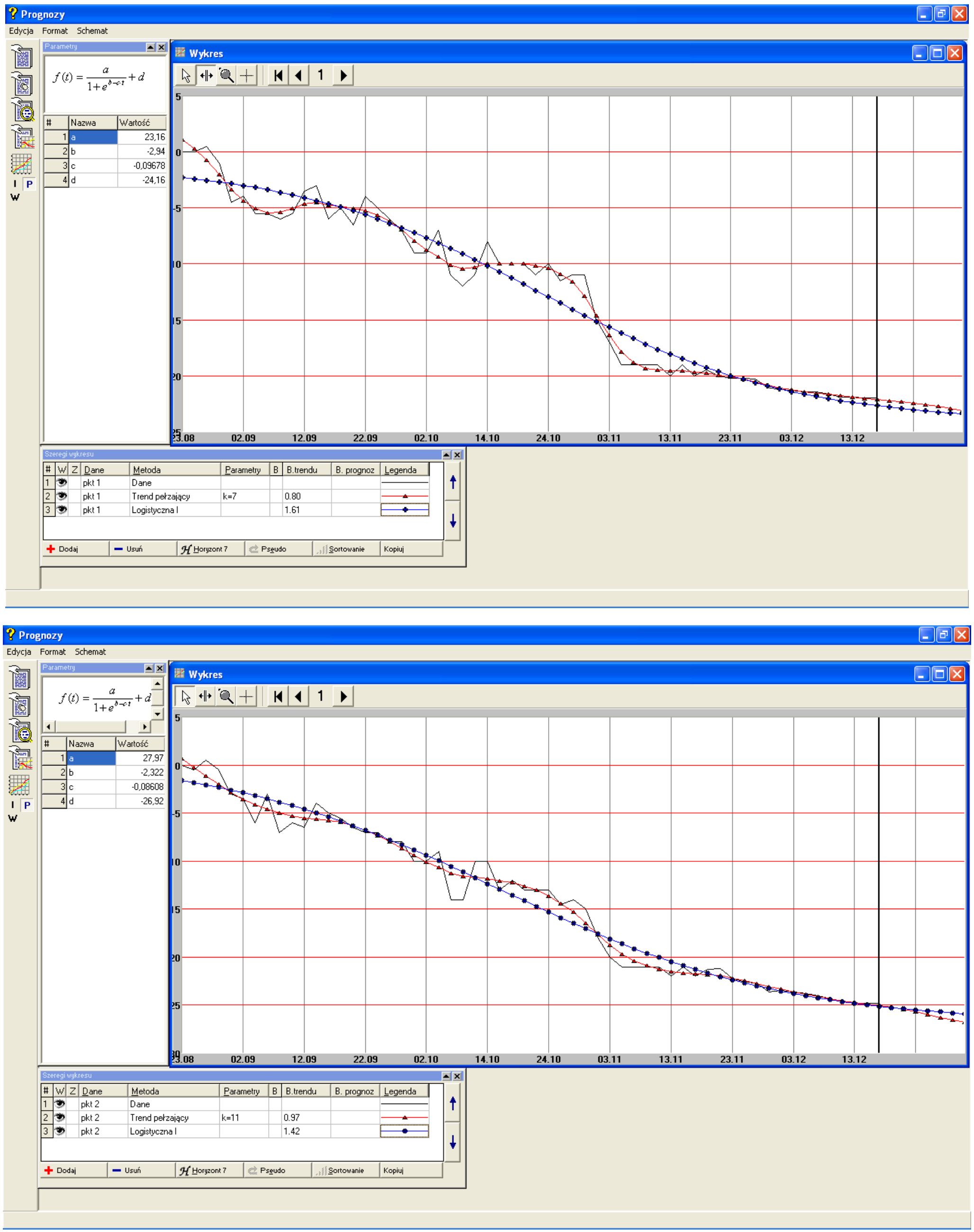

Fig. 2. Time series and trends.

4. SUMMARY AND CONCLUSION

Before research, a graphical analysis of time series is important to carry out because the configuration of empirical points in a proper Cartesian coordinate system enables one to make a decision which class the researched trend function belongs to.

When verifying the model it should be checked whether received values of structural parameters estimation are reasonable. The method of extrapolation of time series can be used to forecast only in case when the mechanism of development of the researched

effect does not change in time considerably or in case if the mechanism of development of the researched effect is not known and we cannot recognize it.

The forecast horizon can be dependent on inertia of researched variables. For variables with a big inertia the forecast horizon can be considerably longer. Longer forecast horizon corresponds with less probability of occurrence of the provided state and simultaneously certainty of the forecast is less.

Another question is a stability of the model. It does not mean that the model will be stable after the period the model was estimated, i.e. it will be stable in future.

Adaptive models are useful in case of lack of stability in the researched period. Forecast procedures based on those models assume that the effect intensification in time can be segmental, i.e. "smooth", only in some intervals of time. Such models are particularly important for short-term forecasts.

Taking one of the following models into account depends on:

- **clear interpretation of model's parameters,**
- **possibility of a simple estimation of the model's parameters,**
- **the level of accuracy that the model describes the effect's development in time.**

It should be noticed that building a „good" model describing the given effect on the base of data from the past not always can be proper in future. To enlarge forecast certainty (especially for short time series) several forecasting methods should be used and their results should be compared.

REFERENCES

Aczel A.D., 2000. Statystyka w zarządzaniu. PWN, Warszawa.

Box G.E.P.,1983. Analiza szeregów czasowych. Prognozowanie i sterowanie. PWN, Warszawa.

Bryś H., Zielina L., 1995. Osiadanie reperów głębinowych w warunkach eksploatacji Kombinatu Hutniczego. Materiały Konferencji Katedr i Zakładów Geodezji, Poznań.

Cieślak M,, 2001, Prognozowanie gospodarcze. Metody i zastosowanie, PWN, Warszawa.

Gadomska M., Gadomski J., 1998. Identyfikacja modelu przemieszczeń punktów kontrolowanych na podstawie pomiarów geodezyjnych. Aktualne problemy budownictwa, ATR Bydgoszcz.

Kolenda K., Kolenda M., 1999. Prognozowanie szeregów czasowych. Agencja Wydawnicza Placet, Warszawa.

Przewłocki S., Tarabichi A., 1991. Uwarunkowania geodezyjne i konstrukcyjne w procesie wyznaczania deformacji płyt fundamentowych (zastosowanie praktyczne), Geodezja i Kartografia t. XXXIX, z. 1.

Toś C., Wolski B., Zielina L., 2003. Algorytm interpretacji wyników wieloletnich pomiarów przemieszczeń pionowych komina przemysłowego. Współczesna geodezja w rozwoju nauk technicznych, przyrodniczych i ekonomicznych, Wydawnictwo SGGW, Warszawa.

Wolski B., 2006. Monitoring metrologiczny obiektów geotechnicznych, Wydawnictwo Politechniki Krakowskiej.

Wójcik M., 2001. Badanie przemieszczeń pionowych obiektów zabytkowych na Ostrowie Tumskim w Poznaniu, Materiały XVI Konferencji Katedr i Zakładów Geodezji na Wydziałach Niegeodezyjnych, Zielona Góra – Łagów.

Zeliaś A., 1997. Teoria prognozy, Polskie Towarzystwo Ekonomiczne, Warszawa.

Zeliaś A., Pawełek B., Wanat S., 2003. Prognozowanie ekonomiczne. Teoria, przykłady, zadania. PWN, Warszawa.