

KINEMATIC GPS BATCH PROCESSING, A CONSTRAINED SOLUTION APPLIED TO ANTENNA ARRAY

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Kinematic GPS observation processing requires robustness, especially in noisy environments. Ambiguity resolution robustness can be improved with multi base approach and parameters constraining. We consider a discrete-time linear systems with known dynamics, produced by a GPS antenna array mounted on a vehicle. Simulated and real data examples are given, where both system dynamics and geometric constraints strengthen ambiguities fixing. The developed algorithm works in a multi-base approach with the simplifying hypothesis of short baselines.

1. INTRODUCTION

Kinematic GPS observation processing requires robustness against cycle slips, especially in noisy environments, that is the case of mobile antennas, working in urban areas, roads, forest areas, etc. Ambiguity resolution robustness can be improved with multi base approach and parameters constraining. Has been shown that constraining system dynamic strengthen ambiguities fixing, performed by LAMBDA method (Roggero, 2006). Constraining dynamic in Least Square estimation of float ambiguities the search volume for LAMBDA method is reduced, and the robustness of cycle slip fixing increased. ADOP and Success Rate are also improved. The effect of noise on the estimated trajectory can be reduced, according to the kinematic model. Moreover in antenna arrays of known geometry is possible to constrain the baseline length, reducing subsequently the search volume.

We consider a discrete-time linear systems with known dynamics, produced by a GPS antenna array mounted on a vehicle. The developed algorithm, implemented in Fortran90 programming language, works in a multi-base approach with the simplifying hypothesis of short baselines. This hypothesis greatly simplify the observation equations, allowing to neglect the atmospheric effects in the inter antenna array baselines. Thanks to that, it is also possible to use only single frequency observations efficiently. The estimated baselines are then used to compute the attitude angles of the vehicle, that are necessary for sensor direct orientation.

Simulated and real data examples are given, where both system dynamics and geometric constraints strengthen ambiguities fixing.

2. ESTIMATION OF FLOAT AMBIGUITIES

Let us consider a discrete-time linear system described by a finite state vector x and constant biases b evolving with known dynamics trough the epochs t ($t = 1, \dots, T$):

$$\begin{aligned}
x_{t+1} &= T_{t+1}x_t + v_{t+1} \\
y_{t+1} &= H_{t+1}x_{t+1} + C_{t+1}b_{t+1} + \varepsilon_{t+1}
\end{aligned} \tag{2.1}$$

where y are the observations and the last equation shows that the bias vector b is a constant. The bias vector can be constant or constant with steps, that is the case of carrier phase ambiguities affected by cycle slips; steps are taken in account by matrix C . The matrix C , that link the bias vector to the observations, must be known a priori. The system generates the normal matrix N

$$N = D^T W_\omega D + M^T W_\varepsilon M \tag{2.2}$$

that includes the contribution of the dynamic and of the geometry, as described in (Albertella et al., 2006) and (Roggero, 2006). The dynamic model is described by the epoch by epoch transition matrix T , and by the overall matrix D , with the weight matrix W_ω . The contribution of geometry is given by the epoch by epoch design matrix H , and by the overall matrix M , with the weight matrix W_ε . Must be noted that the weight matrices depend on the variance covariance matrices of the system noise and of the observation errors. As the terms in the sum ($D^T W_\omega D$ and $M^T W_\varepsilon M$), also the normal matrix $N = D^T W_\omega D + M^T W_\varepsilon M$ is in the quasi triangular Schur form, and can be partitioned

$$\begin{bmatrix} N_x & N_{xb} \\ N_{xb}^T & N_b \end{bmatrix} = \begin{bmatrix} W_\omega + T^T W_\omega T & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} H^T W_\varepsilon H & H^T W_\varepsilon C \\ C^T W_\varepsilon H & C^T W_\varepsilon C \end{bmatrix} \tag{2.3}$$

Thanks to domain decomposition is possible to estimate the float ambiguities with their variance-covariance matrix, without solving for the other unknown parameters (i.e. trajectory). Then, this result is used as input to search for the integer ambiguities through LAMBDA method. Finally, the double differenced GPS observations can be corrected for the carrier phase integer ambiguities and the system can be solved for the non constant unknowns (positions, velocities, etc.); this last step does not require to invert the sub matrix N_x that has already been inverted to compute the Schur complement to N_b . Note that we need only to invert N_x , that is large because has dimension $(p \cdot T) \times (p \cdot T)$ but is tridiagonal, and S_b that has dimension $n \times n$.

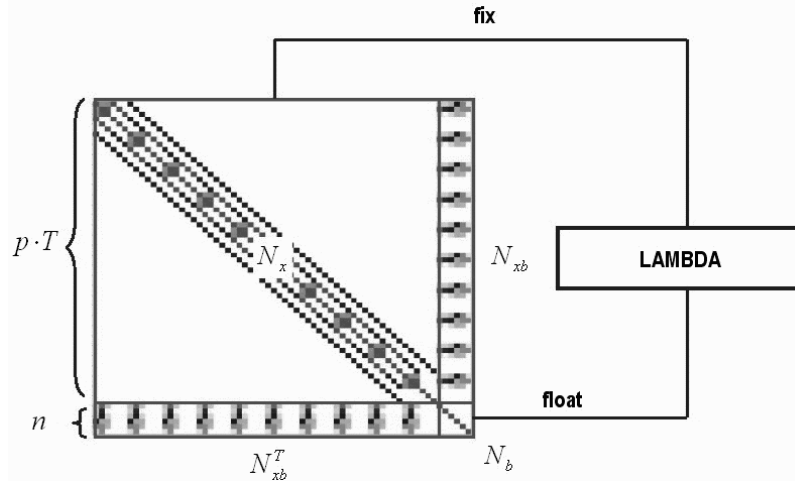


Fig. 1. Domain decomposition allows to solve for the float ambiguities, neglecting the other system parameters. After applying the LAMBDA method, the integer fixed ambiguities are used to correct the observations and to solve for the non constant system parameters (i.e. trajectory)

3. CONSTRAINING THE BASELINE LENGTH

In some steps of the algorithm, the inter antenna array baseline length can be constrained, with the main advantage of strengthening ambiguity fixing. We will show that baseline precision is not greatly improved.

Let us consider now an antenna array composed by three antennas, disposed in the vertices of an equilateral triangle. The baseline lengths are known and constrained. We want to see the effect of baseline length constraining on the baseline precision, and of course on the attitude angles precision, that is our goal. With the a priori hypothesis that the three GPS baselines have the same length (2 m) and 3D precision ($\sigma = 1$ cm), if the baseline length is not constrained we obtain a mean square error of 0.32 gon for the three attitude angles. Constraining the baseline length with $\sigma = 1$ mm, the simulated network results are characterized by small improvements in horizontal precision and only in the baseline directions, while the vertical precision is unchanged. This is due to the fact that the antenna array lies approximately in a horizontal plane. As a consequence, only the drift angle precision has been improved by constraining, obtaining in the simulation a mean square error of 0.14 gon.

Tab. 1 Static test: standard deviation of the estimated coordinates, in meters. Two minutes of single frequency observation, acquired at one second data rate. The baseline is 50 m length and approximately in the direction E – W, so the baseline length has the main influence on the east component.

		non constrained				constrained			
		σ_E	σ_N	σ_h	σ_{base}	σ_E	σ_N	σ_h	σ_{base}
C/A	DD	0,45	1,07	1,02	0,41	0,10	1,16	1,05	0,02
L1	TD	0,010	0,020	0,025	0,010	0,001	0,008	0,026	0,0003
L1	DD float	0,011	0,010	0,030	0,012	0,001	0,007	0,011	0,0003
L1	DD fix	0,003	0,007	0,009	0,003	0,001	0,006	0,010	0,0003

The standard deviation of the estimated coordinates has also been evaluated in a static test, comparing the results obtained with and without the application of the constrain. The tested baselines is 50 m length and approximately in the E – W direction, so the baseline length has the main influence on east component. Baseline processing has been performed in five steps: single point positioning (of course constraining is not possible), code pseudorange double differences, carrier phase double differences with float ambiguities and finally with fixed ambiguities. Every step has been performed with and without constraining the baseline length. The results are summarized in table 1, and are estimated as standard deviation of the position time series (three minutes of data, at one second data rate). They show that the main precision improvements are in the east component, that is also clear in the figure 2.

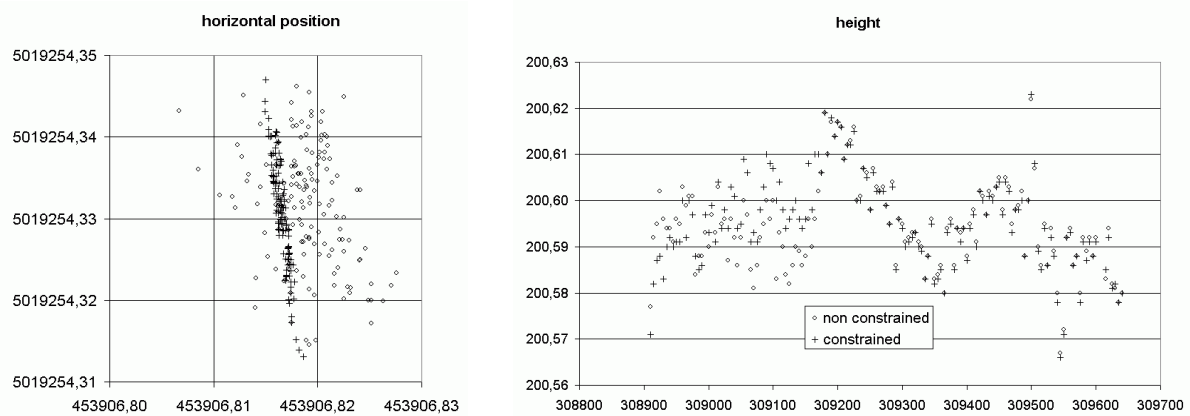


Fig. 2. Static test: horizontal and vertical position. The crosses represent the constrained solution, while the circles represent the non constrained solution

4. ALGORITHM

The implemented software package, based on GPS kinematic data batch processing, includes routines for data reading and preprocessing. Observation files in RINEX format are imported, while is possible to use both the broadcast ephemeris in RINEX NAV format and the precise (rapid or ultrarapid) ephemeris in SP3 format.

The approximate parameters are estimated epoch by epoch, neglecting time correlation. At first, the approximate trajectory is computed in single point positioning, using the code pseudorange only. Then a priori coordinates are assigned to the master station, and the baselines are formed, preparing all the differenced observations that are required. The estimated trajectory is refined processing the code pseudorange double differences, where the biases (receiver and satellite clock errors, ionospheric and tropospheric delays) are eliminated. Residual differential biases are usually present in double differences, but their effect is very small in the short baselines formed between the antennas of the array mounted on the moving vehicle. Finally, the approximate trajectory is refined processing the carrier phase triple differences. In this two steps is already possible to constrain the baseline length by pseudo observations.

Before estimating the float ambiguities, it is necessary to detect the cycle slips in the observations. Then, estimating the float ambiguities the system can be solved for carrier phase ambiguity only, using Schur decomposition. As result we obtain the approximate values of the ambiguities and their variance covariance matrix.

The LAMBDA method is used to search for the integer ambiguities. The estimation of the fix solution is obtained after that the bias term has been removed from the observation vector.

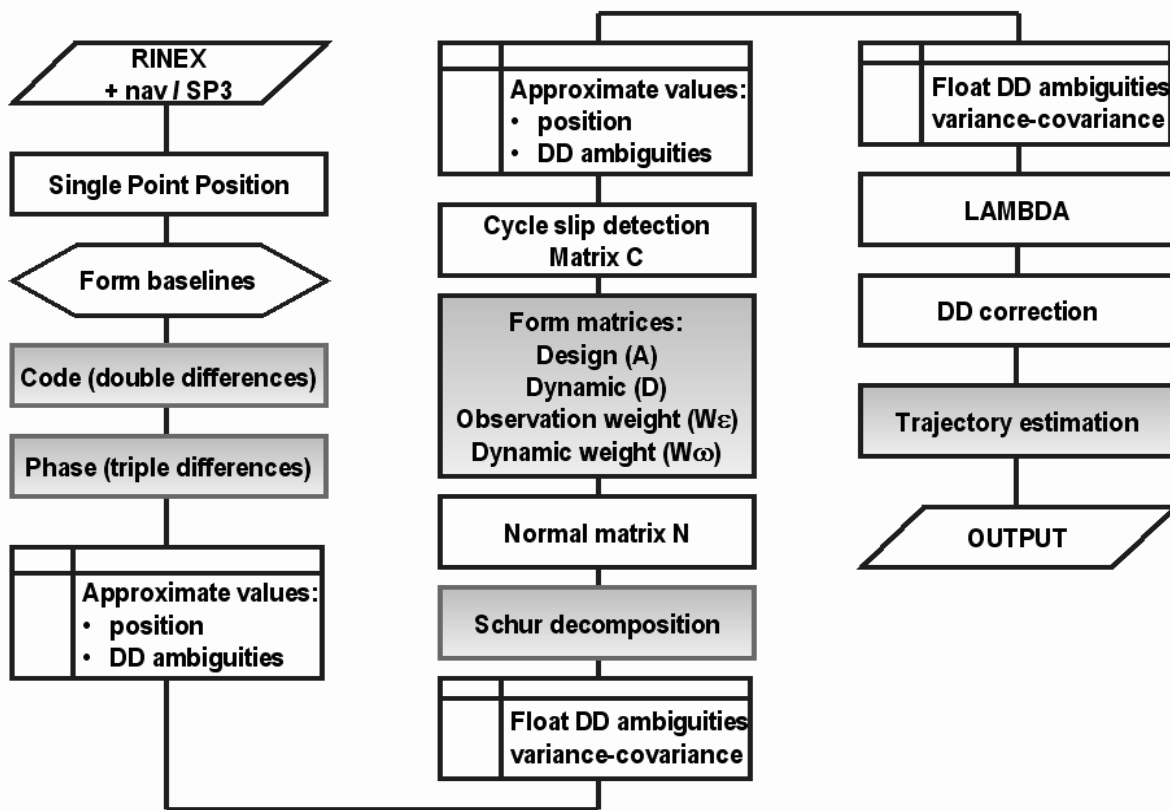


Fig. 3. Flow chart of the implemented algorithm. The baseline length can be constrained in the greyed blocks

The algorithm has been implemented using the Fortran90 programming language. To consider time correlation require large matrices, so it is necessary to treat the matrices as sparse. Matrices are stored in CSR (Compact Sparse Row) format or equivalent specific compact format. As example: a data set of 15' at 1" data rate and double precision (8 byte) GPS single frequency observations produce a design matrix of ~120 Mbyte. Writing this design matrix in CSR format, it reduces to ~1 Mbyte. Moreover, matrix - vector operations (sum, multiplication, transpose, inversion) are quite time consuming, but they speed up thanks to the sparse structure. Only to give an idea on the improvements obtained in practical applications, without taking advantage of sparse structure the algorithm is almost unusable in practice, because you need some hours or more waiting for the results. Since the CPU time required for the inversion of a $n \times n$ matrix scales as n^3 , a brute force analysis of this type of data set will hardly be feasible. Applying CRS and specific matrix operation routines, the processing time reduces up to few seconds or minutes, anyway not more than a coffee break. The algebraic background on sparse matrices is due to (Saad , 2000).

5. APPLICATIONS

The data available for the test have been acquired by two multi antenna systems for mobile mapping applications, a Mobile Mapping Vehicle equipped by three antennas and a boat equipped by two antennas. So the algorithm has been applied to process data for road cadastral surveying and river bathymetry.

The software has been designed to handle GPS single frequency observations, over short baselines (2 -10 km); moreover the software can process the data in multi-base approach, and allows to constrain the baseline length between the antennas of the antenna array. As final products the software provides trajectories and orientation angles of the antenna array.

6. CONCLUSIONS

We take advantage of the least squares approach, applying static constraints to kinematic observations. The multi base approach is necessary to correctly apply the constraints to the baseline length. Constraining the base line length in kinematic GPS data processing the robustness of cycle slip fixing have been increased, and the accuracy of estimated drift has been improved. However, the overall accuracy of the antenna array system is not improved by baseline length constraining; the main effect of constraining is only the robustness against cycle slips.

7. REFERENCES

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