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EVIDENCE-BASED MODEL FOR 2-UNCERTAIN RULES AND INEXACT REASONING

In empirical sciences, among others – in medicine, domain data – stored in different repositories – are the most important source of domain information. There is a great number of methods, including semantic data integration, that enable to acquire domain knowledge from such data and express it in a convenient form. In the paper we propose a model for rules with uncertainty (2-uncertain rules) that can be obtained from somewhat heterogeneous data, written in a common format of tuples. The rules are uncertain implications, with complex premises and single conclusions, and two specific reliability factors. In addition, we propose functions for propagating uncertainty through reasoning chains in Rule-Based Systems (RBSs) with such rules in their knowledge base.

1. INTRODUCTION

In the last decades, it has been observed a growing interest in the field of machine learning. By applying inductive learning to real data stored in various data warehouses, it is possible to acquire and structuralize valuable domain knowledge. Much of this knowledge is represented declaratively, by means of rules. We mean here as well decision rules as production rules. In general, a rule determines the conditional dependence of some fact (conclusion) on another ones (conjunction of premises). Hardly ever the dependence is an absolute one, more often it happens only with some frequency (first order probability). Depending on the number and provenance of source data, we can also say about the reliability of the rule as a whole (second order probability) [2].

In the paper we propose a model for 2-uncertain rules. It is based on the interpretation of uncertainty as a kind of evidential, second order probability. The model is based on a classical implication, provided with two reliability factors:

internal (irf), stating the level of the dependence of a rule's conclusion on rule's premises,

external (grf), stating the quality of the underlying rule.

If factor irf has its equivalents in most of models of uncertainty, then factor grf can be found in a clear form only rarely. Its specific counterparts occur in well-known D-S [8] model (span of the range IR) and statistic research (span of confidence interval CI). In both cases, in order to estimate the correctness of the value of interest (belief value or risk value), one needs to calculate the proportion between this value and the span of an appropriate range of uncertainty. Some other proposals of calculating the rule's quality can be found, among others, in [11, 12]. A specific metrics of quality, in fact - fuzzy metrics, is given by Zadeh [13] to his Z numbers. We would like to use factor grf of rule's quality for ordering rules in the agenda and, next, for differentiating the influence of successively fired rules on the final results of reasoning.

The proposal of 2-uncertain rules is comprehensive. First, the syntax and semantics of the rules are defined. They are general enough to express also 2-uncertain facts. The definition is an extension of the previous one, published in [4]. In the new version, we added the possibility to write rule's components (premises, conclusion) in the negated form. The most important and completely new in the paper is a set of combination functions for propagating uncertainty. The functions, attached to the RBS inference engine, allow to correctly reason with the proposed 2-uncertain rules, as well by means of forward chaining as by means of backward chaining [3, 7]. It is proven that they do not violate the semantic constraints imposed on the knowledge base, in particular – on the reliability factors of its rules and facts.

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The new model is called as 'evidence based' because of the recommended method of acquiring 2-uncertain rules from data stored in repositories. An exemplary algorithm for designing 2-uncertain rules from data of attributive representation is given in [4]. A detailed proposal of calculating the values of factors irf and grf has been put forward in [5]. In [9] it is demonstrated how to make use of 2-uncertian rules in medical diagnostics.

2. 2-UNCERTAIN RULE – SYNTAX AND SEMANTICS

In the light of the above considerations, let us propose the following, formal and useful from the practical point of view, form of 2-uncertain rule:

it is declared with grf (p_r) :

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \quad C \text{ with irf } (p_c), \tag{1}$$

where P_1, P_2, …, P_n stand for the rule's premises; C – its conclusion; grf and irf – names of reliability factors, relating to: conclusion C given certain occurrences of all premises P_1, P_2, …, and P_n, and the considered rule itself given a set of distributed domain data, respectively; p_r and p_c – values of reliability factors grf and irf, respectively, such that $0 < p_r, p_c \le 1$. The rule's premises and its conclusion have to be formulas of the form:

$$A \in \{v_1, v_2, \ldots, v_m\} \text{ or} \tag{2}$$
$$\neg(A \in \{v_1, v_2, \ldots, v_m\}) \text{ or} \tag{3}$$
$$\{v_1, v_2, \ldots, v_m\} \subseteq A \text{ or} \tag{4}$$
$$\neg(\{v_1, v_2, \ldots, v_m\} \subseteq A)\ , \tag{5}$$

where A stands for one of the domain data attributes, and v_1, v_2, …, v_m stand for values coming from the domain over which attribute A ranges. In order to unify these four forms, let us make for the formulas the following notational agreement:

$$A \blacksquare \{v_1, v_2, \ldots, v_m\} q_A \tag{6}$$

where $\blacksquare \in \{=, \neq\}$ stands for one of the two relational symbols of equality; $q_A \in \{\odot, \oplus\}$ gives the formula an interpretation: for q_A being $\oplus$ and $\blacksquare$ being = – the interpretation (2), for q_A being $\oplus$ and $\blacksquare$ being ≠ – (3), for q_A being $\odot$ and $\blacksquare$ being = – (4), for q_A being $\odot$ and $\blacksquare$ being ≠ – (5). The using in (6) a set of attribute values instead of one individual value increases expressive power of the formula, and next – of the rule (1) as a whole. Obviously, such an extension entails the necessity of using an expanded Truth Maintenance System (TMS). The admission to use in (6) both relation = and its negation ≠ means that the rules of the form (1) describe the reality based on the open world assumption (the lack of knowledge on holding the relation is not equivalent to non-holding the relation).

A tacit assumption of our model is that a rule expresses a positive monotonic dependence between its premises and conclusion. It can be summarized as follows: the lower the level of fulfillment of the premises, the lower the level of fulfillment of the conclusion. The both reliability factors used in the rule (1) declare the level of some evidential probability, also called subjectivist probability. Whereas factor irf represents an internal conditional probability (dependence of the rule's conclusion on its premises), then factor grf – an external conditional probability (dependence of the rule itself on the evidence from which it has been derived). Factor irf has a significance influence on the probabilities of conclusions being derived in the process of reasoning. For a change, factor grf influences mainly the quality of derivations of the conclusions. However, by determining the order of firing active rules from agenda, it influences – slightly and indirectly – also on the probabilities of the conclusions.

The format (1) is to be used also for representing 2-uncertain facts. Such a fact, written in short as follows:

$$\text{with grf } (p_r) : C \text{ with irf } (p_c) \tag{7}$$

where C, grf, irf, p_r and p_c preserve their syntax and semantics from definition (1), is practically an instance of a specific rule, with the premise equal true:

$$\begin{gathered}\textit{it is declared with grf}\,(p_r):\\ \textit{true} \rightarrow\\ C\ \textit{with irf}\,(p_c)\,.\end{gathered} \tag{8}$$

An important constraint assumed in the model is the one imposed on the values p_c and p_r:

$$BEL_c \leq p_c \leq PLS_c;\ BEL_r \leq p_r \leq PLS_r\,. \tag{9}$$

It means that experimentally determined values p_c and p_r should come from fixed ranges. Let us observe that facts can change their factors (both grf and irf) during reasoning in a RBS with 2-uncertain rules. At the beginning, before starting reasoning, the fact's p_r is axiomatically assumed to be 1 (it does not depend on the contents of given repositories). Next, as the reasoning proceeds, the value of p_r gradually decreases. Differently, the value of p_c can change in both directions. We will demand from p_c and p_r that they satisfy the constraint (9) along the whole reasoning process. In Section 3, we will define a set of functions for propagating uncertainty through reasoning chains in a RBS with 2-uncertain rules. We will prove that these functions do not violate the requirement (9).

If domain data are stored in an attributive form, then one can derive from them a set of rules of the form (1) that fulfill some elementary criteria of semantic correctness. This possibility has been shown in [4], where an algorithm of semantic data integration and designing 2-uncertain rules has been proposed. Now, one more question should be answered: is the format (1) optimum for 2-uncetain rules? In particular, would it be possible to increase the usefulness of the rules by enabling disjunctions of premises or conjunctions/disjunctions of conclusions? Or, by extending the set of operators used in formulas defining rules' premises and conclusions? Let us observe that a compound rule of the form: it is declared with grf (p_r) : $P_1 \vee P_2 \rightarrow C$ with irf(p_c) can be replaced by means of the equivalent pair of simple rules: it is declared with grf (p_r) : $P_1 \rightarrow C$ with irf(p_c) and it is declared with grf (p_r) : $P_2 \rightarrow C$ with irf(p_c). Such a replacement cannot be done for a compound conclusion: neither disjunction, nor conjunction of simple conclusions distributes over reliability factor irf. Thus, compound conclusions would be semantically justified, but they would result in an exponential growth in the number of investigated hypotheses and, as a consequence, an essential growth of computational complexity.

Let us observe that operators used in formulas of premises and conclusions of the rule (1) constitute a right subset of the set used in Attributive Logic with Set Values over Finite Domains (ALSV(FD)) [6]. The remaining ALSV(FD) operators are not used either due to low applicability of rules with those operators (e.g. operator =), or due to difficulties in implementation (e.g. operator ~).

Yet, let us briefly comment the role of the operator of logical negation ¬. Having enabled to use it in formulas of the rule (1), we have increased the expressive power of our method of knowledge representation. On the other hand, due to the obligatory condition (9) and the obvious requirement $\mathrm{PLS}(\neg C) = 1 - \mathrm{BEL}(C)$, its usage would normally result in the necessity of performing additional tests – for all the hypotheses considered while reasoning – by the module TMS. A satisfactory solution of this difficulty will be proposed in Section 3.

The above remarks show that the set of operators for usage in the rule (1) should be selected very carefully. Among others, one should take into account the existence of attributes ranging over wide domains. Due to such attributes, the number of possible hypotheses HYP can be really great. We calculate it from the following formula:

$$HYP = 2^{|D_{A1}|+1} + 2^{|D_{A2}|+1} + \cdots + 2^{|D_{An}|+1}\,, \tag{10}$$

where n stands for the number of considered attributes, A_i – an attribute of the ordinal number i $(1 \leq i \leq n)$, D_{Ai} – the attribute's domain, and $|D_{Ai}|$ – the cardinality of this domain (1's in the exponents are an effect of using formulas in both positive and negative form).

3. FUNCTIONS FOR PROPAGATING UNCERTAINTY THROUGH REASONING CHAINS

In order to correctly reason in RBSs with 2-uncertain rules, one should precisely define the influence of the both reliability factors on the course and the final result of reasoning. Therefore, for the case of forward chaining [3, 7], we assume the agenda (a set of rules that are active at the current step of reasoning) to consist of all these and only these rules that satisfy the condition:

$$min(grf_{P1}, grf_{P2}, \dots, grf_{Pn}, p_r) \geq \tau_1; \quad min(irf_{P1}, irf_{P2}, \dots, irf_{Pn}) \geq \tau_2, \tag{11}$$

where irf_{Pi} and grf_{Pi}, $1 \leq i \leq n$, stand for current values of reliability factors of premise P_i, and τ_1, τ_2 – for thresholds of reliability that are necessary for rule firing. Conflicts in the agenda will be resolved in favor of rules of highest priorities, and the dependency between the rule's priority and the rule's factor grf will be set as monotonically increasing. The complete process of forward chaining, starting at some initial state of the system's database (a set of axiomatic 2-uncertain facts), finishes after coming to an empty agenda. Because of the proposed method of resolving conflicts in the agenda, we can let the forward chaining process stop earlier – just after firing a desired number of rules. Obviously, the sets of hypotheses obtained after these two processes of reasoning can differ from each other. First, the set obtained after the shortened process of reasoning may not contain some hypotheses obtainable after the complete process of reasoning. Next, in pairs of corresponding hypotheses from the two sets, the factors grf or irf can differ from each other.

Let us assume that ne stands for one of the two non-equality operators (≤, ≥), and $\mathrm{i}(ne)$ stands for the reverse of operator ne. In case of backward chaining [3, 7], the proof of the goal: (it is declared with grf(p_{r1}) : true → C_1 with irf (p_{c1}), ne) starts from choosing one of the rules whose conclusion C_2 with irf(p_{c2}) satisfies the requirement: $\big((C_2 = C_1) \wedge (p_{c2}\, ne\, p_{c1})\big) \vee \big((C_2 = \neg C_1) \wedge (p_{c2}\, \mathrm{i}(ne)\, p_{c1})\big)$, and whose reliability factor grf satisfies the inequality: $p_{r2} \geq p_{r1}$. Again, the first place should be given to this one of all the matching rules which has the greatest priority. The described procedure should be continued - as a search with backtracking - for the premises of the appointed rule, next for the premises of the subsequent rules, and so on. The whole process will be finished after proving the goal (successfully), or after searching the whole space of potential solutions (with failure).

When forward chaining, the uncertainty will be propagated in the system according to a few principles, having the form of functions similar to those in [1] and [10]. These are: an opposite function and three combination functions, intended for:

- propagating uncertain evidence,
- operating on complex conjunctive hypotheses,
- managing with multiple production rules.

We will shortly discuss these functions in the successive subsections. In order to precisely explain their semantics, we introduce an additional notion of 'virtual rule', admitting a conjunctive form of the conclusion. Virtual rules will be used only when necessary. Speaking shortly 'rule', we continue to mean 2-uncertain rule of the form consistent with the definition (1). Only such rules can be stored in the system's knowledge base.

The opposite function. In order to make the database of our RBS as homogeneous as possible, we will restrict 2-uncertain facts stored in the database to positive only, i.e. the ones based on the formulas (2) and (4). However, since our rules can contain also premises and conclusions in negative form (based on the formulas (3) and (5)), we have to know how to derive the necessary negative premises and how to process the obtained negative conclusions. The both problems will be solved by using the opposite function of during reasoning. The function is defined over the set of 2-uncertain facts, and its calculation for the following fact R_i:

$$R_i = it\ is\ declared\ with\ grf\,(p_{ri}) : true\ \rightarrow C_i\ with\ irf\,(p_{ci}) \tag{12}$$

is being performed according to the formula:

$$of(R_i) = R_{of} = \text{it is declared with grf } (p_{rof}) : \text{true } \rightarrow C_j \text{ with irf } (p_{cof}), \quad (13)$$

where $C_j = \neg C_i$, $p_{rof} = p_{ri}$, and $p_{cof} = 1 - p_{ci}$. By the assumption (9), we have: $BEL_{ci} \leq p_{ci} \leq PLS_{ci}$. In this case: $(1 - PLS_{ci}) \leq p_{cof} \leq (1 - BEL_{ci})$ and, in view of the obvious equalities: $BEL_{cof} = (1 - PLS_{ci})$ and $PLS_{cof} = (1 - BEL_{ci})$, we obtain the expected double inequality: $BEL_{cof} \leq p_{cof} \leq PLS_{cof}$. From the other hand, in view of the equalities $BEL_{rof} = BEL_{ri}$ and $PLS_{rof} = PLS_{ri}$, the constraint $BEL_{rof} \leq p_{rof} = p_{ri} \leq PLS_{rof}$ is satisfied. The domain of the function of cannot be extended to the full set of 2-uncertain rules; such an extension would result in violating the positive monotonic dependence between rule's premises and its conclusion.

The function of will be called every time after obtaining a fact (R_i) in negative form (C_i of the form (3) or (5)). Then, if the system database does not contain a fact with conclusion $C_j = \neg C_i$, then the result R_{of} will be physically stored in it. If the database contains an axiomatic fact with conclusion C_j, then R_{of} will have no influence on its state, otherwise - R_{of} will be subjected to the operation of function mr defined below. Besides, the function of will be called when a negative premise for a rule intended for firing is necessary. Obviously, a negative fact R_{of} will not be considered for storing in the database.

The combination function for propagating uncertain evidence. When forward chaining, it often happens that conclusion C_i, assumed/derived in step i by means of virtual fact R_i:

$$\text{it is declared with grf } (p_{ri}) : \text{true } \rightarrow C_i \text{ with irf } (p_{ci}) \quad (14)$$

becomes a premise of rule R_j, to be fired in step j such that $j > i$:

$$\text{it is declared with grf } (p_{rj}) : P_j \rightarrow C_j \text{ with irf } (p_{cj}) \quad (15)$$

where $P_j = C_i$. Then, immediately after having fired rule R_j, one should call the combination function ue for propagating uncertain evidence. Its calculation for the above arguments R_i and R_j will be performed according to:

$$ue(R_i, R_j) = R_{ue} = \text{it is declared with grf } (p_{rue}) : \text{true } \rightarrow C_j \text{ with irf } (p_{cue}) \quad (16)$$

where $p_{rue} = \min(p_{ri}, p_{rj})$, and $p_{cue} = p_{ci} \cdot p_{cj}$, giving the fact R_{ue} (similarly as we did it for the opposite function op, also for the proposed function ue and the next combination functions ch and mr, one can easily prove that they calculate irfs and grfs fulfilling the requirement (9); because of lack of place, the proofs are here omitted). Let us remark that, according to expectations, together with the increase of reasoning chains both internal (irf) and global (grf) reliability factors of successively generated facts decrease.

Next, in case C_j has negative form (3) or (5), the obtained fact R_{ue} should be subjected to the operation of the function of. Otherwise, if the system database does not contain a fact with conclusion C_j - then result R_{ue} should be physically stored in it; if the database contains an axiomatic fact with conclusion C_j – then R_{of} has no influence on its state; if the database contains a non-axiomatic fact with conclusion C_j – then R_{of} should be subjected to the operation of function mr defined below. The proposed combination function ue harmonizes well with the assumption on positive monotonic dependence between rule's premises and its conclusion.

The combination function for complex conjunctive hypotheses. In order to correctly propagate uncertainty through reasoning chains, also a method for evaluating reliability factors for conjunctions of premises is necessary. For this purpose, we provide the combination function ch for complex conjunctive hypotheses. Having obtained virtual fact R_i of the form (14), and then fact R_j of the form:

$$R_j = \text{it is declared with grf } (p_{rj}) : \text{true } \rightarrow C_j \text{ with irf } (p_{cj}) \quad (17)$$

and such that $C_j \neq C_i$ and $C_j \neq \neg C_i$, one can apply function ch. Its calculation for R_i and R_j will be performed according to the formula:

$$ch(R_i, R_j) = R_{ch} = \text{it is declared with grf } (p_{rch}) : \text{true } \rightarrow C_i \wedge C_j \text{ with irf } (p_{cch}), \tag{18}$$

where $p_{rch} = \min(p_{ri}, p_{rj})$, and $p_{cch} = \min(p_{ci}, p_{cj})$. The formula (18) declares that the factor irf of a compound conjunctive hypothesis should be calculated as the minimum of all factors irf assigned to the component hypotheses. Similarly, the factor grf of the derivation as a whole should be calculated as minimum of all factors grf assigned to the derivations of the component hypotheses. The function $\mathrm{ch}(R_i, R_j)$ will be called only if necessary, i.e. if a rule intended for firing has as its premises both C_i and C_i. An obtained virtual fact R_{ch} will not be physically stored in the system knowledge base.

The combination function for multiple production rules. The most difficult to estimate is the probability of a conclusion that can be derived from the given evidence in many different ways. Each such a way can be represented by a particular reasoning chain, having at the first item - the formula true, and at the last item - the underlying conclusion. The probability of the conclusion should be the resultant of reliability factors irf obtained in all those derivations. Remind that the order of firing rules depends in our RBS on the rules' priorities, and these priorities depend monotonically on the rules' reliability factors grf. Let us agree that the influence of multiple rules concluding the same conclusion on the factor irf of its hypothesis should be differential and decreasing with the course of reasoning. It was proposed in [5] to differentiate it by means of weight v_i that is dependent on: the number d of all multiple rules (the greater the number, the smaller the weight) and the relative position i of the rule (the earlier the position, the greater the weight). The weights can be calculated from the following system of equations:

$$\begin{cases} \frac{v_i}{v_{i-1}} = t, \quad for\ 2 \leq i \leq d, \\ v_1 + v_2 + \cdots + v_d = 1, \end{cases} \tag{19}$$

where d stands for the number of multiple rules, v_i – the weight of that one from among all multiple rules which was obtained as i-th from them, t – a constant of proportion between the weights of reliability factors irf of two successive multiple rules. It is assumed that t should be greater than 1 (e.g. $t = 1.1$, and $t = 2$ to make slight and significant differentiation, respectively, in the weights of multiple rules).

Let us now define the function mr for multiple rules concluding the same conclusion. Assume that R_i stands for fact (12) in positive form (C_i of the form (2) or (4)), and R_j stands for fact (17) such that $C_j = C_i$, and the both facts have been obtained: R_j - in the current step of reasoning j, as n-th ($2 \leq n \leq j$) such fact with conclusion C_i, and R_i - in a preceding step of reasoning i, $i < j$, as $(n-1)$-th such fact with conclusion C_i. The constraints imposed on facts R_i and R_j mean that, during the considered reasoning process, none fact R_k with conclusion $C_k = C_i$ and reliability factor grf(p_{rk}) such that $p_{rk} \in \langle p_{ri}; p_{rj})$ has been obtained. The pair (R_i, R_j) is an argument for function mr, whose calculation should be performed according to the formula:

$$mr(R_i, R_j) = R_{mr} = \text{it is declared with grf } (p_{rmr}) : \text{true } \rightarrow C_i \text{ with irf } (p_{cmr}), \tag{20}$$

where $p_{rmr} = \min(p_{ri}, p_{rj})$; $p_{cmr} = (1 - v_n) \cdot p_{ci} + v_n \cdot p_{cj}$, and v_n stands for the weight of fact R_j, that was obtained as n-th fact with conclusion C_i. The value v_n should be calculated based on index n, according to the formula (19). The function $\mathrm{mr}(R_i, R_j)$ is to be called just after having obtained fact R_j. The resultant fact R_{mr} is in positive form (C_i of the form (2) or (4)) and, as such, it will be physically stored in the system knowledge base, replacing fact R_j there.

In fact, forward chaining can be seen as the dynamic process of building a virtual derivation graph. In our RBS, it is implemented as a process with a kind of backtracking: there is no possibility to delete or modify once obtained conclusions, but it is possible to repeatedly correct once estimated values of their factors irf and grf. Thus, the derivation graph is being built incrementally: once added nodes and edges cannot disappear; at most, the nodes are slightly modified. In the end, each hypothesis from the graph has its factor irf equal to a weighted-average of all irfs obtained in the multiple rules concluding its conclusion.

For a change, backward chaining can [3, 7] be seen as the process of searching an appropriate path in a virtual derivation graph. It is implemented as a process with full backtracking. Here, the uncertainty is propagated in the opposite direction, from a fact-hypothesis and required values of its factors **irf** and **grf**, to facts-axioms that are physically stored in the system knowledge base. In our RBS, the propagation will be performed by means of an extended opposite function (eo) and two next combination functions, intended for:

- back-propagating uncertain evidence (bu),
- operating on complex conjunctive conditions (cp).

The mentioned functions differ from their counterparts from forward chaining as well in domains as in codomains. The next difference lies in that their results (facts obtained and examined while backward chaining) are not being stored in the database, and are not subjected to the constraints (9). Because of lack of place, the definitions are here omitted.

4. TWO EXAMPLES OF INEXACT REASONING

Let us now illustrate the above considerations by means of medical examples of the both methods of reasoning, first – forward chaining, next – backward chaining [3, 7].

Assume that the thresholds τ_1 and τ_2 of reliability that are necessary for firing rules are set to 0.7 and 0.5, and the constant t of proportion between the weights of successive irfs is equal to 1.5. Assume we have a RBS with 2-uncertain rules to support general medical diagnostics. Let its knowledge base contain the following axiomatic 2-uncertain facts of the patient's state of health:

R_1 = it is declared with grf (1) :
true → General_Diagnosis = {asthma}⊕ with irf (0.7)
R_2 = it is declared with grf (1) :
true → Current_Health_State = {asthma_attack}⊕ with irf (0.3)

and the following 2-uncertain rules of the domain:

R_3 = it is declared with grf (0.8) :
General_Diagnosis = {asthma}⊕
→ Symptoms = {wheezing, shortness_of_breath} ⊙ with irf (0.8)
R_4 = it is declared with grf (0.7) :
Symptoms = {wheezing, shortness_of_breath} ⊙
→ Diagnosis = {bronchitis}⊕ with irf (0.6)
R_5 = it is declared with grf (0.9) :
Symptoms = {wheezing, shortness_of_breath} ⊙ ∧
¬Current_Health_State = {asthma_attack}⊕
→ Diagnosis = {bronchitis}⊕ with irf (0.7) .

At the beginning, from among these three rules, only R_3 will enter the agenda. As a result of firing, it will add a new fact R_6 to the knowledge base:

R_6 = it is declared with grf (0.8) :
true → Symptoms = {wheezing, shortness_of_breath} ⊙ with irf (0.56) .

Its reliability factors, obtained by means of using the function ue, are as follows: $\mathrm{grf}(0.8) = \min(1.0, 0.8)$, and $\mathrm{irf}\,(0.56) = 0.7 * 0.8$.

Now, the two remaining rules fulfill the conditions (11), but R_5 has greater priority than R_4 ($0.9 > 0.7$). However, in order to become fully active, it needs, first – determining the value of function op for fact R_2, and next – determining the value of function ch for rule R_6 and the newly obtained rule R':

$R' = \mathrm{op}(R_2)$ = it is declared with grf (1) :
true → ¬(Current_Health_State = {asthma_attack}⊕) with irf (0.7)
$R'' = \mathrm{ch}(R_6, R')$ = it is declared with grf (0.8) :
true → (Symptoms = {wheezing, shortness_of_breath} ⊙ ∧
¬Current_Health_State = {asthma_attack}⊕) with irf (0.56) ,

where $\mathrm{grf}(0.8) = \min(1.0, 0.8)$, and $\mathrm{irf}\,(0.56) = \min(0.56, 0.7)$.
The obtained fact R' is in negated form, and the obtained fact R'' is a virtual one, and – as such – they will not be stored in the system knowledge base.

Due to the absence of a fact with conclusion Diagnosis = {bronchitis}⊕ in the system knowledge base, the firing of rule R_5 will result in adding the following fact R_7 to this base:

R_7 = it is declared with grf (0.8) :
 true → Diagnosis = {bronchitis}⊕ with irf (0.39) ,

where $\mathrm{grf}(0.8) = \min(0.8, 0.9)$, and $\mathrm{irf}\,(0.39) \approx 0.56 \cdot 0.7$.

The firing of the last active rule R_4 will entail calling the function mr and, as a consequence, the replacement of R_7 in the system knowledge base by the following fact R_{7-1}:

R_{7-1} = it is declared with grf (0.7) :
 true → Diagnosis = {bronchitis}⊕ with irf (0.37) ,

where factor $\mathrm{grf}(0.7) = \min(0.8, \min(0.8, 0.7))$, and factor $\mathrm{irf}(0.37) \approx$
$(1 - 1/(1.5 + 1)) * 0.39 + 1/(1.5 + 1)) * (0.56 * 0.6)$.

Assume again the primary contents $R_1 - R_5$ of the system knowledge base. Let us now answer the following uncertain question H_1:

H_1 = (it is declared with grf (0.7) :
 true → ¬(Symptoms = {wheezing, shortness_of_breath} ⊙) with irf (0.1), ≤) .

The only rule with conclusion Symptoms = {wheezing, shortness_of_breath} ⊙ is R_3. Considering a positive form of the conclusion in R_3, the RBS will use the extended opposite function eo to obtain the complement of question H_1. It will be as follows:

H' = (it is declared with grf (0.7) :
 true → Symptoms = {wheezing, shortness_of_breath} ⊙ with irf (0.9), ≥) .

The calling of function eo is synchronized with the reversal of the constraint imposed on factor irf. Next, the inference engine concentrates on premise General_Diagnosis = {asthma}⊕ of rule R_3. By using function bu for back-propagating uncertain evidence, it makes the following uncertain question H'':

H'' = (it is declared with grf (0.7) :
 true → General_Diagnosis = {asthma}⊕ with irf (1.0), ≥) ,

where 0.7 is the smallest acceptable value of factor grf of the fact from H'', and $1.0 = \min(1.0,\ 0.9/0.8)$ – the smallest acceptable value of its factor irf. Let us observe, that fact R_1 fulfills the constraint imposed on grf but does not fulfill the constraint imposed on irf. As a result, the initial uncertain question H_1 will be answered with no, that is compatible with the result obtained in the process of forward chaining.

5. CONCLUSIONS

The proposed 2-uncertain rule model can be a base for construction of a RBS with uncertainty, in particular – a RBS to support medical diagnostics. In general, diagnostic efforts can be classified in two categories:

- initial efforts, that consist in drawing a general picture of patient's condition (it is created based on the anamnesis – if possible, and some physical examinations, e.g. blood pressure measurement, temperature measurement, bronchial auscultation, heart auscultation),
- advanced efforts, that consist in verifying different diagnostic hypotheses.

In the first case, a RBS with 2-uncertain rules can be used to generate as much as possible medical conclusions from the data acquired while initial diagnosing. In order to do this, the system should operate in the mode of forward chaining, with an optional constraint on either the threshold of reliability of the conclusions (factors grf), or the maximum number of the conclusions (remark, that successively generated conclusions have their grfs smaller and smaller). The conclusions will be stored together with their probabilities (factors irf) estimated by the functions for propagating uncertainty. From a medical point of view, the most interesting and important will be conclusions with very small (close to 0) and very great (close to 1) values of factor irf.

For a change, in case of the advanced diagnostics, the RBS can be used to advise with a concrete hypothesis: is it no less (no more) probable than we would expect it to be? In this case, the system should operate in the mode of backward chaining, in cooperation with the functions for back-propagating of uncertainty.

The proposed in the paper strategy for qualification 2-uncertain rules to the agenda is a simple one. On request, it can be modified in any way. Similarly, the proposed algorithm for resolving conflicts in the agenda can be replaced by a more sophisticated one. For instance, apart from the factor grf of the rule as a whole, it can also take factors grf of its premises into account.

Regardless of its operational mode, our RBS to support medical diagnostics will use still the same knowledge base. The base will consist of thousands of 2-uncertain rules, that were induced from the real medical evidence. In the paper, we did not raise the problem of the base quality, which is very important for an effective and efficient system operation. As it was proposed in [4], the knowledge base – with concern for its quality – will be refined from the pairs of contradictory rules and from all rules subsumed by any other ones to be found in it. The refinement will be performed at once during the construction of the base.

The practical advantages and practical difficulties of using 2-uncertain rules are widely illustrated in [9].

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