POLISH ASTRO-GEODETIC GEOID – FINAL RESULTS

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1. INTRODUCTION

In 2005 the project aimed at the geoid determination in Poland was completed. It was assigned by the Polish State Committee for Scientific Research and its assumption was to be interdisciplinary, so several Polish research centres took part to create combined geoid of a centimetre level of accuracy. One of them was the Institute of Geodesy and Geodetic Astronomy of the Warsaw University of Technology which was responsible for astro-geodetic geoid determination according to the deflections of vertical. The archival data consisted of two parts: deflections from astro-geodetic and astro-gravimetric levelling. The researches performed within this study encompassed: archival data storage, theoretic elaboration upon the transformation of existing deflections to the valid standards, supplementary astrometric observations and their processing and the levelling network adjustment as well. The paper presents final results of astro-geodetic geoid determination on the territory of Poland.

2. DATA

Accessible astro-geodetic data kept in resources of CODGiK (Central Geodetic and Cartographic Database) and IGIK (Institute of Geodesy and Cartography) are stored in 10 catalogues elaborated in 1967-1981. There are kept only catalogues including geodetic coordinates, geographic coordinates, astro-geodetic and astro-gravimetric vertical deflections. Archival astrometric data includes mean values of determined coordinates with observation epoch. These data has information about: quantity observed couples of stars, methods of observations, instruments used for observations and catalogue of stars coordinates used to the reduction. Furthermore there are stored values of reduction to conventional mean pole and correction to UT1. Relative vertical deflections presented in catalogues are in JSAG (Unified Astro-Geodetic Network) system (touchdown point in Pulkowo and Krasowki's ellipsoid 1974). Geodetic coordinates used for determination of components of vertical deflections were also derived from JSAG catalogue. Heights in this network are normal heights referenced to mareograph in Kronsztad on mean epoch '66. In our analyses there are points so called points of gravimetric densification. For these points there were made interpolation of relative vertical deflections using gravimetric information: this is intermediate interpolation based on gravimetric components of vertical deflection computed by integration gravimetric Faye's anomalies in limits to 102 km or sometimes to 304 km. No information about accuracy was available.

3. UNIFICATION OF ASTROMETRIC DATA

3.1. Existing astrometric data used for vertical deflection determination

The information about astrometric determinations which are saved in resources of CODGiK and IGiK contains the average values of determined coordinates with giving the epochs of their determination. The information about observed pairs, methods of observation, instruments which were used and also star's coordinates catalogue used for reduction are saved. Saved information also contains the values of reduction to conventional average pole and correction to UT1 time system according to conventions of International Astronomical Union (IAU), but observation's programmes were not saved (catalogue numbers of stars used in observations). Observations have mostly been reduced using the FK3 catalogue. FK4 catalogue has been used for 10 determination and the Chelberger catalogue for 3 of them.

3.2. The possibility to account with catalogue's differences in observation methods used.

Theoretic analyse of fundamental star's catalogue used to elaborate astrometric observations does not give unique bases to determine both individual and systematic corrections to determined geographic coordinates. In connection to this fact the trial has been undertaken to analyse influence of changing coordinates of stars which were observed to determined geographic coordinates.

<u>Talcott and Sterneck method</u>
If we take off that the influence of changing the coordinates of stars on reduction beyond meridian observations shouldn't exceed 0.03", the correction to determined latitude will be:

$$\Delta\varphi = \frac{1}{2}(\Delta\delta_S + \Delta\delta_N) \qquad \textbf{(1)}$$

<u>Piewcow method</u>
From differentiating of basic equation of Piewcow method we can get formula changing the geographical latitude :

$$d\varphi = -\frac{\sin q_S \cos\delta_S d\alpha_S - \sin q_N \cos\delta_N d\alpha_N + \cos q_S d\delta_S - \cos q_N d\delta_N}{\cos a_S - \cos a_N} \qquad \textbf{(2)}$$

Assuming that observation was conducted with keeping the symmetry reflection in local meridian $\cos q_S = \cos q_N = \cos q$ **and also** $\delta_S \cong \delta_N$ **the equation given above can be rearranged:**

$$d\varphi \cong -\frac{\sin q \cos\delta(d\alpha_S - d\alpha_N) + \cos q(d\delta_S - d\delta_N)}{2\cos a_S} \qquad \textbf{(3)}$$

Making also the assumption that in our latitudes: 30° < δ **< 70° ; 15° <** z **< 60°; 6° <** α **< 40°, and in the same time the differences between the right ascension and declination are not high in this case, we can describe maximum influence of star's coordinates difference using the equation:**

$$d\varphi \cong (d\alpha_S - d\alpha_N) + (d\delta_S - d\delta_N) \qquad \textbf{(4)}$$

Knowing that systematic changes of coordinates in the fundamental catalogues are differencing each other the strongest to right ascension it can be said that in case of Piewcow method influences on determined systematic difference on position will be not significant.

Mayer method

The assumption of Mayer method is determination of the time correction of star meridian passages were clock correction will be equal during upper and lower culmination to:

$$u = \alpha - S$$
$$u = \alpha - S \pm 12^h \qquad (5)$$

The effect on determined longitude will have only differences in right ascension catalogue in $d\lambda = d\alpha$. However we have to deal with noticed individual correction for each observed star.

3.3. Systematic differences between star catalogue's FK3 FK5 coordinates

Systematic differences between following editions of fundamental FK catalogues generally differ only from definition of equinox point and also assumed precession and nutation models. Those differences are published in form of corrections which depends on star declination and right ascension in form of corrections ($\Delta\alpha_\alpha, \Delta\alpha_\delta, \Delta\delta_\alpha, \Delta\delta_\delta$). Corrections depended on δ are significantly small and change slowly in function of declination as for right ascension as also star declination. More important are changes of systematic corrections in function of right ascension. During the research over unification the observation materials in astrogeodetic network, the tables of systematic corrections $d\alpha$ and $d\delta$ for declination range in right ascension function from 30 to 70° were prepared. Developed procedures which make possible to converse coordinates from FK3, FK4 and Hipparcos catalogues to the FK5 J2000 epoch turn out to be unuseful. If we have given information the conception of correcting particular observation (position of star or the couples of stars) is impossible. It can be only estimated the maximum influences of fundamental catalogue's systematic errors, used into preparation of astrometric observations on value 0.2-0.4". To conclude: we can claim that the average value of mean error for astrometric determinations could reach of about 0.3".

3.4. The reduction of astrometric observation to the IERS pole and UT1 time scale

The accessible documentation allows on precision conversion of determined astronomic coordinates to an obligatory ITRF frame (pole position and time scale). The geographic coordinates are correcting using substitution of reductions which were made in epoch of observation to valid now (according to IERS conventions).

4. UNIFICATION OF GEODETIC DATA

4.1. Transformation of geodetic coordinates from JSAG to ETRF'89

Geodetic coordinates used for evaluation of components of vertical deflections which are from the working out made from 1965 till 1981 are stored in catalogue "The Uniform AstroGeodetic Network" (JSAG). The heights in this network are normal heights in Kronsztad'66 system. Nowadays the reference frame in Poland is ETRF'89. The heights of sites in this network are ellipsoidal (EVLN'97). In the country's base frame of heights the Kronsztad'86 system is obligatory. On Polish territory quasi systematic difference between the Kronsztad'66 and '86 system is about 7 cm. The transformation of coordinates between two references usually is occurred with using formulas of spatial 7-parametric transformation using Helmert method, which can be written as:

$$\begin{vmatrix} X_B \\ Y_B \\ Z_B \end{vmatrix} = \begin{vmatrix} X_A \\ Y_A \\ Z_A \end{vmatrix} + \begin{vmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{vmatrix} + \begin{vmatrix} s & \omega_z & -\omega_y \\ -\omega_z & s & \omega_x \\ \omega_y & -\omega_x & s \end{vmatrix} \begin{vmatrix} X_A \\ Y_A \\ Z_A \end{vmatrix} \qquad (6)$$

The parameters for this transformation were determined using 348 sites of horizontal frame which have coordinates in both systems (JSAG and ETRF'89). To determine coordinates *XA, YA, ZA* the knowledge of normal heights and undulations between quasi-geoid and ellipsoid is necessary. The parameters of spatial transformation have the values:

ΔX= 33.4297 m, ΔY= -146.5746 m, ΔZ= -76.28 m,

ω_x= 0."358671, ω_y= 0."052837, ω_z= -0."843542, s= -8.840767 ppm

Finding the heights of points in base of JSAG (the basic catalogue does not include the heights) and undulations between ellipsoid and quasi-geoid is an obstruction in counting coordinates in transformation from the JSAG to ETRF'89 systems. The other method in enumerating geodetic coordinates between different systems and projection on different ellipsoidal references is the Molodenski method described by formulas (7):

$$\Delta B = \left\{ -\Delta X \sin B \cos L - \Delta Y \sin B \sin L + \Delta Z \cos B + \Delta a N e^2 \sin B \cos b / a \Delta f \left[M \frac{a}{b} + N \frac{b}{a} \right] \sin B \cos B \right\} \frac{1}{M+N}$$

$$\Delta L = \left[-\Delta X \sin B \cos L + \Delta Y \cos L \right] \frac{1}{(N+H)\cos B}$$

$$B_{ETRF'89} = B_{JSAG} + \Delta B$$

$$L_{ETRF'89} = L_{JSAG} + \Delta L$$

where *N, M* - radius of curvature, and:

$H = h_{norm}$+2 m, ΔX= 23.74m, ΔY= -123.83m, ΔZ= -81.81m,

Δa= -108 m, Δf=4.80812*10^{-7}

The advantage of using it is that having data about undulations is not necessary. The replacement value of undulations on the positions of frame by using the average value for Poland (+2 m) have not got any influence neither on precision of geodetic coordinates nor on calculated components of vertical deflections.

5. UNIFICATION OF GRAVIMETRIC DATA

In the database of vertical deflections many points of gravimetric densification for which the interpolation of relative vertical deflections using gravimetric information (indirect interpolation based on gravimetrical components of vertical deflection calculated for the whole Faye's gravimetric anomaly in acceptable limits till about 102 km and sometimes till 304 km) was conducted were stored. In the indirect interpolation gravimetric deflections were used, which are characterized by the average error assessed about 0.6".

5.1. Influence of changes of gravity's reference level (tide up with transformation to the International Gravimetric Standard Network IGSN'71 or the European Unified Gravimetric Network EUGN'2002) on geoid undulation and deflections of vertical as well

The change of established value of reference in determination of gravity occurred in 1972, when the decision about reducing of the Potsdam level of about 140 $\mu m*s^{-2}$ was made. The Eastern and Middle Europe countries decided simultaneously to correct the Helmert's formula into normal gravity using the constant element ε = -140 $\mu m*s^{-2}$. It can be proved that this transformation has not got any influence on value of characteristic the field of gravity. This situation insists, because of properties of *G* Stokes's integral extended of the whole globe, where the Stokes's weighted function $S(\psi)$ and Faye's gravimetric anomaly are occurred. However the transformation to the normal gravity, what should be expected in Poland for few years, for sure will cause significant - in compare to values of given errors - changes in components of vertical deflections and geoid's undulations. We should add, that the IGSN'71 network which the Polish gravimetric network and astro-gravimetric deflections should be referred to, is identify with the formula on normal gravity, appropriative to the global geodetic system GRS'1967 but it should not be bounded up with Helmert's formula modified in 1971. Possibility of rebinding and adjusting the Polish gravimetric network to the European system EUGN will cause the necessity of changing the formula on normal gravity, which is referenced to the ellipsoid in global system GRS'80. The solution of raised point can be found after transforming formulas into general form:

$$\gamma = a + b\sin^2 B + c\sin^2 2B \qquad \textbf{(8)}$$

properly for current until- and now-used ones. In next step we can determine normal gravity change:

$$\Delta\gamma = \Delta a + \Delta b\sin^2 B + \Delta c\sin^2 2B. \qquad \textbf{(9)}$$

Theory of physical geodesy gives formulas for geoid's undulation and meridian component of vertical deflections determined gravimetrically. There is appropriately:

$$\Delta N = -\frac{R}{\gamma}\left[\frac{2}{3}\Delta b + \frac{8}{105}\Delta c - \left(\Delta b - \frac{20}{21}\Delta c\right)\cos^2 B - \frac{4}{3}\Delta c\cos^4 B\right] \qquad \textbf{(10)}$$

and

$$\Delta\xi = \frac{\rho''}{\gamma}\Delta c \sin 2B\left(\frac{\Delta b}{\Delta c} - \frac{20}{21} + \frac{8}{3}\cos^2 B\right) \qquad \textbf{(11)}$$

In Poland changing the undulation in cases of passing into GRS'80 system is contained in range till 25 cm however changing the component ξ till 0.4". Because in determination of the "centimetre" geoid gravimetric information were used for interpolation of relative components of vertical deflections, so why the influence of right changes on normal gravity, that is non identifying formula on g_0 and other datum shape, determination of geodetic coordinates can be included in the right of interpolation characteristics values and bigger error than astrometric observation can't be made.

5.2. Influence of tidal system changes on geoid undulations

First part is consideration of changes in geoid undulations and vertical deflections in case of enforcing tidal corrections to Earth's gravity according to others, beyond this time, formulas. Such situation in Poland appeared in moment of including to IGSN71 system using absolute gravity measurements, and also during adjustment of national gravimetric network POGK1997. Worldwide network was elaborated including I.M. Longman's formula on tidal correction with putting into effect T. Honkasalo's segment. Changing of formula does not include bigger corrections than 3 μGal into observed gravity values. That Honkasalo's correction has numerical form:

$$c_H = -(-15.14 + 45.62 \cdot \cos 2B + 0.007 \cdot \sin 2B) \cdot 10^{-8}\, m \cdot s^{-2} \qquad \textbf{(12)}$$

Because absolute value of tidal correction does not exceed 3 μm*s^{-2}, including the Honkasalo's correction, we can define conclusion that change of formula on tidal corrections will not cause any changes in undulations and vertical deflections calculated using integration of the anomaly. The discrepancy between corrections are lover than the influence of other factors in undulation and vertical deflection errors.
The second direction of analysis is showing the difference between "average geoid" in relation to so-called "zero-geoid" on Polish territory. Both terms are connected with introducing to gravimetric and satellite observations parts according to averaged in time tidal deformations. According to Rapp, Ekman and Groten proposal the permanent component of Earth's gravity pollution from celestial bodies is eliminated, however the tidal geoid deformation is being restored itself. That is the way how the "average geoid" is getting. This is the consequence of operate the average horizontal gravity component caused by celestial bodies. Analogically to Honkasalo correction in Earth's gravity the permanent meridian component can be marked as:

$$\left(F_{NS}\right)_{aver} = \left(-45.59 \sin 2B + 0.03 \sin 4B\right) \cdot 10^{-8}\, m \cdot s^{-2}. \qquad \textbf{(13)}$$

Physics of this phenomenon explains the interpretation of this expression as the creation of permanently deformed Earth. After putting the condition of equality of deformed globe's volume and after total removal of tidal effect, Heikkinen got formula for distance of these surfaces :

$$N_{def} - N_{n.def} = 0.148\left(\cos 2B - \frac{1}{3}\right) \quad \textbf{(14)}$$

On territory of Poland value of such discrepancies can be accepted as linear variable and in consequence of this data interpolation received from relative deflections will remove this influence. On the entire globe these differences have range from +30 cm on the equator to -60 cm on pole. Influence of differences of deformations on meridian component of deflection has been estimated in Poland in range from 0.009" to 0.013".

6. THE PROBLEM OF VERTICAL DEFLECTION UNIFYING ISSUE THRU TRANSFORMATION INTO ETRF'89 FRAME

Transforming procedures must consist of changes of reference ellipsoid parameters and also changes in orientation parameters (rotation angles). Besides translation of system's origin and scale change should be included. During the analyses we have separated two ways. Using H. Moritz conclusions geodetic coordinates transformation can be defined by formulas:

$$\begin{aligned} B^{89} &= B^{42} + \Delta B^{ellips} + \Delta B^{transf} \\ L^{89} &= L^{42} + \Delta L^{ellips} + \Delta L^{transf} \end{aligned} \quad \textbf{(15)}$$

Substituting values shown above we have received following formulas for vertical deflection changes in passing from 1942 to ETRF'89 on Polish territory:

$$\begin{aligned} \Delta\xi_{89-42} &= -0.0841''\sin 2B + 1.0811''\sin B\cos L - 4.7401''\sin B\sin L + 2.4671''\cos B \\ &\quad - 0.05283''\cos L + 0.3587''\sin L \\ \Delta\eta_{89-42} &= -0.8435''\cos B + 1.0811''\sin L + 4.7401''\cos L - 0.05283''\sin B\sin L \\ &\quad - 0.3587''\sin B\cos L \end{aligned} \quad \textbf{(16)}$$

This attempt and that one given by Kadaj appeared after numerical experiments conduction, bring results in accordance with 0.01" agreement for meridian and 0.1" for prime vertical component.

7. NEW ASTROMETRIC OBSERVATIONS

On time of duration of project was foreseen about 25 points for measurements on terrain of whole country. It was decided to design first of all points in places of incompatibilities with models (13 points). The remaining points were designed in order to densify the existing net (9 points). Borowa Gora served as the reference point. Additionally 7 points were established in Tatra mountains as a separate scientific project made together with the Slovak University of Technology in Bratislava. For the observations we have used circumzenithal 50/500, the original Czech astronomic instrument invented at the beginning of 20th century. It was constructed for simultaneous determination of geographical latitude and longitude by the Gauss' method of equal altitudes of 40° zenith distance. The time measurements and time registration were gradually developed following the evolution of electronic timekeeping devices. The actual configuration uses the portable PC as the relative time measurement device. PC is used also for the processing and registration of the star observation. Realization of absolute time scale is based on two approaches.

The first method uses for time synchronization one-second impulses broadcasted by time signal DCF 77. The other method uses the one-second impulses generated by GPS geodetic receiver, which are decoded from received signals of GPS satellites. Both methods ensure one millisecond accuracy, which is fully sufficient for astronomical field observations. Detailed description of the data processing could be found in the paper of Bogusz et. al., 2005.

8. UNIFIED DEFLECTIONS OF VERTICAL IN POLAND

New determined deflections of vertical were then stored to the database together with the previously made observations (totally 561 points – fig. 1). The figures 2 and 3 present distribution of the vertical deflection in meridian and prime vertical respectively.

Fig. 1. New astrometric measurements

Fig. 2. Meridian component of vertical deflection [“]

Fig. 3. Prime vertical component of vertical deflection [“]

9. ASTRO-GEODETIC LEVELLING

Having 561 points with the vertical deflections determined we have created the net of triangles. 2099 levelling lines were adjusted using GeoLab software.
At the stage of adjustment we made the following assumptions:

1. **reference ellipsoid: GRS'80 (ETRS'89 reference system);**
2. **geoid undulation at BOGO point (N=30.76 m) was considered as errorless;**
3. **level of confidence : 98%;**
4. **test for convergence: 0.0001.**

As the results we have obtained:

1. **adjusted differences of undulations and their standard deviations;**
2. **adjusted undulations and their standard deviations;**
3. **variance-covariance matrix (dimension 560x560).**

m_0 of such constructed network reached 1.0070, standard deviations of the adjusted undulation differences was in the range from 0.00 to 0.23 m, on the average: 0.06 m. Many of point from astro-gravimetric network had to be rejected due to their errors in coordinates.

10. POLISH ASTRO-GEODETIC GEOID

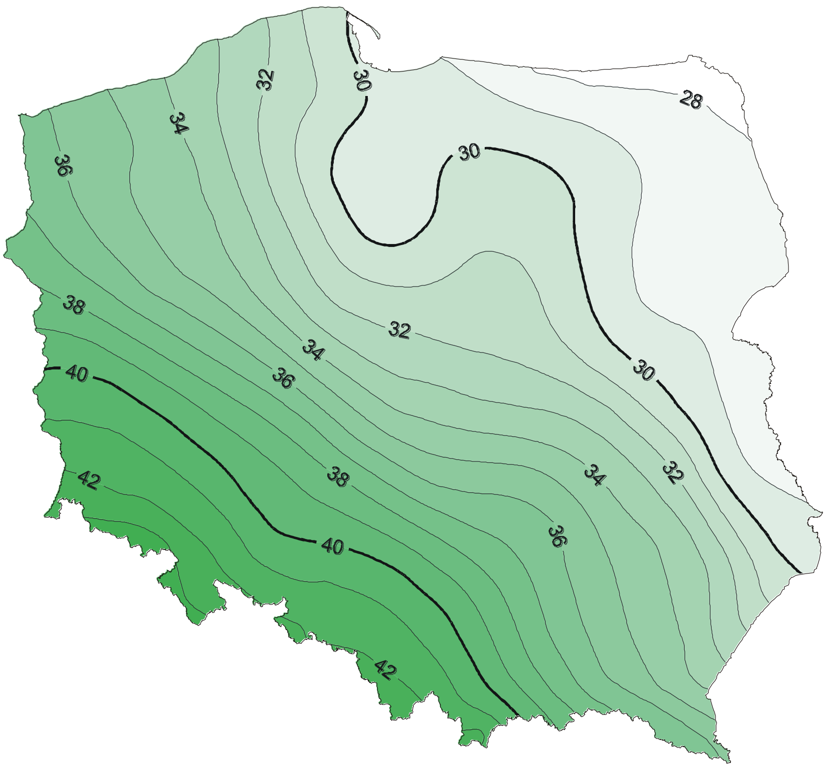

Fig. 4. Polish astro-geodetic geoid [m]

11. CONCLUSIONS

1. The data used for the adjustment was not homogenous. It was divided into three parts: astro-gravimetric, old astro-geodetic and new astro-geodetic deflections. Additionally the observations were made in different epochs with different methods and reduced variously.
2. There are no documentation about astro-gravimetric levelling so we can not preclude some systematic errors in the results. The tilt between astro-geodetic geoid and existed models (about 0.4") could be explained this way, but not unambiguous.
3. From preliminary analyses we can see that elimination of astro-gravimetric point and densyfing with new astrometric determinations will increase the accuracy of the model and diminish the systematic differences.
4. New astrometric determinations fit quite well with old ones despite of different epochs, methods and catalogues (no significant differences occurred).
5. For proper geoid determination information from neighbouring countries has to be used.

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