ON ACCURACY OF CURRENT GEOPOTENTIAL MODELS ESTIMATED THROUGH COMPARISON OF QUASI-GEOID MODELS AND GPS/LEVELLING DATA

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Abstract. Gravity-dedicated satellite missions CHAMP and GRACE provide accurate data that can be inverted into geopotential coefficients forming global geopotential models (GGM). Through an estimation process, mean square errors of these coefficients can be evaluated. In most cases, even the entire variance-covariance matrices are computed. However, these errors are only formal and do not represent well the actual accuracy of estimated coefficients, thus also of various gravity field parameters synthesized from these coefficients. For this reason, validation procedures for the new GGMs are being sought since the classical validation methods reached their limits. This article discusses the validation procedure of the GGM through its comparison with independent data estimated at selected GPS/levelling stations over the area of Central Europe. Due to a different spectral content of height anomalies synthesized from the GGM and of those derived from combination of ellipsoidal and normal heights at selected points of the European Vertical Reference Network, the GGM-based low-frequency height anomaly is completed for a missing high-frequency component based on high resolution and high accuracy ground gravity and elevation data. The methodology is also applied on a set of GPS/levelling stations in the Czech Republic. In accordance with previous validation tests of GGMs based on data of crossover altimetry, obtained results indicate that the current GGMs estimated from single/dual-satellite data seem to have significant problems namely with low-order and degree coefficients.

1. Introduction

Global mapping of the Earth's gravity field based on gravity-dedicated satellite missions CHAMP and GRACE belongs to one of the most important tasks of contemporary geodesy. However, there is another difficult step in global mapping of the Earth’s gravity field sometimes neglected or underestimated: validation and accuracy assessment of estimated geopotential parameters using (preferably) independent data. The problem of new satellite data is in their high accuracy that makes some of the traditional verification techniques obsolete and available independent data relatively inaccurate.

In this contribution, we attempt to evaluate external accuracy of a newest generation of GGMs by comparison of high-resolution quasi-geoid models with GPS/levelling data. A high resolution and accuracy local gravity and elevation database is combined with a selected GGM for evaluation of the local quasi-geoid that is then tested against available GPS/levelling data. Height anomalies derived at selected stations of the European Vertical Reference Network (EUVN) as well as Czech trigonometric stations with heights estimated both by GPS and precise levelling are used as an independent benchmark for testing the new generation of the GGMs.

Testing the new GGMs using the GPS/levelling data has become quite popular in the recent years. Combining geodetic (GPS) and normal (levelled) heights, discrete values of the height anomaly can be obtained. However, some problems may be related to this method: 1- possible contamination of levelled heights by systematic observation errors and 2- strictly point values of the height anomaly derived at the GPS/levelling benchmarks versus surface means synthesized from the GGM. It is namely the latter problem that is tackled in this contribution. The spatial resolution of the GGM-based height anomalies is increased using local gravity and elevation data. Thus, tested (height anomalies based on combination of GGM/local gravity) and reference values (height anomalies derived from GPS/levelling data) become more comparable in respect of their frequency content. Obtained differences are used for validation of a respective GGM in the test area.

2. Methodology

The methodology applied for validation of the new GGMs is relatively simple and well known. Generally, discrete values of height anomalies derived from normal heights and geodetic (ellipsoidal) heights are directly compared with height anomalies synthesized from the geopotential coefficients. The main problem of this test is in the different spatial resolution of values that are being compared. Thus, the values synthesized from the GGM are completed for the missing high-frequency information using the local gravity and elevation data. This is the key issue discussed in this section, namely the evaluation of the high-frequency height anomaly from local ground gravity.

The estimation of the high-frequency height anomaly relies upon local ground gravity and elevation data. In this contribution, discrete surface mean values of both functions on the regular geographical grid of 30x30 arcsec (approximately 1x1 km) from the database VUGTK2002 compiled and verified in the Research Institute of Geodesy, Topography and Cartography in Prague (Kostelecký (jr.) 2004) were used, see Section 3 for its description. The spectral decomposition of the height anomaly was applied with a distinction made between the low-frequency (reference) and the high-frequency (residual) component. The threshold harmonic degree 120 corresponds to the maximum degree and order available in the GGMs derived from data of the satellite missions CHAMP and GRACE tested in this contribution. The reference quasi-geoid was synthesized directly from the spherical harmonic coefficients of the GGM (Heiskanen and Moritz 1967). The residual quasi-geoid was evaluated from ground gravity data using the surface integral (Molodensky 1960) that represented a well-known solution to the geodetic boundary-value problem defined for the surface of the Earth. This apparatus requires no masses outside the Earth that can be achieved by additional reduction of ground gravity data for the gravitational effect of atmospheric masses; see e.g. (Novák 2000). Due to the geographically limited ground gravity data, the spheroidal Stokes integral could not be evaluated over the full spatial angle. The contribution of the spherical cap was evaluated by discrete numerical integration over mean values of gravity anomalies given on a regular geographical grid within the spherical cap with radius of 3 arcdeg. The contribution of far-zone data was evaluated from the GGM by using the so-called Molodensky coefficients (Molodensky 1960) that accounted for the influence of gravity data omitted from the truncated surface integration. To keep their magnitude as small as possible, the integral kernel was modified according to (Vaníček and Kleusberg 1987).

3. Data description

Input elevation and ground gravity data in the database VUGTK2002 are given at a uniform geographical grid with the spatial resolution of 30x30 arcsec (approximately 1x1 km). The database was compiled from available national gravity and elevation data sets. Namely detailed and accurate ground gravity and elevation data from the Czech Republic, Slovakia, parts of Germany, Austria and Poland entered this database. However, observed values of gravity were available only for the territory of the former Czechoslovakia. Outside its political boundaries, mean and differently reduced values of gravity were available at a lower spatial resolution. Thus, the quality of gravity data in the database is not the same over the entire area. Generally, the best gravity data cover the Czech Republic and Slovakia. The area covered by the database spans between the parallels of 36 and 60 arcdeg northern latitude, and the meridians of 0 and 30 arcdeg eastern longitude. The smaller computation area covers the territory of Central Europe with the boundaries at 46 and 54 arcdeg northern latitude, and 7 and 23 arcdeg eastern longitude, respectively. This area corresponds to 1,843,200 computation points. Due to the cap integration used for numerical evaluation of the residual quasi-geoid (3 arcdeg), the actual input data area was bounded by 42 and 58 arcdeg northern latitude, and 0 and 30 arcdeg eastern longitude. This area corresponds to 6,912,000 data points.

40 stations of the European Vertical Reference Network (EUVN) and 300 Czech stations provided a nice sample of independent data, although covering only a relatively small region of Central Europe. Ellipsoidal heights of the EUVN stations were obtained through a one-week GPS campaign that was organized at 1st-order reference levelling stations across Europe. The accuracy of the estimated heights can be characterized by the RMS error at the level of 1 cm. This accuracy can further be degraded by systematic errors originating in different height systems used in various European countries. The 300 Czech stations are trigonometric stations with known Molodensky-normal heights estimated using very precise levelling. GPS heights of these trigonometric stations were estimated using static 8-hour GPS observation campaigns. The accuracy of height anomalies computed at these stations can be characterized by the RMS error of 2 cm. Although smaller in the geographical coverage, the Czech stations have the advantage of being homogeneous with respect to one height system used in the area.

The following GGMs were validated through the methodology described in this article: EGM96, EGG97, CSR GGM02C, CSR GGM02S, JPL GRACE GSM02, EIGEN-2, EIGEN-GRACE02S, EIGEN-CG03C and EIGEN-CHAMP03S. The European Gravimetric Geoid/Quasi-geoid model EGG97 was used for the testing purposes. This model combines the low-frequency component based on EGM96 with the high-frequency information coming from local gravity data. The coefficients up to maximum degree and order 120 were only considered from all these global models for numerical calculations considered in this contribution. This limit was selected with respect to available local gravity data; it also represents the minimum resolution common for all the tested GGMs. Main characteristics of the tested GGMs are summarized in Table 1.

Table 1. Characteristics of the tested GGMs

model	institution	data source	max degree	accuracy
EGM96	NASA/DMA	combined	360	—
GGM02C	CSR	GRACE/ground	200	formal
GGM02S	CSR	GRACE	160	formal
GSM02	JPL	GRACE	120	—
EIGEN-2	GFZ	CHAMP	140	formal
EIGEN-GRACE02S	GFZ	GRACE	150	calibrated and formal
EIGEN-CG03C	GFZ	CHAMP/ground	360	calibrated
EIGEN-CHAMP03S	GFZ	CHAMP	140	calibrated

4. Results

Point values of the height anomaly obtained at the GPS/levelling stations were compared against height anomalies estimated through the local quasi-geoid modelling. The quasi-geoid model is represented by point values of the height anomaly given on a mesh of geographical coordinates with resolution of 30x30 arcsec, i.e., approximately one value per km^2. They had to be first interpolated (quadratic interpolation) to the location of the GPS/levelling stations. Although the interpolated function is smooth and given on a relatively dense grid of geographical coordinates, interpolation errors at the mm level can still arise. Mean differences were then computed and tabulated separately for the EUVN stations and Czech GPS/levelling stations along with their respective standard deviations. Obtained results for the 40 EUVN stations can be found in Table 2, and for the 300 Czech

GPS/levelling stations in Table 3, respectively. Figures 1 and 2 then represent a surface computed from the residual differences estimated at the 40 EUVN stations for the “best” model EGG97 and “worst” model GFZ EIGEN-2. Figures 3 and 4 then similarly represent differences obtained at the 300 Czech GPS/levelling stations for the “best” model EGM96 and “worst” model GFZ EIGEN-2. Due to their large number, thus also high density, neither their distribution nor the estimated differences are shown at these two figures. A distribution of the reference stations with a corresponding value of the reduced difference can be seen at these figures. Obviously the EUVN stations cover a much larger area than the Czech stations. The EUVN stations may suffer from additional complications caused by three different height systems used in Central Europe. However, these systematic differences practically cancel each other due to the relative comparison among the different GGMs. Thus, the EUVN stations are more suitable for investigations of eventual long-wavelength biases in the GGMs.

Table 2. Differences at the selected 40 EUVN stations

model	mean difference	standard deviation	units
EGM96	46.44	± 19.57	cm
CSR GGM02C	47.31	± 29.41	
CSR GGM02S	47.35	± 29.47	
JPL GRACE GSM02	47.45	± 29.39	
EIGEN-2	48.60	± 31.12	
EIGEN-GRACE02S	49.70	± 29.28	
EIGEN-CG03C	49.83	± 29.34	
EIGEN-CHAMP03S	49.72	± 28.71	
EGG97	-14.28	± 17.07	

Looking at the statistical values in Table 2, namely at the standard deviations, it is obvious that the EGM96 and EGM96-based solution EGG97 perform better than those based on the new generation of the GGMs based solely on CHAMP and/or GRACE data. The fit of the tested and reference values is almost by 50% better in the case of the former solutions. The EGG97 model is comparable with the EGM96 model: the slightly better performance of the EGG97 model can be attributed to more accurate local gravity data used in its solution.

Table 3. Differences at the 300 GPS/levelling stations in the Czech Republic

model	mean difference	standard deviation	units
EGM96	36.68	± 3.47	cm
CSR GGM02C	26.34	± 4.15	
CSR GGM02S	26.35	± 4.23	
JPL GRACE GSM02	26.66	± 4.17	
EIGEN-2	19.92	± 10.03	
EIGEN-GRACE02S	28.85	± 4.11	
EIGEN-CG03C	28.97	± 7.18	
EIGEN-CHAMP03S	27.42	± 7.44	
EGG97	-22.07	± 3.56	

This is more obvious if the results for the Czech GPS/levelling stations are reviewed: in this case, the EGM96 model has a better fit, although the difference between the EGM96 and EGG97 is rather insignificant. Over the small area covered by the Czech stations, it is almost impossible to investigate the long-wavelength errors in the GGMs. In this case, the quasi-geoid models based on the new GGMs perform as well as the EGM96 or EGG97 models. Only some of the GFZ models, namely EIGEN-2, EIGEN-CHAMP03S and EIGEN-CG03C, show a significantly worse fit in terms of the standard deviation: 7-10 cm compared to 3-4 cm for all other models. It is interesting to compare these results with those based on the LCC, see Figures 2 and 4 for the "worst" model EIGEN-2 in regards of our results. This figure clearly indicates problems of this model at low frequencies. Also the surfaces computed from the estimated differences at the Czech stations show similar features: a clearly visible meridian slope of the quasi-geoid models. This slope indicating some flaw in the low-frequency spectrum of the new GGMs is not so significant for the EGM96-based solutions. This slope may originate in the orbital geometry of the single-satellite (CHAMP) or dual-satellite (GRACE) missions that results in a higher consistency of the sampled data along the north-south direction compared to the west-east direction. Note that both the CHAMP and GRACE missions are on near-polar orbits.

The results in Tables 2 and 3 are surprisingly good: the fit of the gravimetric quasi-geoid models with the independent GPS/levelling data is at the level of 4 cm for the Czech part of the test where the high quality gravity data were available. If the errors related to the GPS/levelling data, quasi-geoid models, and their interpolation are combined; the fit of the two sets of data seems to be excellent. One can hardly expect a fit at the sub-centimetre level of accuracy since levelled heights, GPS-based heights, and height anomalies in the gravimetric quasi-geoid have all the accuracy at the centimetre level. However, their comparison such as the one in this article can help to identify possible problems with one of the data source and give some idea on the real performance (outer accuracy) of the current and future quasi-geoid models. This is quite important in the view of their application in a transformation between geometric and physical heights.

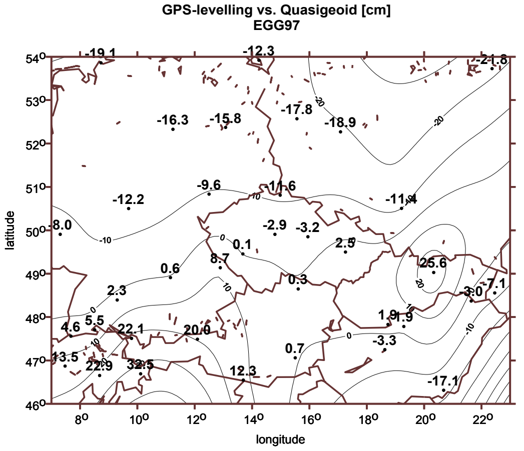

Figure 1

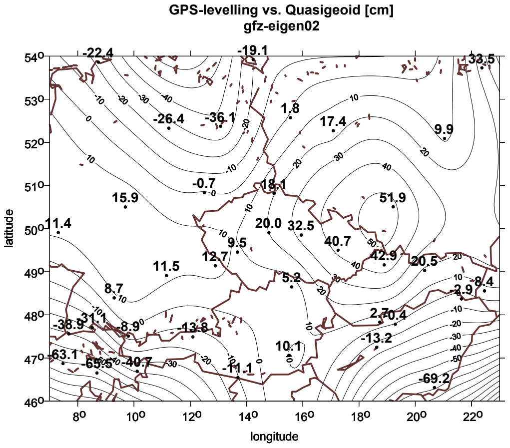

Figure 2

A comment on the results of the tests using the SSC by LLC follows since it confirms the results of the investigations presented in this article. We have checked various scaling or calibration factors of the formal variance-covariance matrices of several GGMs, older, pre-CHAMP as well as recent CHAMP/GRACE-based GGMs (e.g., Klokočník et al. 2005). As a by-product, we have discovered a significant decrease in accuracy of the CHAMP or GRACE-only based models for the lowest orders, namely for m = 2. We discuss this result in more details in (Kostelecký et al. 2006). These tests also confirm the results obtained in this article: the current GGMs based on single/dual-satellite data seem to exhibit deficiencies in their low degree and order coefficients.

5. Conclusions

The validation procedure for testing the GGM through the high-frequency quasi-geoid modelling and independent GPS/levelling data was discussed in this article. Due to a relatively low-frequency gravity signal contained in the GGM, local high accuracy and resolution gravity and elevation data were used to add the missing medium and high frequency components. In such a way, values of the height anomaly become comparable to respective values obtained by comparison of GPS and levelling data.

The procedure was applied to several GGMs based mainly on data of recent gravity-dedicated satellite missions CHAMP and GRACE. The EGM96 was also used due to its general acceptance among geodesists worldwide, as well as the European quasi-geoid model EGG97. However, the new satellite-based GGMs were the central topic of the numerical investigations. Obtained results in a form of mean differences and corresponding standard deviations were computed for the 40 EUVN and 300 Czech GPS/levelling stations. These results indicate a comparable accuracy of all models that is at the level of 30 cm for the EUVN stations and 4 cm for the Czech GPS/levelling stations. Results for the new GGMs based on data of the CHAMP and/or GRACE missions have a significantly worse (50%) fit at the EUVN stations than those based on the EGM96. The results in terms of the standard deviation based on the differences for all the models are comparable in the case of the Czech GPS/levelling stations. However, the geometric representation of the differences reveals deficiencies in the low-frequency spectrum of the new models. These results are supported by independent studies based on the latitude lumped coefficients with independent single satellite crossovers from long-term altimetry.

Generally, the results presented in this contribution supported by independent tests indicate that the GGMs based on the single-satellite and/or dual-satellite missions seem to include systematic errors. These errors may not be significant for low-resolution monthly solutions used for determination of temporal variability of the Earth's gravity field. However, despite their high spatial resolution, these GGMs do not seem to represent the actual gravity field as accurately as expected. To avoid this remedy, a combination of different data collected through independent techniques will most likely be the solution. Other accuracy improvements can be expected through enhanced modelling and estimation procedures used by processing centres for development of future GGMs.

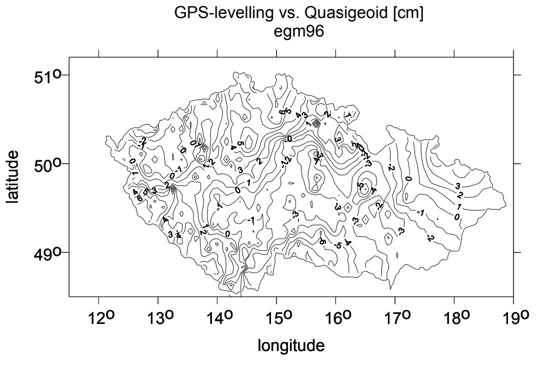

Figure 3

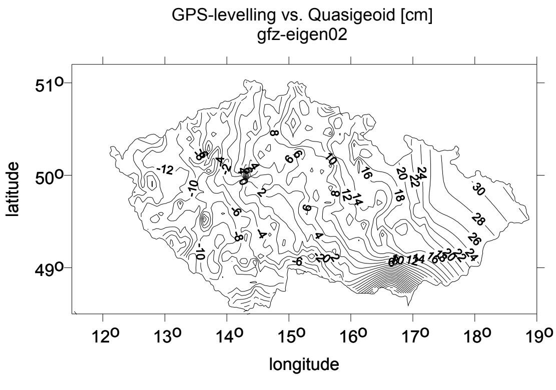

Figure 4

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