JOURNAL OF MEDICAL INFORMATICS & TECHNOLOGIES Vol.3/2002, ISSN 1642-6037

image enhancement,
noise reduction,
anisotropic diffusion

Bogdan SMOLKA*

ON THE APPLICATION OF THE FORWARD AND BACKWARD DIFFUSION SCHEME FOR IMAGE ENHANCEMENT

In this paper a novel approach to the problem of edge preserving smoothing is proposed and evaluated. The new algorithm is based on the combined forward and backward anisotropic diffusion with incorporated time dependent cooling process. This method is able to efficiently remove image noise, while preserving and enhancing its edges.

1. ANISOTROPIC DIFFUSION

Perona and Malik [1] formulate the anisotropic diffusion filter as a process that encourages intraregional smoothing, while inhibiting interregional denoising. The Perona-Malik (P-M) nonlinear diffusion equation is of the form [1-5]:

$$\frac{\partial}{\partial t} I(x,y,t) = \nabla\left[c(x,y,t)\nabla I(x,y,t)\right], \tag{1}$$

where $\mathbf{I}(x,y,t)$ denotes the color image pixel at position (x,y), t refers to time or iteration step in the discrete case and $c(x,y,t)$ is a monotonically decreasing conductivity function, which is dependent on the image gradient magnitude $c(x,y,t) = f\left(\|\nabla \mathbf{I}(x,y,t)\|\right)$ such as:

$$c_1(x,y,t) = \exp\left\{-\left(\frac{\|\nabla I(x,y,t)\|}{\beta}\right)^2\right\}, \qquad c_2(x,y,t) = \left\{1+\left(\frac{\|\nabla I(x,y,t)\|}{\beta}\right)^2\right\}^{-1} \tag{2}$$

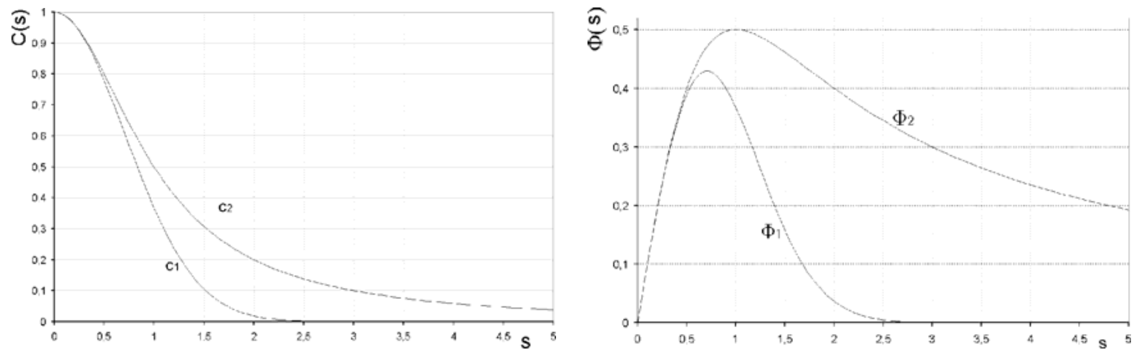
which were introduced in the original paper of Perona and Malik [1]. The parameter β is a threshold parameter, which influences the anisotropic smoothing process. Using the notation $g = \|\nabla I(x,y,t)\|$, $s=g/\beta$, where $\|\cdot\|$ denotes the vector norm, we obtain following formulas for the conductivity functions: $c_1(x,y,t)=\exp(-s^2)$, $c_2(x,y,t)=1/(1+s^2)$. It is evident that the behavior of the anisotropic diffusion filter depends on the gradient threshold parameter β. To show the influence of β it is

* Silesian University of Technology, Department of Automatic Control, Gliwice, Poland

helpful to define a flux function: $\Phi(x,y,t)=c(x,y,t)\,\|\nabla I(x,y,t)\|$. With the flux function defined above, Eq. 1 can be rewritten as $\frac{\partial}{\partial t} I(x,y,t) = \nabla \Phi(x,y,t)$. The flux functions $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ corresponding to conduction coefficients c_1 and c_2 are shown in Fig. 1.

As it is easy to notice in Fig. 1b, the flow increases with the gradient strength to reach a maximum and then decreases slowly to zero. This behavior implies that the diffusion process maintains homogenous regions since little smoothing flow is generated for low image gradients.

In the same way, edges are preserved because the flow is small in regions where the image gradient is high.



a) b)

Fig.1. Dependence of the conductivity functions c_1 and c_2 (**a**), and the respective flux functions Φ_1 and Φ_2 (**b**), on the value of the normalized image gradient s.

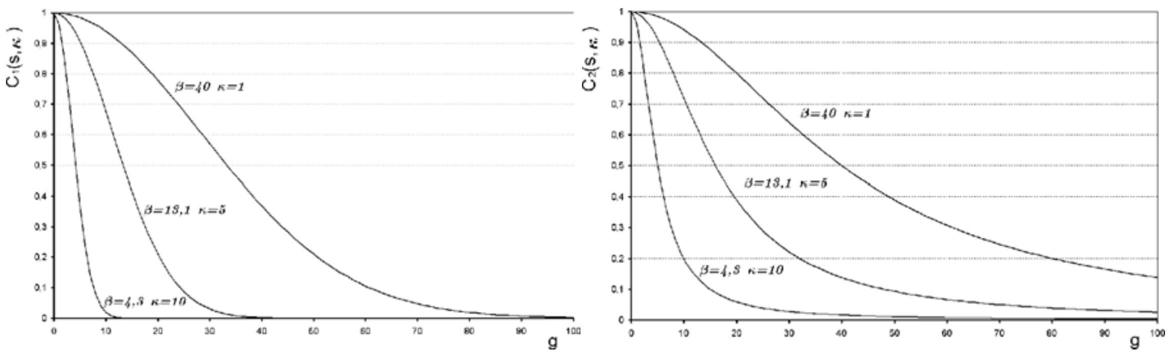


Fig.2: Dependence of the conductivity functions on the iteration step and the image gradient g for the c_1 and c_2 conductivity functions, (forward diffusion, β_1=40, γ=0.8).

2. DISCRETE IMPLEMENTATION

Although not obvious from Eq. (1), the discrete implementation of the nonlinear anisotropic diffusion filter is straightforward. In one dimension, the gradient and divergence expressions reduce to derivatives:

$$\frac{\partial}{\partial t} I(x,t) = \frac{\partial}{\partial x}\left[c(x,t) \frac{\partial}{\partial x} I(x,t) \right] \tag{3}$$

Substituting discrete approximations for the derivatives and introducing the flow functions we get:

$$\frac{\partial}{\partial t} I(x,t) \approx \frac{\partial}{\partial x}\left\{ c(x,t)\frac{1}{\Delta x}\left[I\left(x+\frac{\Delta x}{2},t \right) - I\left(x-\frac{\Delta x}{2},t \right)\right]\right\} \approx$$

$$\frac{1}{(\Delta x)^2}\left[c\left(x+\frac{\Delta x}{2},t \right)(I(x+\Delta x,t)-I(x,t)) - c\left(x-\frac{\Delta x}{2},t \right)(I(x,t)-I(x-\Delta x,t))\right]$$

The conductivity values **Błąd!**and **Błąd!** are easily computed by substituting the discrete approximation of the gradient :

$$c(x,t) \approx f\left(\frac{1}{\Delta x}\left| I\left(x+\frac{\Delta x}{2},t \right) - I\left(x-\frac{\Delta x}{2},t \right)\right| \right),$$

$$c(x+\frac{\Delta x}{2},t) \approx f\left(\frac{1}{\Delta x}\left| I(x+\Delta x,t) - I(x,t) \right| \right),$$

$$c(x-\frac{\Delta x}{2},t) \approx f\left(\frac{1}{\Delta x}\left| I(x,t) - I(x-\Delta x,t) \right| \right),$$

Introducing the notation:

$$c_R = c(x+\tfrac{\Delta x}{2},t)\cdot\tfrac{1}{\Delta x^2}, \qquad c_L = c(x-\tfrac{\Delta x}{2},t)\cdot\tfrac{1}{\Delta x^2}$$
$$\nabla_R I(x,t) = I(x+\Delta x,t) - I(x,t); \nabla_L I(x,t) = I(x-\Delta x,t) - I(x,t),$$

we obtain :

$$\frac{\partial}{\partial t} I(x,t) = c_L \cdot \nabla_L I(x,t) + c_R \cdot \nabla_R I(x,t) = \Phi_L + \Phi_R,$$
$$I(x,t+\Delta t) \approx I(x,t) + \Delta t \cdot \frac{\partial}{\partial t} I(x,t) = I(x,t) + \Phi_L + \Phi_R$$

The 1-D discrete formulation of the diffusion process is straightforwardly extended to the 2-D case:

$$\frac{\partial}{\partial t}\mathrm{I}(x,y,t) = \frac{\partial}{\partial x}\left[c(x,y,t)\cdot\frac{\partial}{\partial x}\mathrm{I}(x,y,t)\right] + \frac{\partial}{\partial y}\left[c(x,y,t)\cdot\frac{\partial}{\partial y}\mathrm{I}(x,y,t)\right] \approx$$

$$\approx \frac{1}{(\Delta x)^2}\left[c(x+\frac{\Delta x}{2},y,t)\cdot(\mathrm{I}(x+\Delta x,y,t)-\mathrm{I}(x,y,t))-c(x-\frac{\Delta x}{2},y,t)\cdot(\mathrm{I}(x,y,t)-\mathrm{I}(x-\Delta x,y,t))\right]+$$
$$+\frac{1}{(\Delta y)^2}\left[c(x,y+\frac{\Delta y}{2},t)\cdot(\mathrm{I}(x,y+\Delta y,t)-\mathrm{I}(x,y,t))-c(x,y-\frac{\Delta y}{2},t)\cdot(\mathrm{I}(x,y,t)-\mathrm{I}(x,y-\Delta y,t))\right]=$$
$$=c_N(x,y,t)\cdot\nabla_N\mathrm{I}(x,y,t)+c_S(x,y,t)\cdot\nabla_S\mathrm{I}(x,y,t)+c_W(x,y,t)\cdot\nabla_W\mathrm{I}(x,y,t)+c_E(x,y,t)\cdot\nabla_E\mathrm{I}(x,y,t)=$$
$$=\Phi_N+\Phi_S+\Phi_W+\Phi_E\ ,$$

where

$$c_N=c\left(x,y+\frac{\Delta y}{2},t\right)\cdot\frac{1}{\Delta y^2},\qquad c_S=c\left(x,y-\frac{\Delta y}{2},t\right)\cdot\frac{1}{\Delta y^2},$$
$$c_E=c\left(x+\frac{\Delta x}{2},y,t\right)\cdot\frac{1}{\Delta x^2},\qquad c_W=c\left(x-\frac{\Delta x}{2},y,t\right)\cdot\frac{1}{\Delta x^2},$$

$$\nabla_N I(x,y,t)=I(x,y+\Delta y,t)-I(x,y,t),$$
$$\nabla_S I(x,y,t)=I(x,y-\Delta y,t)-I(x,y,t),$$
$$\nabla_E I(x,y,t)=I(x+\Delta x,y,t)-I(x,y,t),$$
$$\nabla_W I(x,y,t)=I(x-\Delta x,y,t)-I(x,y,t),$$

The filtering process consists of updating each pixel in the image by an amount equal to the flow contributed by its nearest neighbors, (we assume 4-neighborhood system, the extension to 8-neighborhood is a trivial task),

$$I(x,y,t+\Delta t)\approx I(x,y,t)+\Delta t\cdot(\Phi_N+\Phi_S+\Phi_W+\Phi_E)..$$

3. FORWARD-AND-BACKWARD DIFFUSION

The conductance coefficients in the P-M process are chosen to be a decreasing function of the signal gradient. This operation selectively smoothes regions that do not contain large gradients. In the Forward-and-Backward diffusion (FAB), a different approach is taken. Its goal is to emphasize the extrema, if they indeed represent singularities and do not come as a result of noise. As we want to emphasize large gradients, we would like to move "mass" from the lower part of a "slope" upwards. This process can be viewed as moving back in time along the scale space, or reversing the diffusion process. Mathematically, we can change the sign of the conductance coefficient to negative: $\frac{\partial}{\partial t}\mathrm{I}(x,y,t)=\nabla[-c(x,y,t)\nabla I(x,y,t)],c(x,y,t)>0$. However, we cannot simply use an inverse linear diffusion process, because it is highly unstable. Three major problems associated with the linear backward diffusion process must be addressed: explosive instability, noise amplification and oscillations.

One way to avoid instability explosion is to diminish the value of the inverse diffusion coefficient at high gradients. In this way, when the singularity exceeds a certain gradient threshold it does not continue to affect the process any longer. The diffusion process can be also terminated after a limited number of iterations. In order not to amplify noise, which after some pre-smoothing, can be regarded as having mainly medium to low gradients, the inverse diffusion force at low gradients should also be eliminated. The oscillations should be suppressed the moment they are introduced. For this, a forward diffusion force that smoothes low gradient regions can be introduced to the diffusion scheme.

The result of this analysis is that two forces of diffusion working simultaneously on the signal are needed - one backward force (at medium gradients, where singularities are expected), and the other, forward one, used for stabilizing oscillations and reducing noise. These two forces can actually be combined to one coupled forward-and-backward diffusion force with a conductance coefficient possessing both positive and negative values. In [6-8] a conductivity function that controls the FAB diffusion process has been proposed

$$c_{FAB}(g) = \begin{cases} 1-\left(g/k_f\right)^n & ,\ 0 \le g \le k_f \\ \alpha\left[\left(\left(g-k_b\right)/w\right)^{2m}-1\right] & ,\ k_b - w \le g \le k_b + w \\ 0 & ,\ otherwise \end{cases} \tag{4}$$

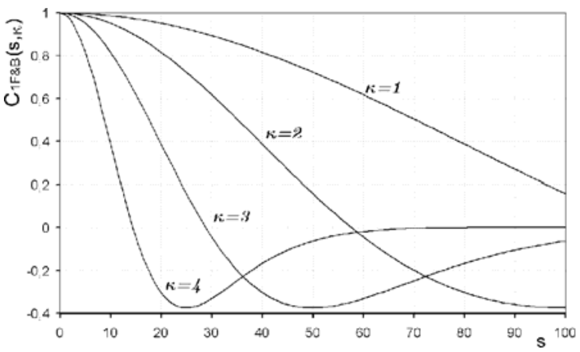
where g is an edge indicator (gradient magnitude or the value of the gradient convolved with the Gaussian smoothing operator), k_f, k_b, w are design parameters and $\alpha = k_f/(2k_b)$, ($k_f \le k_b$) controls the ratio between the forward and backward diffusion. The dependence of such a defined conductance coefficient on the value of the gradient indicator is shown in Fig. 4b.

In the P-M equation, an "edge threshold" β is the sole parameter, the FAB process described in [6-8] is modelled by a parameter which regulates forward force k_f, two parameters for the backward force (defined by k_b and width w), and the relation between the strength of the backward and forward forces α. Essentially k_f is the limit of gradients to be smoothed out and is similar in nature to the role of β parameter of the P-M diffusion equation, whereas the k_b and w define the backward diffusion range.

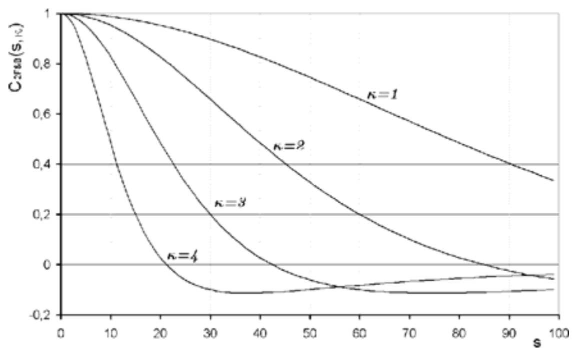
In this study we propose two more natural conduction coefficients directly based on the P-M approach:

$$c_{1_{FAB}}(s) = 2\exp(-s_1^2) - \exp(-s_2^2),\, c_{2_{FAB}}(s) = \frac{2}{1+s_1^2} - \frac{1}{1+s_2^2}, \tag{5}$$

The plots of the c_{1FAB} and c_{2FAB} diffusion coefficients are shown in Figs. 3, 4a.



a)



b)

Fig.3. Dependence of the conductivity functions on the iteration number κ and the normalized image gradient *s* for the c_{1FAB} (**a**) and c_{2FAB} (**b**) conductivity functions, (forward and backward diffusion), for $\beta_1(1)=40$, $\beta_2(1)=80$ and $\gamma=0.5$. Note, that because of low $\gamma=0.5$ already in the second iteration, the conductivity functions have negative values for large enough gradients.

In the diffusion process smoothing is performed when the conductivity function is positive and sharpening takes place for negative conduction coefficient values.

4. COOLING DOWN OF THE DIFFUSION PROCESS

Various modifications of the original diffusion scheme were attempted in order to overcome stability problems. Yet, most schemes still converge to a trivial solution (the average value of the image gray values) and therefore require the implementation of an appropriate stopping mechanism in practical image processing. In case of images contaminated by Gaussian noise, a common way of denoising is the usage of nonlinear cooling, which depends on the gradient, where large gradients cool faster and are preserved. In this study four simple time-dependent conduction coefficients were used:

$$c_1(g,t)=\exp\left[-\frac{g}{\beta(t)}\right]^2, c_2(g,t)=\frac{1}{1+\left(\frac{g}{\beta(t)}\right)^2}, \tag{6}$$

$$c_{1_{FAB}}(s,t)=2\exp\left[-\frac{g}{\beta_1(t)}\right]^2-\exp\left[-\frac{g}{\beta_2(t)}\right]^2, c_{2_{FAB}}(s,t)=\frac{2}{1+\left(\frac{g}{\beta_1(t)}\right)^2}-\frac{1}{1+\left(\frac{g}{\beta_2(t)}\right)^2}. \tag{7}$$

where $g=\|\nabla I(x,y,t)\|$ is the L_1 or L_2 norm of the color image vector in the RGB space, $\beta_i(t+1)=\beta_i(t)\cdot\gamma$, $\gamma\in(0,1]$, $\beta_i(1)$ is the starting parameter, $i=1,2$, $\beta_1(t)<\beta_2(t)$.

The scheme depends only on two (in case of forward or backward diffusion) or three (in case of FAB diffusion) parameters: initial values of starting β_i parameters and the cooling rate γ. Setting

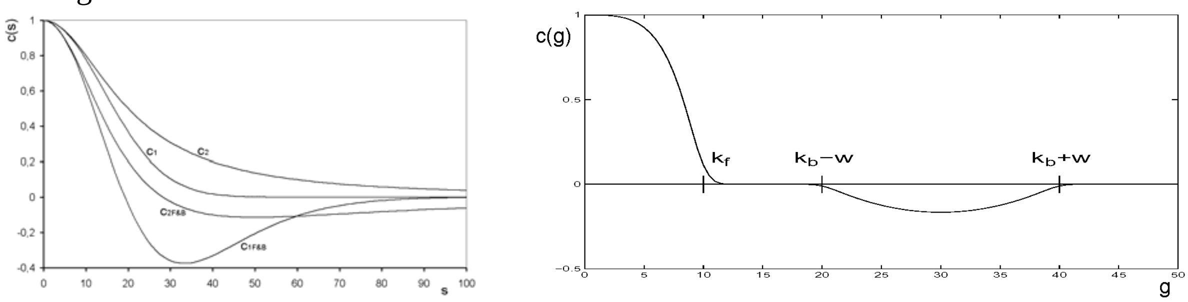
γ to 1 means, that there is no cooling in the system. As γ decreases, the cooling is faster, less noise is being filtered but edges are better preserved. Figures 3a and 3b illustrate the dependence of diffusion coefficients $c_1(s,t)$ and $c_2(s,t)$ on iteration step t. The behavior of the diffusion coefficients $c_{1_{FAB}}(s,t)$ and $c_{2_{FAB}}(s,t)$ are compared in Fig. 3 and 4a.

If the cooling coefficient γ is lower than 1, then the gradient threshold $\beta(t)$ decreases with time, allowing lower and lower gradients to take part in the smoothing process. As time advances, only smoother and smoother regions are being filtered, whereas large gradients can get enhanced due to local inverse diffusion. The scheme converges to a steady state for $\beta \to 0$, which means that no diffusion is taking place.

5. EXPERIMENTATIONS AND RESULTS

In this paper a novel approach to the problem of edge preserving smoothing is proposed. The experiments revealed that better results of noise suppression using the FAB scheme were achieved using the conductivity function c_2 from the original P-M approach (7). This is due to the shape of the coupled forward and backward conductivity shown in Fig. 3b, which allows more effective image sharpening.

The efficiency of the proposed technique and especially its excellent ability to filter out noise and sharpen the image edges is presented in Figs. 5 and 6, where the noisy images are enhanced using the new FAB anisotropic techniques. The results confirm good performance of the new method, which could be used for the enhancement of images in different areas of medical image processing.



(**a**) (**b**)

Fig.4. **a**) Comparison of the shape of the proposed forward and backward diffusion conductivity functions, **b**) the forward and backward conductivity function proposed in [6-8].

BIBLIOGRAPHY

[1] P. PERONA, J. MALIK, Scale space and edge detection using anisotropic diffusion, IEEE PAMI, Vol. 12, No. 7 pp. 629-639, July 1990

[2] R. WHITAKER, G. GERIG, Vector-Valued Diffusion, Ed. B.M. ter Haar Romeny, "Geometry-Driven Diffusion in Computer Vision", Kluwer Academic Press, pp. 93-134, 1994

[3] M.J. BLACK, G. SAPIRO, D.H. MARIMONT AND D. HEEGER, Robust anisotropic diffusion, IEEE Transactions on Image Processing, Vol. 7 (3), pp. 421-432, March 1998

[4] J. WEICKERT, "Anisotropic Diffusion in Image Processing", Stuttgart-Germany: Teubner-Verlag, 1998
[5] P. BLOMGREN, T. CHAN, Color TV: Total variation methods for restoration of vector valued images, IEEE Transactions on Image Processing, March 1998, Special issue on Geometry Driven Diffusion and PDEs in Image Processing
[6] G. GILBOA, Y. ZEEVI, N. SOCHEN, Anisotropic selective inverse diffusion for signal enhancement in the presence of noise, Proc. IEEE ICASSP-2000, Istanbul, Turkey, vol. I, pp. 211-224, June 2000
[7] G. GILBOA, Y. ZEEVI, N. SOCHEN, Resolution enhancement by forward-and-backward nonlinear diffusion process, Nonlinear Signal and Image Processing, Baltimore, Maryland, June 2001
[8] G. GILBOA, Y. ZEEVI, N. SOCHEN, Signal and image enhancement by a generalized forward-and-backward adaptive diffusion process, EUSIPCO-2000, Tampere, Finland, September 2000

Partially supported by KBN Grant PBZ-KBN-040/P04/08

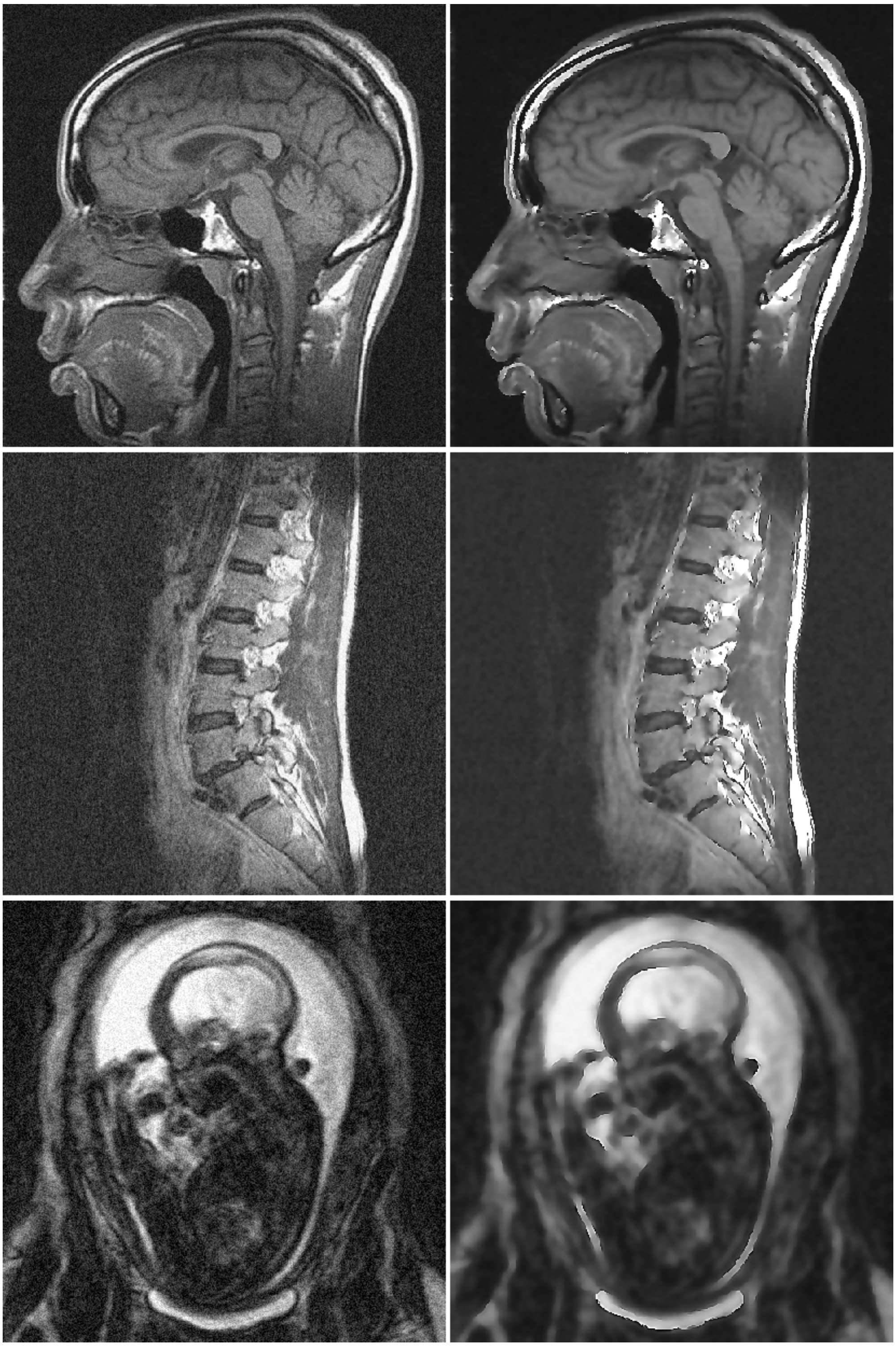


Fig.5. Illustration of the new combined forward and backward anisotropic diffusion scheme applied to gray scale images. To the left: noisy MRI test images, to the right images enhanced with the forward and backward diffusion, (5 iterations, conductivity function c_2).

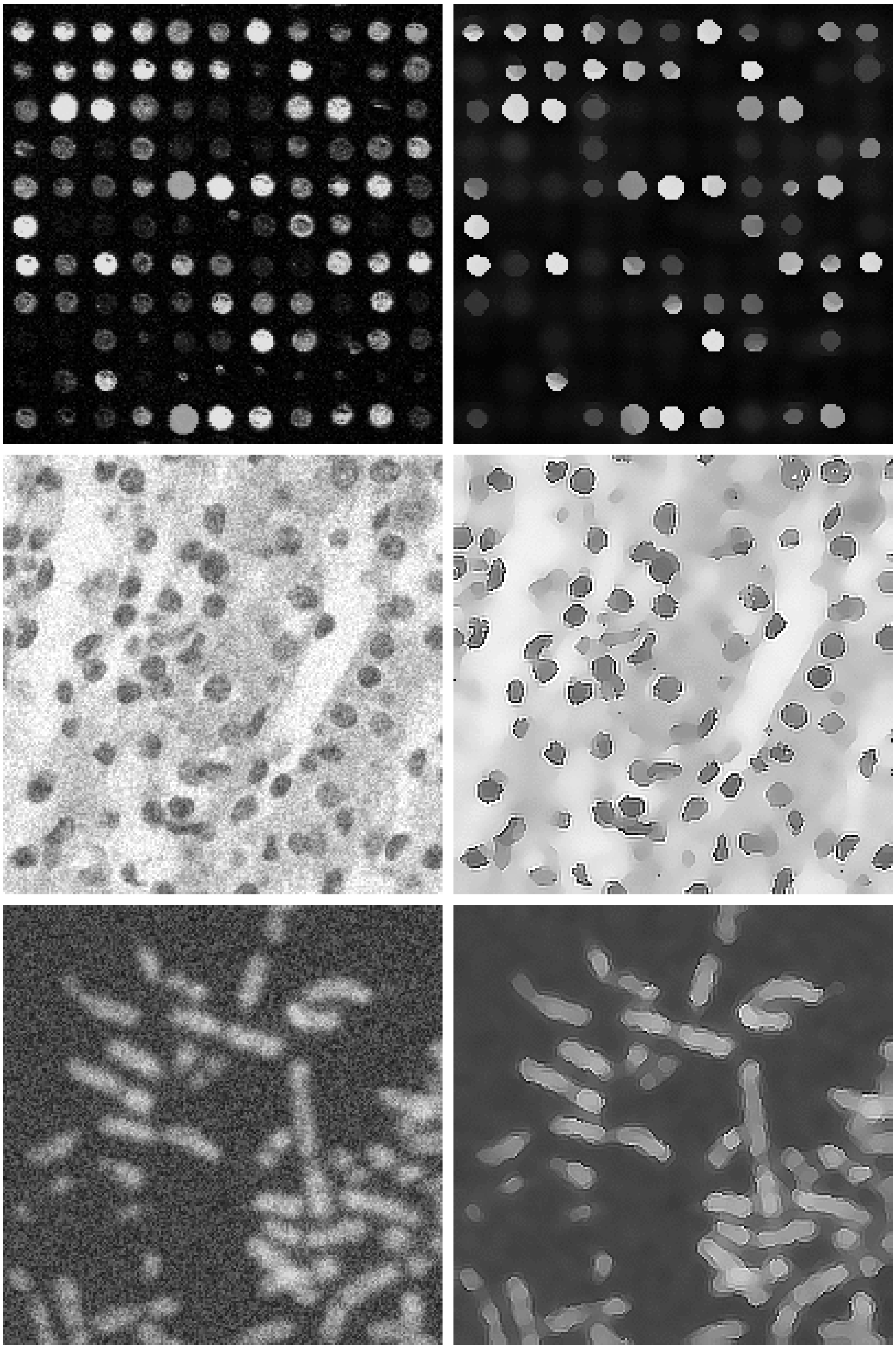


Fig.6. Effectiveness of the new coupled forward and backward anisotropic diffusion scheme. Left column: color, noisy biomedical images, to the right images enhanced with the new FAB anisotropic diffusion scheme, (10 iterations, conductivity function c_1).