ATTENUATION MATRIX IN ROBUST, FREE ADJUSTMENT

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ABSTRACT

Equivalent weight matrix $\acute{\mathbf{P}}_{\mathrm{X}}$ plays a major role in robust, free adjustment. It is contained in the optimization criterion $\Psi(\mathbf{d}_{\mathrm{X}}) = \mathbf{d}_{\mathrm{X}}^{T}\acute{\mathbf{P}}_{\mathrm{X}}\mathbf{d}_{\mathrm{X}}$, where $\mathbf{d}_{\mathrm{X}}$ is an increment vector to approximate coordinates of all network points. Assuming, that $\acute{\mathbf{P}}_{\mathrm{X}} = \mathbf{P}_{\mathrm{X}}\mathbf{T}(\bar{\mathbf{d}}_{\mathrm{X}})$, where $\mathbf{P}_{\mathrm{X}}$ is a priori weight matrix, the paper presents the way how to calculate an attenuation matrix $\mathbf{T}(\bar{\mathbf{d}}_{\mathrm{X}})$($\bar{\mathbf{d}}_{\mathrm{X}}$ is a standardized increment vector). Special attention is paid to the way of increment standardization and to computation of an increment variance matrix.

Key words: Free adjustment, Robustness, Outliers.

1. INTRODUCTION

There are some problems, in geodesy or navigation, where observation sets contain some degree of freedom naturally e. g. (Mittermayer, 1980; Wolf, 1972). A geodetic network consists of some points with approximate coordinates only, is a typical example of such problem. Increments to these coordinates are computed on the base of an optimization criterion. Thus during the estimation process the adjusted network is fitted in the approximate one optimally (according to the assumed optimization criterion). That process could be disturbed by some "badly" computed (or assumed) approximate coordinates (outliers), which forces to apply a robust, free adjustment (such a concept was presented for example in (Czaplewski, 2004; Wiśniewski, 2005). The base for that robust method is application of an equivalent weigh matrix of increments (similarly to usage of an equivalent weigh matrix of observations in classic, robust methods (Wiśniewski, 1999; Yang, 1994). An attenuation matrix, which is computed on the base of attenuation functions, plays a major role in such equivalent solution. The paper presents some suggestions concerning assumptions and computation of such matrices and functions.

2. FREE ADJUSTMENT. COVARIANCE MATRIX

Let a coefficient matrix $\mathbf{A} \in R^{n,u}$, from the residual equation $\mathbf{v} = \mathbf{A}\mathbf{d}_{\mathbf{X}} + \mathbf{l}$, be not a column full rank ($\mathbf{v} \in R^{n,1}$ - residual vector, $\mathbf{d}_{\mathbf{X}} \in R^{u,1}$- parameter vector, $\mathbf{l} \in R^{n,1}$ - vector of free terms). Let it assume that $rank(\mathbf{A}) = r < u$, then $d = u - r > 0$ is a defect of the residual equation.

The following vector can be a solution of the inconsistent equation $\mathbf{A}\mathbf{d}_{\mathbf{X}} + \mathbf{l} = \mathbf{0}$ (with $d > 0$) e.g. (Kubačkova et. al., 1987; Prószyński, 1981; Wiśniewski, 2005)

$$\hat{\mathbf{d}}_{\mathbf{X}} = -\mathbf{A}^{+}_{\mathbf{PP_X}}\mathbf{l} \tag{2.1}$$

Such solution fulfills the conditions:

$$\textit{i)} \quad \left(\mathbf{A}\hat{\mathbf{d}}_{\mathbf{X}} + \mathbf{l}\right)^T \mathbf{P}\left(\mathbf{A}\hat{\mathbf{d}}_{\mathbf{X}} + \mathbf{l}\right) = \hat{\mathbf{v}}^T\mathbf{P}\hat{\mathbf{v}} = \min$$

$$\textit{ii)} \quad \hat{\mathbf{d}}^T_{\mathbf{X}}\mathbf{P}_{\mathbf{X}}\hat{\mathbf{d}}_{\mathbf{X}} = \min$$

where $\mathbf{P} \in R^{n,n}$ is a weight matrix of observations and $\mathbf{P}_{\mathbf{X}} \in R^{u,u}$ is a weight matrix of such approximation $\mathbf{X}^{\circ}$ (for example approximate coordinates of geodetic network points) that

$$\mathbf{X} = \mathbf{X}^{\circ} + \mathbf{d}_{\mathbf{X}}$$

Thus $\mathbf{X}^{\circ}$ is regarded as a pseudo observation vector with the assumed weight matrix ($\mathbf{P}_{\mathbf{X}}$ a priori).

The presented solution (2.1) is based on usage of $\mathbf{A}^{+}_{\mathbf{PP_X}}$ which is a general inverse matrix with the minimum norm

$$\mathbf{d}^T_{\mathbf{X}}\mathbf{P}_{\mathbf{X}}\mathbf{d}_{\mathbf{X}} = \left\|\mathbf{d}_{\mathbf{X}}\right\|^2_{\mathbf{P_X}} = \min$$

in the least squares method ($\mathbf{v}^T\mathbf{P}\mathbf{v} = \min$). Such inverse is of the following, general form

$$\mathbf{A}^{+}_{\mathbf{PP_X}} = \mathbf{P}^{-1}_{\mathbf{X}}\mathbf{Q}_{\mathbf{X}}\left(\mathbf{Q}_{\mathbf{X}}\mathbf{P}^{-1}_{\mathbf{X}}\mathbf{Q}_{\mathbf{X}}\right)^{-}\mathbf{A}^T\mathbf{P} \tag{2.2}$$

($\mathbf{Q}_{\mathbf{X}} = \mathbf{A}^T\mathbf{P}\mathbf{A}$). It can be proved (e. g. Perelmuter, 1979; Wolf, 1972; Wiśniewski, 2005) that, if $\mathbf{A} = \left[\mathbf{A}_1 \in R^{n,r}, \mathbf{A}_2 \in R^{n,d}\right]$, $\mathbf{P}_{\mathbf{X}} = Diag\left[\mathbf{P}_{\mathbf{X}_1} \in R^{r,r}, \mathbf{P}_{\mathbf{X}_2} \in R^{d,d}\right]$ and $rank(\mathbf{A}) = rank(\mathbf{A}_1) = r$ then

$$\left(\mathbf{Q}_{\mathbf{X}}\mathbf{P}^{-1}_{\mathbf{X}}\mathbf{Q}_{\mathbf{X}}\right)^{-} = Diag\left(\mathbf{\Theta}^{-1} \in R^{r,r}, \mathbf{0} \in R^{d,d}\right)$$

and afterwards

$$\mathbf{A}^{+}_{\mathbf{PP_X}} = \mathbf{P}^{-1}_{\mathbf{X}}\mathbf{B}^T\left(\mathbf{B}\mathbf{P}^{-1}_{\mathbf{X}}\mathbf{B}^T\right)^{-1}\mathbf{A}^T_1\mathbf{P}$$

where $\mathbf{B} = \left[\mathbf{A}^T_1\mathbf{P}\mathbf{A}_1 \, , \, \mathbf{A}^T_1\mathbf{P}\mathbf{A}_2\right]$ and $\mathbf{\Theta} = \mathbf{B}\mathbf{P}^{-1}_{\mathbf{X}}\mathbf{B}^T$.

Let $\mathbf{C}_{\mathbf{x}} = \sigma^2_0\mathbf{P}^{-1}$ be a covariance matrix of the observation vector *x*, and $\mathbf{C}_{\mathbf{X}} = \sigma^2_{0\mathrm{X}}\mathbf{P}^{-1}_{\mathbf{X}}$ be a covariance matrix of the vector $\mathbf{X}^{\circ}$. If additionally, $\mathbf{x} = \mathbf{F}(\mathbf{X})$ is a set of observation

equations, then the vector of free terms is $\mathbf{l}=\mathbf{F}(\mathbf{X}^{o})-\mathbf{x}$, and its covariance matrix can be written in the form

$$\mathbf{C}_{\mathbf{l}}=\sigma_{0\mathrm{X}}^{2}\mathbf{A}\mathbf{P}_{\mathrm{X}}^{-1}\mathbf{A}^{T}+\sigma_{0}^{2}\mathbf{P}^{-1} \qquad (2.3)$$

where $\mathbf{A}=\partial\mathbf{F}(\mathbf{X})/\partial\mathbf{X}$. Considering the solution (2.1) and the above formula, the covariance matrix of $\hat{\mathbf{d}}_{\mathrm{X}}$ can be derived as follows

$$\begin{aligned}\mathbf{C}_{\hat{\mathbf{d}}_{\mathrm{X}}}&=\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\mathbf{C}_{\mathbf{l}}\left(\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\right)^{T}=\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\left(\sigma_{0\mathrm{X}}^{2}\mathbf{A}\mathbf{P}_{\mathrm{X}}^{-1}\mathbf{A}^{T}+\sigma_{0}^{2}\mathbf{P}^{-1}\right)\left(\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\right)^{T}=\\&=\sigma_{0\mathrm{X}}^{2}\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\mathbf{A}\mathbf{P}_{\mathrm{X}}^{-1}\mathbf{A}^{T}\left(\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\right)^{T}+\sigma_{0}^{2}\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\mathbf{P}^{-1}\left(\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\right)^{T}\end{aligned} \qquad (2.4)$$

It should be noted that the first term in the above formula is related to errors of the $\mathbf{X}^{o}$ vector and the second one to errors of the observation vector x. Thus, applying the following notation

$$\mathbf{C}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{X}^{o}}=\sigma_{0\mathrm{X}}^{2}\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\mathbf{A}\mathbf{P}_{\mathrm{X}}^{-1}\mathbf{A}^{T}\left(\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\right)^{T}=\sigma_{0\mathrm{X}}^{2}\mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{X}^{o}} \qquad (2.5)$$

$$\mathbf{C}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{x}}=\sigma_{0}^{2}\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\mathbf{P}^{-1}\left(\mathbf{A}_{\mathbf{PP}_{\mathrm{X}}}^{+}\right)^{T}=\sigma_{0}^{2}\mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{x}} \qquad (2.6)$$

the covariance matrix of $\hat{\mathbf{d}}_{\mathrm{X}}$ can be finally written in the form

$$\mathbf{C}_{\hat{\mathbf{d}}_{\mathrm{X}}}=\mathbf{C}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{X}^{o}}+\mathbf{C}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{x}}=\sigma_{0\mathrm{X}}^{2}\mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{X}^{o}}+\sigma_{0}^{2}\mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{x}}=\sigma_{0}^{2}\left(r_{\sigma}\mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{X}^{o}}+\mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{x}}\right) \qquad (2.7)$$

where $r_{\sigma}=\sigma_{0\mathrm{X}}^{2}/\sigma_{0}^{2}$.

3. ROBUST, FREE ADJUSTMENT. ATTENUATION MATRIX

The idea of a robust, free adjustment was formulated in the paper (Czaplewski, 2004) (mainly to identify outlying adjustive points in sea navigation). The further development of the idea, for geodetic adjustment purposes, was shown in the paper (Wiśniewski, 2005).

Classic methods of robust adjustment in geodesy are usually based on the equivalent weight matrix $\overline{\mathbf{P}}=\mathbf{T}(\mathbf{v})\mathbf{P}$ application ($\mathbf{T}(\mathbf{v})$ is an attenuation matrix e. g. (Wiśniewski, 1999; Yang, 1994). The problem of robust, free adjustment can be solved in the similar way. Assuming that $\mathbf{X}^{o}$ is the vector of independent pseudo observations with the diagonal weight matrix $\mathbf{P}_{\mathrm{X}}$, the following equivalent matrix can be proposed

$$\overline{\mathbf{P}}_{\mathrm{X}}=\mathbf{T}(\overline{\mathbf{d}}_{\mathrm{X}})\mathbf{P}_{\mathrm{X}} \qquad (3.1)$$

where $\mathbf{T}(\overline{\mathbf{d}}_{\mathrm{X}})=Diag\left[t(\overline{d}_{\mathrm{X}1}),\ldots,t(\overline{d}_{\mathrm{X}u})\right]$ ($\overline{\mathbf{d}}_{\mathrm{X}}$ is the standardized increment vector). Function $t(\overline{d}_{\mathrm{X}})$, $\overline{d}_{\mathrm{X}}=\overline{d}_{\mathrm{X}1},\overline{d}_{\mathrm{X}2},\ldots,\overline{d}_{\mathrm{X}u}$ is an attenuation function with the following essential properties:

$$\left.\begin{array}{lll} i) & t(\overline{d}_{\mathrm{X}})=1 \quad for & \overline{d}_{\mathrm{X}}\in\Delta_{\mathrm{X}}=\langle -k_{\mathrm{X}},k_{\mathrm{X}}\rangle \\ ii) & t(\overline{d}_{\mathrm{X}})<1 \quad for & \overline{d}_{\mathrm{X}}\notin\Delta_{\mathrm{X}} \\ iii) & \forall\ \overline{d}_{\mathrm{X}i},\overline{d}_{\mathrm{X}j}\notin\Delta_{\mathrm{X}},\ \left|\overline{d}_{\mathrm{X}i}\right|<\left|\overline{d}_{\mathrm{X}j}\right| & :\ t(\overline{d}_{\mathrm{X}i})>t(\overline{d}_{\mathrm{X}j}) \end{array}\right\} \qquad (3.2)$$

where $\Delta_{\mathrm{X}} = \langle -k_{\mathrm{X}}, k_{\mathrm{X}} \rangle$ is an interval accepted for standardized increments $\bar{d}_{\mathrm{X}}$. Referring to the Danish attenuation function, that is widely used in "classic" robust adjustment, one can propose the following attenuation function

$$t\left(\bar{d}_{\mathrm{X}}\right) = \begin{cases} 1 & \textit{for} \quad \bar{d}_{\mathrm{X}} \in \Delta_{\mathrm{X}} \\ \exp\left[-l\left(\bar{d}_{\mathrm{X}} - k_{\mathrm{X}}\right)^{g}\right] & \textit{for} \quad \bar{d}_{\mathrm{X}} \notin \Delta_{\mathrm{X}} \end{cases} \qquad \textbf{(3.3)}$$

(*l, g* are control parameters).

The way of increment standardization is one of the main problems in robust, free adjustment (it usually influences the number of necessary iterative steps). Generally, the standardization should be done on the base on the estimated covariance matrix $\hat{\mathbf{C}}_{\hat{\mathbf{d}}_{\mathrm{X}}} = \hat{\sigma}_0^2\left(\hat{r}_\sigma \mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathrm{X}^{\mathrm{o}}} + \mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{x}}\right)$, where $\hat{\sigma}_0^2 = \mathbf{v}^T\mathbf{P}\mathbf{v}/(n-r+d)$ is an estimator of the covariance coefficient σ_0^2, and $\hat{r}_\sigma = \hat{\sigma}_{0\mathrm{X}}^2/\hat{\sigma}_0^2$. The estimate $\hat{\sigma}_{0\mathrm{X}}^2$ is not precisely defined so far. If one can assume that the matrices $\mathbf{C}_{\mathbf{x}}$ and $\mathbf{C}_{\mathrm{X}}$ are assessed at the same level then $r_\sigma = \sigma_{0\mathrm{X}}^2/\sigma_0^2 = 1$ hence $\hat{r}_\sigma = 1$. Thus, with sufficient approximation

$$\hat{\mathbf{C}}_{\hat{\mathbf{d}}_{\mathrm{X}}} = \hat{\sigma}_0^2\left(\mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathrm{X}^{\mathrm{o}}} + \mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{x}}\right) = \hat{\sigma}_0^2 \mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}} \qquad \textbf{(3.4)}$$

where $\mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}} = \left(\mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathrm{X}^{\mathrm{o}}} + \mathbf{Q}_{\hat{\mathbf{d}}_{\mathrm{X}}/\mathbf{x}}\right)$.

The following vector of the adjusted increments is the solution of the robust, free adjustment with application of the attenuation matrix $\mathbf{T}\left(\bar{\mathbf{d}}_{\mathrm{X}}\right)$

$$\hat{\mathbf{d}}_{\mathrm{X}} = -\mathbf{A}^{+}_{\mathbf{P}\breve{\mathbf{P}}_{\mathrm{X}}}\mathbf{l} \qquad \textbf{(3.5)}$$

whereat

$$\left.\begin{aligned} \breve{\mathbf{P}}_{\mathrm{X}} &= \lim_{j\to\infty} \mathbf{P}_{\mathrm{X}}^{j} \\ \hat{\mathbf{d}}_{\mathrm{X}} &= \lim_{j\to\infty} \mathbf{d}_{\mathrm{X}}^{j} \\ \mathbf{d}_{\mathrm{X}}^{j} &= -\left(\mathbf{A}^{+}_{\mathbf{PP}_{\mathrm{X}}}\right)^{j}\mathbf{l} = -\left(\mathbf{P}_{\mathrm{X}}^{j}\right)^{-1}\mathbf{B}^{T}\left[\mathbf{B}\left(\mathbf{P}_{\mathrm{X}}^{j}\right)^{-1}\mathbf{B}^{T}\right]^{-1}\mathbf{A}_1^{T}\mathbf{P}\mathbf{l} \\ \mathbf{P}_{\mathrm{X}}^{j} &= \mathbf{T}\left(\bar{\mathbf{d}}_{\mathrm{X}}^{j-1}\right)\mathbf{P}_{\mathrm{X}}^{j-1} \end{aligned}\right\} \qquad \textbf{(3.6)}$$

It is important to determine the matrix $\mathbf{T}\left(\bar{\mathbf{d}}_{\mathrm{X}}^{j}\right)$ in such a way that $\left|\mathbf{P}_{\mathrm{X}}^{j+1}\right| > 0$. Thus it is necessary to put in the process a such numerical protection, that if for big standardized increment $\bar{d}_{\mathrm{X}i}$ is $t\left(\bar{d}_{\mathrm{X}i}\right) < e$ then at most $\left[\mathbf{T}\left(\bar{\mathbf{d}}_{\mathrm{X}}\right)\right]_{i,i} = e$ (*e* is a numerical threshold for singularity of $\mathbf{P}_{\mathrm{X}}^{j+1}$).

4. EXAMPLE

The figure 1 presents a free geodetic network taken to the example computations.

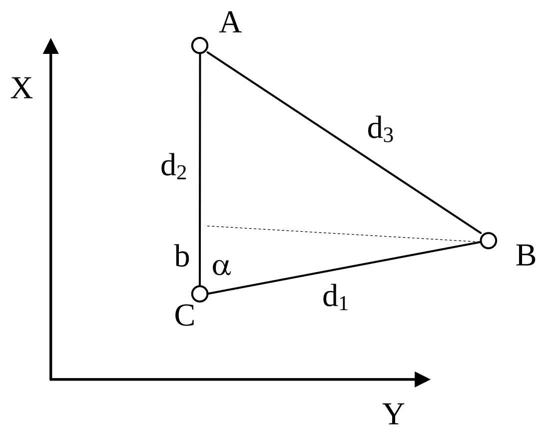

Fig. 1. Free network

The values of the approximate point coordinates and the observations were as follows

$$\mathbf{X}^{o}=\begin{bmatrix}X_A\\Y_A\\X_B\\Y_B\\X_C\\Y_C\end{bmatrix}=\begin{bmatrix}200.00\,m\\100.00\,m\\100.00\,m\\200.00\,m\\100.00\,m\\100.00\,m\end{bmatrix},\ \mathbf{x}=\begin{bmatrix}99.97\,m\\100.02\,m\\141.44\,m\\100.040^{g}\end{bmatrix}\ \Rightarrow\qquad \mathbf{l}=\mathbf{F}\left(\mathbf{X}^{o}\right)-\mathbf{x}=\begin{bmatrix}0.030\,m\\-0.020\,m\\-0.020\,m\\-0.040^{g}\end{bmatrix}$$

It was also assumed that $\mathbf{P}_{\mathrm{X}}=\mathbf{I}$ and $\sigma_d=0.02\ m, \sigma_\alpha=0.020^{g}$.
The following results were obtained applying classic, free adjustment

$$\hat{\mathbf{d}}_{\mathrm{X}}=\begin{bmatrix}-0.015\,m\\-0.009\,m\\0.016\,m\\-0.015\,m\\-0.001\,m\\0.024\,m\end{bmatrix},\qquad \hat{\mathbf{v}}=\begin{bmatrix}\text{-0.002 }m\\\text{-0.002 }m\\0.002\ m\\\text{-0.003}^{g}\end{bmatrix}$$

Let now, the approximate coordinate X_C be disturbed with a large gross error b = 2.00 *m*. It means that $X_C^{o}=102.00\,m$ (other coordinates stay the same) and

$$\mathbf{l}=\mathbf{F}\left(\mathbf{X}^{o}\right)-\mathbf{x}=\begin{bmatrix}0.05\,m\\-2.02\,m\\-0.02\,m\\1.23^{g}\end{bmatrix}$$

Adjusting the network once again one can obtain

$$\hat{\mathbf{d}}_X = \begin{bmatrix} 0.317\,m \\ -0.184\,m \\ 0.851\,m \\ 0.317\,m \\ -1.168\,m \\ -0.133\,m \end{bmatrix}, \quad \hat{\mathbf{v}} = \begin{bmatrix} \text{-}0.002\ m \\ \text{-}0.002\ m \\ 0.002\ m \\ \text{-}0.002^{g} \end{bmatrix}$$

The presented earlier robust, free adjustment can be applied to decrease the influence of the outlying coordinate. Using the attenuation function (3.3) for l = $5*10^{-4}$, g = 2, k_x = 2.5 and the procedure presented in the paper one can obtain the following results

$$\hat{\mathbf{d}}_X = \begin{bmatrix} -0.003\,m \\ -0.026\,m \\ 0.051\,m \\ -0.003\,m \\ -1.967\,m \\ 0.026\,m \end{bmatrix}, \quad \hat{\mathbf{v}} = \begin{bmatrix} \text{-}0.002\ m \\ \text{-}0.002\ m \\ 0.002\ m \\ \text{-}0.003^{g} \end{bmatrix}$$

The iterative process leading to the above solution is presented in the table 1.

Table 1.

		$\mathbf{d}_X$	$\overline{\mathbf{d}}_X$	$Diag\mathbf{T}(\mathbf{d}_X)$	$Diag\mathbf{P}_X$
Iterative steps	**0**	**0.317**	**45.208**	**0.402**	**0.402**
		-0.184	**-18.979**	**0.873**	**0.873**
		0.851	**87.518**	**0.027**	**0.027**
		0.317	**45.208**	**0.402**	**0.402**
		-1.168	**-117.328**	**0.001**	**0.001**
		-0.133	**-13.329**	**0.943**	**0.943**
	1	**0.002**	**1.801**	**1.000**	**0.402**
		-0.027	**-2.876**	**1.000**	**0.873**
		0.064	**3.249**	**1.000**	**0.027**
		0.002	**1.801**	**1.000**	**0.402**
		-1.954	**-92.031**	**0.018**	**$2.5*10^{-5}$**
		0.024	**2.736**	**1.000**	**0.943**
	2	**-0.003**	**-2.493**	**1**	**0.402**
		-0.026	**-2.742**	**1.000**	**0.873**
		0.051	**2.588**	**1.000**	**0.027**
		-0.003	**-2.493**	**1**	**0.402**
		-1.967	**-92.029**	**0.018**	**$4.5*10^{-7}$**
		0.026	**2.879**	**1.000**	**0.943**
	3	**-0.003**	**-2.574**	**1.000**	**0.402**
		-0.026	**-2.740**	**1.000**	**0.873**
		0.051	**2.576**	**1.000**	**0.027**
		-0.003	**-2.574**	**1.000**	**0.402**
		-1.967	**-92.029**	**0.018**	**$8.3*10^{-9}$**
		0.026	**2.882**	**1.000**	**0.943**

The simple example shows than applying the special attenuation matrix it is possible to decrease or even erase the effect of outlier that can occur among approximate coordinates in free geodetic network.

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