-signature recognition,
Hidden Markov Models

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USING HIDDEN MARKOV MODELS IN SIGNATURE RECOGNITION PROCESS

This paper presents a method of recognition of handwritten signatures with the use of Hidden Markov Models (HMM). The method in question consists in describing each signature with a sequence of symbols. Sequences of symbols were generated on the basis of an analysis of local extremes determined on diagrams of dynamic features of signatures. For this purpose, the method proposed by G.K. Gupta and R.C. Joyce has been modified. The determined sequences were then used as input data for the HMM method. The studies were conducted with the use of the SVC2004 database. The results are competitive in relation to other methods known from the literature.

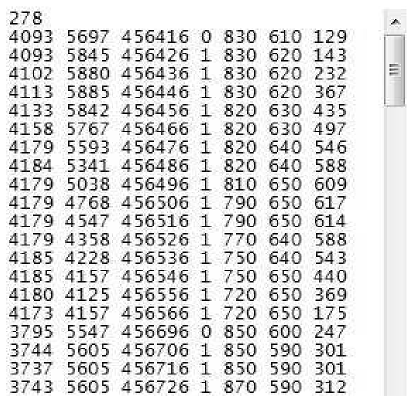
1. INTRODUCTION

Biometric techniques are currently among the most dynamically developing areas of science. They prove their usefulness in the era of very high requirements set for security systems. Biometry can be defined as a method of recognition and personal identification and verification based on physical and behavioural features [1, 2, 3, 13, 16]. Physiological biometrics covers data coming directly from a measurement of a part of human body, e.g. a fingerprint, a shape of face, a retina, etc. Behavioural biometrics analyses data obtained on the basis of an activity performed by a given person, e.g. speech, handwritten signature.

Data collection within a signature recognition process can be divided into two categories: static and dynamic. The static system collects data using *off-line* devices [6, 7]. A signature is put on paper, and then is converted into a digital form with the use of a scanner or a digital camera. In this case, the shape of the signature is the only data source, without the possibility of using dynamic data. On the other hand, dynamic systems use *on-line* devices, which register, apart from the image of the signature, also dynamic data produced during measurement process [5, 6]. The most popular on-line devices are graphics tablets.

The variety of tablets and the parameters registered by them allows analysing many static and dynamic features that characterize a given signature [6, 14, 20]. They can be recorder in the form of a text file. A sample signature is presented in Fig. 1a, while Fig. 1b shows a fragment of the text file containing numerical values describing dynamic features of the registered signature.

a)



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278
4093 5697 456416 0 830 610 129
4093 5845 456426 1 830 620 143
4102 5880 456436 1 830 620 232
4113 5885 456446 1 830 620 367
4133 5842 456456 1 820 630 435
4158 5767 456466 1 820 630 497
4179 5593 456476 1 820 640 546
4184 5341 456486 1 820 640 588
4179 5038 456496 1 810 650 609
4179 4768 456506 1 790 650 617
4179 4547 456516 1 790 650 614
4179 4358 456526 1 770 640 588
4185 4228 456536 1 750 640 543
4185 4157 456546 1 750 650 440
4180 4125 456556 1 720 650 369
4173 4157 456566 1 720 650 175
3795 5547 456696 0 850 600 247
3744 5605 456706 1 850 590 301
3737 5605 456716 1 850 590 301
3743 5605 456726 1 870 590 312
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b)

Fig. 1 a) A graphical representation of an exemplary signature b) text file with a description of the signature generated by the tablet.

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In the first line of the file (Fig. 1b), the number of all registered points (samples) of a given signature is recorded. In the next lines of the file, the following data can be distinguished:

- *X* feature – a set of coordinates of *x* points of the signature,
- *Y* feature – a set of coordinates of *y* points of the signature,
- the time in which, the point with the coordinates (*x*,*y*) was registered,
- the state of the pen (1 - pen touches the tablet surface, 0 - the pen is lifted),
- azimuth - pen rotation in relation to the *Z* axis (clockwise),
- elevation - pen inclination in relation to the surface of the tablet,
- pressure - pressure of the pen on the tablet surface.

The presented method of signature verification is based on an analysis of the *X* and *Y* coordinate values of the signature points registered in the time domain. Thanks to the registration of time, the data from the tablet not only describe the shape of the signature, but also provide information about the manner of writing it. The features registered by the tablet can be presented as a diagram. Figure 2 presents a diagram that shows the dependence of the *X* and *Y* coordinate values of the pen position on the time of their registration.

a)

b)

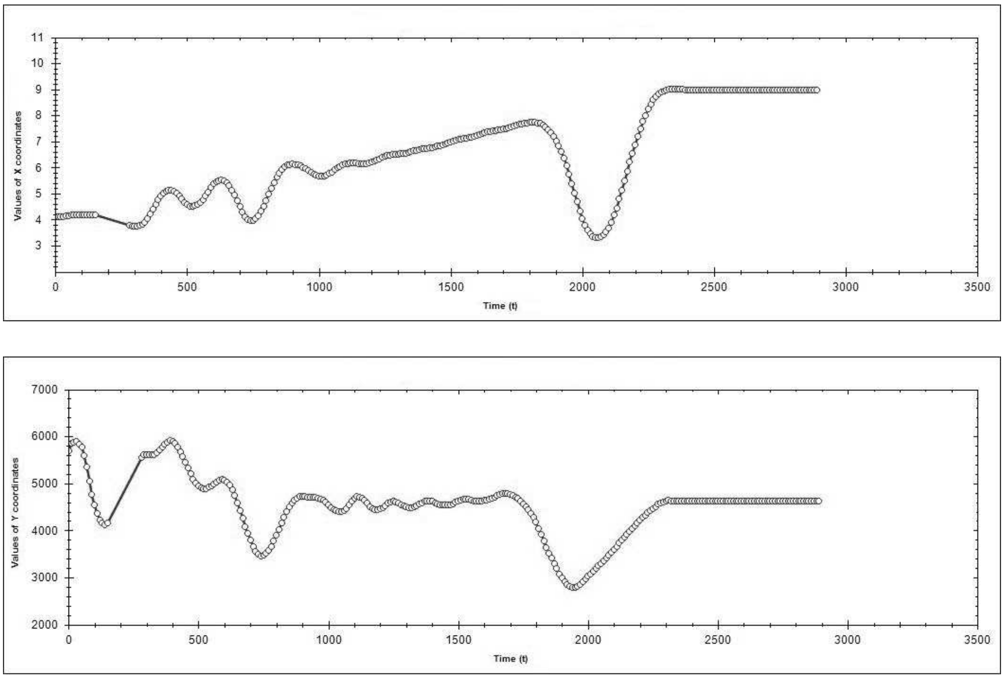

Fig. 2. Courses of a) X feature and b) Y feature in the time domain.

2. MARKOV MODELS AND ESTIMATION OF THEIR PARAMETERS.

Markov statistical methods and hidden Markov modelling were developed and presented in the late 60's and early 70's [4, 15]. However, the popularity of Markov models increased significantly only in the last dozen or so years. Markov models have a very clear mathematical structure, and therefore they provide a theoretical basis for a wide variety of applications [15]. Hidden Markov Models are based on Markov chains.

2.1. MARKOV CHAINS.

Let $Q \neq \phi$ be a finite, non-empty set of states. The state $k_0 \in Q$ is defined as the initial state. The Markov chain is set by the matrix $M = p_{k,l}$, which for $k,l \in Q$ gives the probability $p_{k,l}$ of transition of an object in the system from the state *k* into the state *l*. The matrix M must satisfy the condition [15]:

$$\textit{for each} \quad k \in Q, \qquad \sum_{l \in Q} p_{k,l} = 1. \tag{1}$$

Condition (1) means that M is a stochastic matrix. A Markov chain describes a certain system, which at any given time may only be in one of the $k \in Q$ states. The system is observed in discrete moments of time $t = 0,1,\ldots$. It is assumed that at the beginning, the system is in the initial state k_0. If, at a given moment t the system is in the state k, then at the moment $t+1$ it transitions into the state l with the probability $p_{k,l}$. The main characteristic feature of a Markov chain is the fact that the next state depends only on the current state. At the same time, it does not depend on the value t or on the history of reaching the current state.

2.2. HIDDEN MARKOV MODELS.

Hidden Markov Models (HMM) are an extension of the definition of Markov chains by addition of the alphabet *V* and a sequence of letters of this alphabet, which are emitted in individual states of the model.

In the HMM model, in the state $k \in Q$ the symbol $x \in V$ is emitted with the probability of $e_k(x)$ and the transition into the state l takes place with the probability of $p_{k,l}$. If in each state $k \in Q$ a certain symbol is emitted, it should be assumed that:

$$\sum_{x \in V} e_k(x) = 1. \tag{2}$$

However, a situation, where in certain states, with a certain probability, no symbols from the alphabet are emitted, is admissible. In such a case, a weaker assumption should be adopted:

$$\sum_{x \in V} e_k(x) \leq 1. \tag{3}$$

Hidden Markov Model is characterized by the following parameters:

- number of states of the model *N*,
- the number of the emissions observed (observations),
- the probability of transitions between states ($p_{k,l}$ is possible for some states),
- the probability of emission of a symbol from the alphabet *V*,
- the initial probabilities (determining the current state of the model).

3. SIGNATURES AS HIDDEN MARKOV MODELS.

3.1. SET OF STATES

A technique that describes a signature with the use of a sequence of symbols was employed in this study. A method proposed by G.K. Gupta and R.C. Joyce, described in [4], was used for this purpose, and the results provided the data for Hidden Markov Models. In this method, the generation of adequate symbols is based on the extremes determined on the features diagrams $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ in the time domain $T = \{t_1, t_2, \ldots, t_n\}$. To determine local extremes for the course of a selected feature of the analysed signature, the pen velocity $Vx = \{vx_1, vx_2, \ldots, vx_n\}$ should be determined in the *X* axis direction and the pen velocity $Vy = \{vy_1, vy_2, \ldots, vy_n\}$ in the *Y* axis direction, on the basis of the following formulas:

$$vx_i = \frac{x_{i+1} - x_i}{t_{i+1} - t_i}, \qquad vy_i = \frac{y_{i+1} - y_i}{t_{i+1} - t_i}, \qquad \text{for} \qquad i=1,\ldots,n\text{-}1 \tag{4}$$
$$vx_i = vx_{n-1}, \quad vy_i = vy_{n-1}, \qquad \text{for} \qquad i=n.$$

where:

x_i – the coordinate X of the signature at its i-th point,
vx_i – the instantaneous velocity of the pen in the X axis direction at the i-th point of the signature,
y_i – the coordinate Y of the signature at its i-th point,
vy_i – the instantaneous velocity of the pen in the Y axis direction at the i-th point of the signature,
t_i – the time of registration of the i-th point.

By determining instantaneous velocities at all points of the signature, local extremes can be easily found. To this end, it is enough to indicate the places where a change in the direction of writing takes place:

- change in the sign of velocity from positive to negative → a local maximum was found,
- change in the sign of velocity from negative to positive → a local minimum was found.

Sample diagrams of *X* and *Y* features with extreme points marked are shown in Fig. 3.

a)

b)

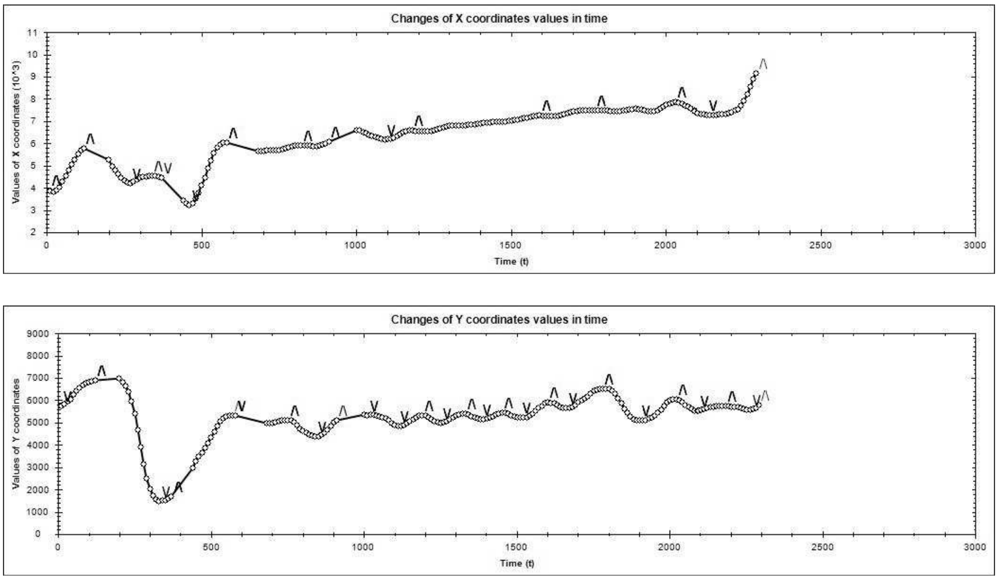

Fig. 3. Extremes in the courses of a) X and b) Y features in the time domain (\/ - minimum, /\ - maximum).

In this study to determine extremes and to describe them with letters the method [4] was used. It has been assumed that the sequences describing a given feature of the signature (*X* or *Y*) consist of the following letters: *P*, *A*, *B*, *C*, where:

- *P* - the moment, when the pen touched the scanning surface of the tablet after a prior interruption in writing.
- Letters *A*,*B*,*C* describe the differences between the previous state (a local extreme or *P*) and the current state (also a local extreme or *P*) in relation to the biggest difference determined for global extremes.

The criteria, according to which the adequate extremes are designated with individual letters, are presented in Algorithm 1.

Algorithm 1. Designation criteria for individual extremes:

Step 1. Count all extremes in the course of the *X* or *Y* feature (depending on which feature the model is built).

$$Ex = \sum Ex_{\max} + \sum Ex_{\min} \;, \tag{5}$$

where:

Ex	–	sum of the all extremes,
$\sum Ex_{\min}$	–	sum of local minimum,
$\sum Ex_{\max}$	–	sum of local maximum.

Step 2. Divide the obtained value by the number of letters that should be assigned. Remember only the integer part.

$$Lex = \left\lfloor \frac{Ex}{k} \right\rfloor, \tag{6}$$

where:

Lex	–	number of extremes described by one letter,
Ex	–	sum of the extremes,
k	–	number of letters, with which extremes are described (excluding P).

Step 3. Select from the set of all extremes the number of the biggest differences between extremes equal to the Lex value determined in **Step 2**.

Step 4. Assign a letter to the selected extremes (in the first iteration C, in the second iteration B).

Step 5. Exclude from the set of search for the biggest differences the extremes, which have already been determined.

Step 6. If all letters from the set (except for A and P) have been allocated, assign the letter A to the remaining undetermined states and end the algorithm. Otherwise go back to **Step 3**.

Courses of features with determined local extremes are shown in Fig. 4

a)

b)

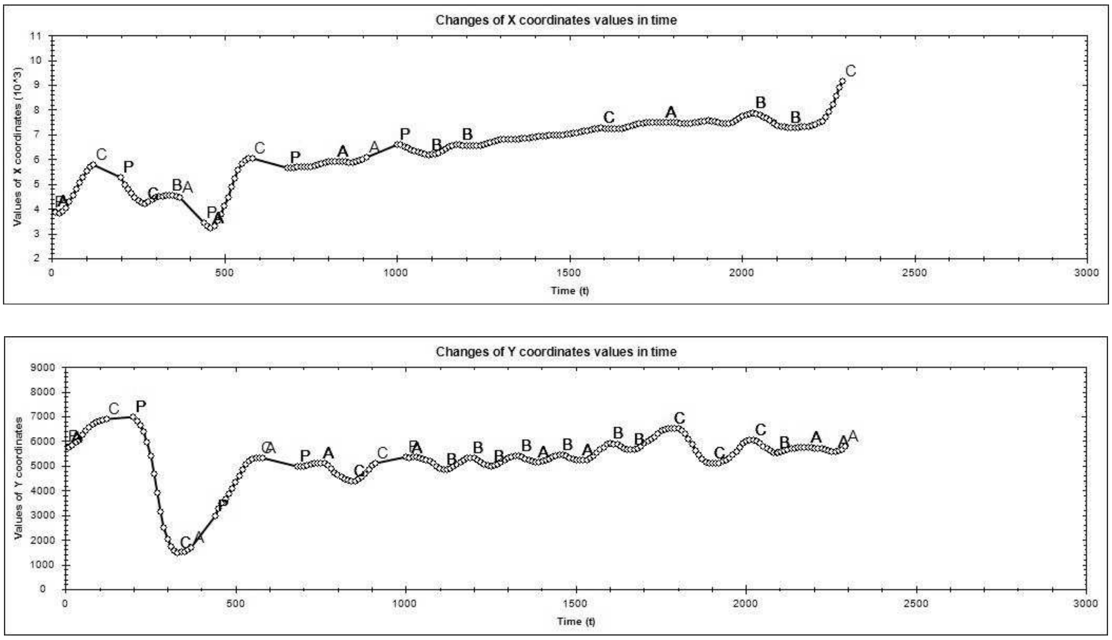

Fig. 4. Courses of features with determined local extremes a) feature X, b) feature Y.

As a result of the operation of the Algorithm 1, there is obtained a sample set of states Q:

$$P - A - C - P - A - C - P - A - A - P - B - B - C - A - B - B - C.$$

The transition matrix is established by determining the number of transitions from a particular state to any other state in relation to the total number of transitions from that state:

$$M = \begin{bmatrix} p_{P,P} & p_{P,A} & p_{P,B} & p_{P,C} \\ p_{A,P} & p_{A,A} & p_{A,B} & p_{A,C} \\ p_{B,P} & p_{B,A} & p_{B,B} & p_{B,C} \\ p_{C,P} & p_{C,A} & p_{C,B} & p_{C,C} \end{bmatrix} = \begin{bmatrix} \frac{0}{4} & \frac{3}{4} & \frac{1}{4} & \frac{0}{4} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{0}{4} & \frac{0}{4} & \frac{2}{4} & \frac{2}{4} \\ \frac{2}{3} & \frac{1}{3} & \frac{0}{3} & \frac{0}{3} \end{bmatrix} = \begin{bmatrix} 0 & 0{,}75 & 0{,}25 & 0 \\ 0{,}2 & 0{,}2 & 0{,}2 & 0{,}4 \\ 0 & 0 & 0{,}5 & 0{,}5 \\ 0{,}67 & 0{,}33 & 0 & 0 \end{bmatrix}$$

3.2. SET OF EMISSIONS.

After defining the set of states Q and the transition matrix M of the Hidden Markov Model, a set of emissions (alphabet V) should be determined. In order to define emissions basing on the pen velocity in the *X* axis direction, mean values of velocity at the extreme points found for the *X* course should be determined:

$$\overline{vx_i} = \frac{vx_{i-1} + vx_i + vx_{i+1}}{3}, \qquad i=2,\ldots,n-1, \tag{7}$$

where:

$\overline{vx_i}$ – mean values of pen velocity in the *X* axis direction at the *i*-th point of signature,

vx_i – pen velocity in the *X* axis direction at the *i*-th point of signature.

On the basis of the values determined, a set of emissions of the Hidden Markov Model can be found. The set of emissions *V* consists of the following letters of the alphabet: *S*, *M*, *L*. They describe the smallest (*S*), medium (*M*) and largest (*L*) values determined by this method. The process of creating a set of emissions is presented in Algorithm 2.

Algorithm 2. The process of creating a set of emissions**.**

Step 1. Count all states, in which the model was, basing on the course of the *X* or *Y* feature (depending on which feature the model is built).

Step 2. Divide the obtained value by the number of letters that should be assigned and remember only the integer part.

$$Lem = \left\lfloor \frac{NS}{k} \right\rfloor, \tag{8}$$

where:

Lem – number of emissions described by one letter,

NS – sum of states determined in **Step 1**,

k – number of letters, with which emissions are described.

Step 3. Select the number of biggest extremes equal to the *Lem* value determined in the **Step 2**.

Step 4. Assign a letter to the selected values (in the first iteration *L*, in the second iteration *M*).

Step 5. Exclude from the search set the values, which have already been assigned.

Step 6. If all letters from the alphabet V (except for S) have been allocated, assign the letter S to the remaining undetermined values and end the algorithm. Otherwise go back to **Step 3**.

The emissions based on the writing speed in the *Y* axis direction (vertical direction), the speeds in *X* and *Y* directions, the pressure, the rate of changing the pressure, and the ratio of the pressure to the writing speed are determined in the same way as when determining the writing speed in the *X* direction, so the formula (7) is also used.

3.3. EMISSION MATRIX.

Having defined the manner of determining the emissions of the Hidden Markov Model, its emission matrix E can be determined. For this purpose, the number of emissions of a given symbol in each state is counted, and the obtained values are divided by the total number of emissions of a given letter.
For the sample sequence of states:

P – A – C – P – A – C – P – A – A – P – B – B – C – A – B – B – C

which was supplemented with the following sequence of observations:

L – M – L – M – S – L – S – S – L – S – M – M – L – S – S – M – S

the number of emissions in individual states is:

$P \to S = 2$	$A \to S = 3$	$B \to S = 1$	$C \to S = 1$
$P \to M = 1$	$A \to M = 1$	$B \to M = 3$	$C \to M = 0$
$P \to L = 1$	$A \to L = 1$	$B \to L = 0$	$C \to L = 3$

Then the number of emissions in individual states is divided by the total number of emissions in a given observation.

$$E = \begin{bmatrix} (P \to S)/7 & (A \to S)/7 & (B \to S)/7 & (C \to S)/7 \\ (P \to M)/5 & (A \to M)/5 & (B \to M)/5 & (C \to M)/5 \\ (P \to L)/5 & (A \to L)/5 & (B \to L)/5 & (C \to L)/5 \end{bmatrix}$$

The E emission matrix for this signature has therefore the following form:

$$E = \begin{bmatrix} e_P(S) & e_A(S) & e_B(S) & e_C(S) \\ e_P(M) & e_A(M) & e_B(M) & e_C(M) \\ e_P(L) & e_A(L) & e_B(L) & e_C(L) \end{bmatrix} = \begin{bmatrix} 0{,}29 & 0{,}43 & 0{,}14 & 0{,}14 \\ 0{,}2 & 0{,}2 & 0{,}6 & 0 \\ 0{,}2 & 0{,}2 & 0 & 0{,}6 \end{bmatrix}$$

The emission matrix defined in such a way is another element of the Hidden Markov Model that describes the analysed signature.
The next stage of the presented method was estimation of HMM parameters with the use of the Baum-Welch algorithm. Additionally, there was used the Viterbi algorithm, which determines the most probable path (sequence of states), which should be passed in a given model to generate a sequence of emissions for a specific signature. It also gives the probability of the occurrence of that path. Detailed descriptions of these algorithms can be found in [15].

4. THE COURSE AND RESULTS OF THE STUDIES.

The studies aiming at determination of the effectiveness of the proposed method were carried out with the use of the SVC2004 signature database [22]. This database contains signatures of 40 users. Each of them put 20 original signatures. During the studies, both original signatures and random forgeries were used. The research procedure consisted of several stages. The first stage included a process of registering the signatures of each user, which consisted in determining the Hidden Markov Models for these signatures. To this end, basing on the *X* and *Y* features of the signature, appropriate matrices of states (transitions) were determined. In the case of the matrix of emissions they have been determined on the basis of the following features:

- writing speed $\overline{V_X}$ in the X axis direction,
- writing speed $\overline{V_Y}$ in the Y axis direction,
- general writing speed $\overline{V_{XY}}$,
- pen pressure $\overline{Pn}$,
- the rate of changing the pressure $\overline{V_{Pn}}$,
- and the ratio of the pressure to the writing speed *R*.

In the proposed method, when creating the matrix of states, very small differences between the extremes (approx. 1%) were ignored already at the time of their determination. This allowed eliminating small changes that resulted rather from tremble of a hand and did not provide important information about a signature. The experiments have shown that ignoring the major differences, at the level of 2% or more, resulted in the loss of important details describing the shape of a signature.

Then the assessment of the effectiveness of the signature verification method was started. For this purpose, a signature selected at random was compared with all twenty signatures of a given user from the database. For each comparison, the obtained value of the probability was compared with the value of the probability for the selected signature determined in the user registration process. If this value was higher, the person was positively verified. EER values for the adopted research methodology were calculated and presented in Table 1.

Table 1. Signature verification errors depending on the analysed behavioural feature and a combination of such features.

Emission matrix determined based on the feature	*EER* [%] The matrix of states determined based on the feature:	
	X	***Y***
$\overline{V_X}$	7	8
$\overline{V_Y}$	8	9,5
$\overline{V_{XY}}$	7,5	7
$\overline{Pn}$	9	8,5
$\overline{V_{Pn}}$	**6,5**	**6,5**
R	11	11,5

On the basis of the results presented in Table 1 it can be concluded that the analysis based on the medium rate of changing the pen pressure was characterized by the smallest EER value. The features associated with the process of putting a signature, such as the velocity of the pen or the time of registering consecutive signature points, are difficult to be forged. Linking the pen velocity characteristics with the pressure of the pen on the base ($\overline{V_{Pn}}$) allows obtaining EER = 6.5%. The results of the study show that such a set of behavioural features is the best among the analysed ones and can be successfully used in a user verification process.

5. CONCLUSIONS

The method described in this paper allows obtaining good results of the classification against the background of other methods known from the literature. A comparison with other methods is presented in Table 2.

Table 2. Comparison of methods of signature recognition.

Method	*EER* [%]	Signature database
Proposed metod	**6,5**	**SVC2004**
Gupta G. K., Joyce R. C. [4]	4,8	Own database (60 users, 1200 signatures)
Lei H., Govindaraju V. [8]	33	SVC2004 database
Li B., Zhang D., Wang K. [9]	1,9	Own database (94 users, 1410 signatures)
Lumini A., Nanni L. [10]	4,5	MCYT database
Maiorana E. [11]	8,3	MCYT database
Nanni L., Lumini A. [12,13]	21	MCYT database
Travieso C. M., Alonso J. B. [17]	12,8	Vargas J. F., Ferrer M. A., MCYT database
Velez J., Sanchez A., Moreno B., Esteban J. L. [18]	12.5	Own database (56 users, 336 signatures)
Wen J., Fang B., Tang Y. Y., Zhang T. [19]	11,4	Own database (55 users, 2640 signatures)
	15,3	MCYT database
Yasuda K., Muramatsu D., Shirato S., Matsumoto T. [21]	4,1	Own database (13 users, 1885 signatures)

In later stages of the study it is planned to extend the set of the analysed features by other features, which have not been used so far. The effectiveness of the method will be determined for other available signature databases. The study will also be extended by identification of persons on the basis of handwritten signatures.

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