vector median filter,
image enhancement,
noise suppression

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MODIFIED CENTRAL WEIGHTED VECTOR MEDIAN FILTER

A new filtering approach designed to eliminate impulsive noise in color images, while preserving fine image details is presented. The computational complexity of the new filter is significantly lower than that of the Central Weighted Vector Median Filter (CWVMF). The comparison shows that the new filter outperforms the CWVMF, as well as other standard procedures used in color image filtering for the removal of impulsive noise.

1. STANDARD NOISE REDUCTION FILTERS

Multichannel signal processing has been the subject of extensive research during the last years, primarily due to its importance to color image processing. The amount of research published to date indicates a growing interest in the area of color image filtering and analysis. The most common image processing tasks are noise filtering and image enhancement. These tasks are an essential part of any image processing system, whether the final image is utilized for visual interpretation or for automatic analysis [9]. It has been widely recognized that the processing of color image data as vector fields is desirable due to the correlation that exists between the image channels, and that the nonlinear vector processing of color images is the most effective way to filter out noise. For this reasons, the new filtering technique presented in this paper is also nonlinear and utilizes the correlation among the color image channels.

A number of nonlinear, multichannel filters, which utilize correlation among multivariate vectors using various distance measures, have been proposed [1-12]. The most popular nonlinear, multichannel filters are based on the ordering of vectors in a predefined sliding window. The output of these filters is defined as the lowest ranked vector according to a specific ordering technique.

Let $\mathbf{F}(x)$ represent a multichannel image and let W be a window of finite size n. The noisy image vectors inside the window will be denoted as $\mathbf{F}_j$, $j=0,1,..,n-1$. If the distance between two vectors $\mathbf{F}_i, \mathbf{F}_j$ is $\rho(\mathbf{F}_i, \mathbf{F}_j)$ then the scalar quantity :

$$R_i = \sum_{j=1}^{n} \rho(\mathbf{F}_i, \mathbf{F}_j) \tag{1}$$

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is the distance associated with the noisy vector $\mathbf{F}_i$ in W. An ordering of the R_i's implies the same ordering to the corresponding vectors $\mathbf{F}_i$'s into a sequence $\mathbf{F}_{(0)}, \mathbf{F}_{(1)},\ldots,\mathbf{F}_{(n-1)}$. Nonlinear ranked type multichannel estimators define the vector $\mathbf{F}_{(0)}$ as the filter output. This selection is due to the fact that vectors that diverge greatly from the data population usually appear in higher indexed locations in the ordered sequence. However, the concept of input ordering, initially applied to scalar quantities is not easily extended to multichannel data, since there is no universal way to define ordering in vector spaces. To overcome this problem, different distance functions are often utilized to order vectors. As an example, the *Vector Median Filter* (VMF) makes use of the L_1 or L_2 norm to order vectors according to their relative magnitude differences [1]. The output of the VMF is the pixel $\mathbf{F}_k \in W$ for which the following condition is satisfied:

$$\sum_{j=0}^{n-1} \rho(\mathbf{F}_k, \mathbf{F}_j) < \sum_{j=0}^{n-1} \rho(\mathbf{F}_i, \mathbf{F}_j), \; i = 0,\ldots,n-1. \qquad (2)$$

In this way the VMF consists of computing and comparing the values of R_i and the output is the vector $\mathbf{F}_k$ for which R_k reaches its minimum. In other words if for some k the value: $R_k = \sum_{j=0}^{n-1} \rho(\mathbf{F}_k, \mathbf{F}_j)$ is smaller than $R_0 = \sum_{j=0}^{n-1} \rho(\mathbf{F}_0, \mathbf{F}_j)$, then the original pixel $\mathbf{F}_0$ in the filter window W is being replaced by $\mathbf{F}_k$ which satisfies the above condition, ($k = \arg\min_i R_i$).

The *Basic Vector Directional Filter* (BVDF) is a ranked-order, nonlinear filter which parallelizes the VMF operation [11]. The output of the BVDF is that vector from the input set, which minimizes the sum of the angles with the other vectors. To improve the efficiency of the directional filters, another method called *Directional-Distance Filter* (DDF) was proposed in [3]. This filter retains the structure of the BVDF but utilizes a new distance criterion to order the vectors inside the processing window.

In [12] the vector median concept has been generalized and the so called weighted vector median has been proposed. Using the weighted vector median approach, the filter output is the vector $\mathbf{F}_k$, for which the following condition holds:

$$\sum_{j=0}^{n-1} w_j \rho(\mathbf{F}_k, \mathbf{F}_j) < \sum_{j=0}^{n-1} w_j \rho(\mathbf{F}_i, \mathbf{F}_j), \quad i = 0,..,n-1, \qquad (3)$$

where the w_j are the weights associated with vectors $\mathbf{F}_j$. In this paper we assume, that the only nonzero weighting coefficient is the w_0 . In this way, we obtain the so called Central Weighted Vector Median Filter (CWVMF), [12].

2. MODIFIED CENTRAL WEIGHTED VECTOR MEDIAN FILTER

The construction of the new filter is very similar to that of the Central Weighted VMF (CWVMF) proposed in [12], in which the filter output is the vector $F_k \in W$, which satisfies:

$$\sum_{j=0}^{n-1} w_j \rho(\mathbf{F}_k, \mathbf{F}_j) < \sum_{j=0}^{n-1} w_j \rho(\mathbf{F}_i, \mathbf{F}_j), \; i = 0, \ldots, n-1. \tag{4}$$

where $w_j \neq 0$ for $j = 0$ and equals 0 otherwise. So the output of the CWVMF is $\mathbf{F}_0$ (Fig. 1), if the sum of distances between $\mathbf{F}_0$ and all its neighbors $R_0 = \sum_{j=0}^{n-1} \rho(\mathbf{F}_0, \mathbf{F}_j)$ is smaller than $R_k = \sum_{j=0}^{n-1} w_j (\mathbf{F}_k, \mathbf{F}_j)$ for k=1,2, …, n-1, otherwise the CWVMF output is the vector $\mathbf{F}_k$ (Fig. 2), for which R_k is minimal. The difference between the VMF and CWVMF is that the distance between the central pixel $\mathbf{F}_0$ and its neighbors is multiplied by the weighting coefficient w_0, as shown in Fig. 2. The weighting privileges the central pixel $\mathbf{F}_0$ as the $w_0 \cdot \rho(\mathbf{F}_0, \mathbf{F}_0)$ is 0. However the weighting can be performed in a much simpler way which describes the new filtering approach.
Let the distance associated with the center pixel be:

$$R_0 = w_0 \sum_{j=0}^{n-1} \rho(\mathbf{F}_0, \mathbf{F}_j), \tag{5}$$

where $w_0 \leq 1$ is a weight assigned to the sum of distances between the central vector $\mathbf{F}_0$ and its neighbors. Then let the sum of distances associated with other pixels be like in VMF:

$$R_i = \sum_{j=0}^{n-1} \rho(\mathbf{F}_i, \mathbf{F}_j), \quad i = 1, \ldots, n-1. \tag{6}$$

If for some k, R_k is smaller than R_0

$$R_k = \sum_{j=0}^{n-1} \rho(\mathbf{F}_k, \mathbf{F}_j) < R_0, \tag{7}$$

then $\mathbf{F}_0$ is being replaced by $\mathbf{F}_k$. It happens when

$$\sum_{j=0}^{n-1} \rho(\mathbf{F}_k, \mathbf{F}_j) \; < w_0 \sum_{j=0}^{n-1} \rho(\mathbf{F}_0, \mathbf{F}_j), \tag{8}$$

which is the condition for the replacement of the central pixel by one of its neighbors.

The center pixel $\mathbf{F}_0$ will be replaced by its neighbor $\mathbf{F}_k$, if the distance R_k associated with $\mathbf{F}_k$ is smaller than R_0 and is the minimal distance associated with the vectors belonging to W. The weight w_0 is a design parameter. For w_0 =0 no changes are introduced to the image, and for w_0 =1 we obtain the standard vector median filter as proposed by Astola.

If $w_0 \in (0,1)$, then the new filter has the ability of noise removal, while preserving fine image details. It is easy to notice that the new filter is faster than the CWVMF, as the only weighting is applied to the sum of distances R_0. As a result the new filter needs only one additional multiplication compared with the VMF. The CWVMF needs 7 additional multiplications to perform the weighting of the distances between $\mathbf{F}_0$ and all its neighbors. As a result the new filtering scheme is significantly faster than CWVMF and is more efficient as will be shown in the next section.

	$\mathbf{F}_1$	
$\mathbf{F}_4$	$\mathbf{F}_0$	$\mathbf{F}_2$
	$\mathbf{F}_3$	

a)

	$\mathbf{F}_1$	
$\mathbf{F}_4$	$\mathbf{F}_2$	$\mathbf{F}_0$
	$\mathbf{F}_3$	

b)

Fig.1. In the vector median approach the sum of distances between the center pixel F_0 and its neighbors is computed. Then the neighbors of $\mathbf{F}_0$ are being put to the center of the window *W* and the appropriate distances are being computed. Fig. **b)** shows the situation where the R_2 distance associated with the $\mathbf{F}_2$ vector is determined.

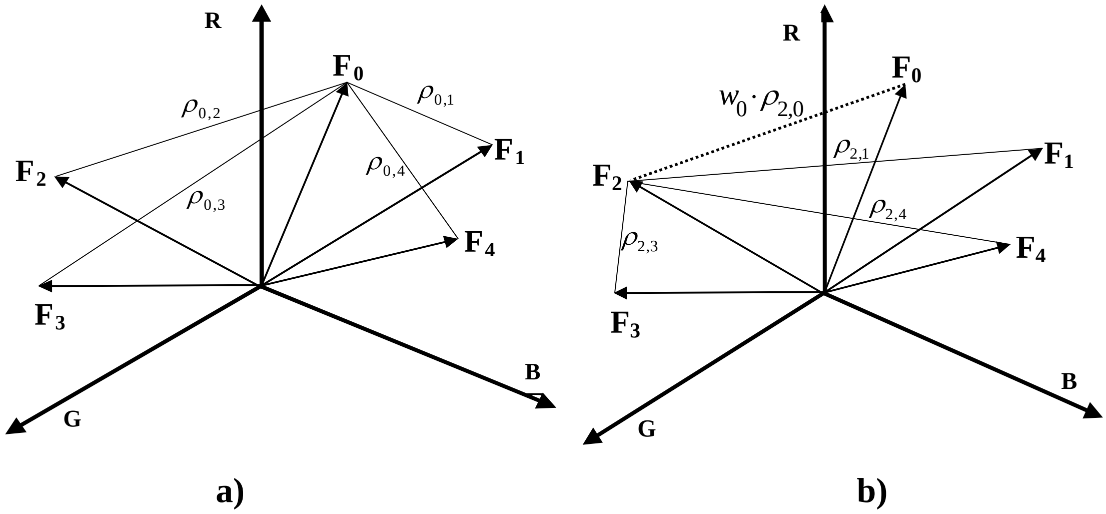

Fig.2. a) The distance R_0 equals $R_0 = \rho(0,1) + \rho(0,2) + \rho(0,3) + \rho(0,4)$, b) the distance R_2 associated with F_2 equals: $R_2 = \rho(2,0) + \rho(2,1) + \rho(2,3) + \rho(2,4)$ in the case of the vector median approach and $R_2 = w_0\, \rho(2,0) + \rho(2,1) + \rho(2,3) + \rho(2,4)$ when applying the Central Weighted Vector Median Filter.

3. EFFICIENCY OF THE NEW FILTER

The color test image *LENA* has been contaminated by 4% impulsive noise (each RGB channel was distorted by impulsive *salt and pepper* noise with probability 0.04 so that the correlation of noise in each channel was equal to 0.5). The root of the mean squared error (RMSE), signal to noise ratio (SNR), peak signal to noise ratio (PSNR), normalized mean square error (NMSE) and normalized color difference (NCD) [9], were taken as measures of image quality. The comparison shows that the new filter outperforms significantly the standard Central Weighted Vector Median Filter, when the impulsive noise has to be eliminated. The efficiency of the new filtering technique is shown in Tabs. 2 and 3.

Figure 3 presents the PSNR value obtained with the CWVMF and with the new *Modified Central Weighted Vector Median Filter* (MCWVMF) in dependence on the w_0 weighting parameter when using the *PEPPERS* image contaminated by the 4% impulsive noise. Figure 4 shows the optimal (maximal possible) PSNR values for the new filter in comparison with the standard technique, when using the test color image *LENA* distorted by impulsive noise ranging from 1 to 6 % (each RGB channel was contaminated independently with *salt and pepper* impulsive noise with probability 0.0x, where x=1,2,..,6). Figure 5 shows the dependence of the PSNR on the weighting coefficient w_0 for images distorted by impulsive noise of different intensities. As can be seen the optimal value of the weighting coefficient is dependent on the noise intensity and increases toward 1 (VMF) with growing noise intensity. Figure 6 illustrates the efficiency of the modified CWVMF as compared with the standard VMF filter using a color image of human retina. As can be seen the vector median performs too much pixel substitutions, which can lead to blurring and corruption of important texture-like structures.

NOTATION	FILTER	REF.
AMF	Arithmetic Mean Filter	[5]
VMF	Vector Median Filter	[1]
BVDF	Basic Vector Directional Filter	[10]
GVDF	Generalized Vector Directional Filter	[10]
DDF	Directional-Distance Filter	[3]
HDF	Hybrid Directional Filter	[2]
AHDF	Adaptive Hybrid Directional Filter	[2]
FVDF	Fuzzy Vector Directional Filter	[8]
ANNF	Adaptive Nearest Neighbor Filter	[7]
ANP-EF	Adaptive Non Parametric (Exponential) Filter	[9]
ANP-GF	Adaptive Non Parametric (Gaussian) Filter	[9]
ANP-DF	Adaptive Non Parametric (Directional) Filter	[9]

Tab.1. Filters taken for comparison with the new Modified Central Weighted Vector Median Filter (MCWVMF).

METHOD	NMSE [10^{-3}]	RMSE	SNR [dB]	PSNR [dB]	NCD [10^{-4}]
NONE	514.720	32.166	12.884	17.983	79.165
AMF	79.317	12.627	21.006	26.105	82.745
VMF	**18.766**	**6.142**	**27.266**	**32.365**	**40.467**
CWVMF	**12,105**	**4,933**	**29,170**	**34,269**	**19,019**
BVDF	24.587	7.030	26.093	31.192	41.151
GVDF	19.474	6.257	27.105	32.204	41.773
DDF	18.872	6.159	27.242	32.340	40.237
HDF	18.610	6.116	27.303	32.401	41.275
AHDF	18.310	6.067	27.373	32.472	41.166
FVDF	22.251	6.688	26.527	31.625	44.686
ANNF	26.800	7.340	25.719	30.817	48.009
ANP-E	78.601	12.570	21.046	26.144	82.457
ANP-G	78.623	12.571	21.045	26.143	82.478
ANP-D	24.178	6.971	26.166	31.264	46.070
MCWVMF	**8,095**	**4,034**	**30,918**	**36,017**	**10,753**

Tab.2. Comparison of the new algorithm with the standard techniques (Tab. 1), using the *LENA* standard image contaminated by 4% impulsive noise with 0.5 correlation between the RGB channels.

4. CONCLUSIONS

The new algorithm presented in this paper can be seen as a modification and improvement of the commonly used Vector Median Filter. The computational complexity of the new filter is lower than that of the Central Weighted Vector Median Filter. The comparison shows that the new filter outperforms the standard and central weighted VMF, as well as other basic procedures used in color image processing, when the impulse noise is to be eliminated. The future work will be focused on finding a way of automatic setting of the optimal weighting parameter, which should be adjusted to the intensity of the noise process, as clearly seen in Fig. 5. This could be achieved by first estimating the noise intensity [13] and then setting the appropriate value of the weighting parameter w_0.

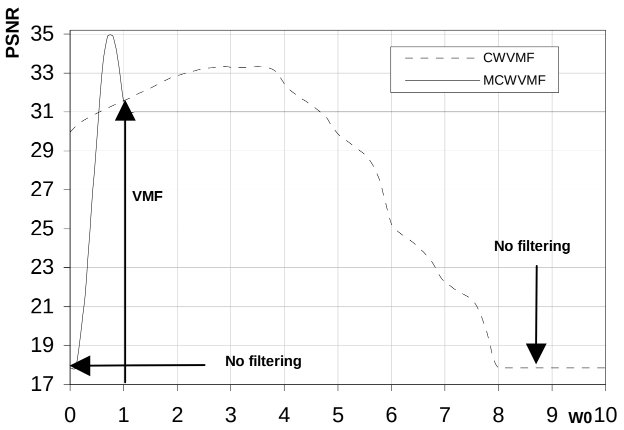

Fig.3. PSNR dependency on w_0 value for the new filter and CWVMF (LENA image corrupted with 4% impulsive noise).

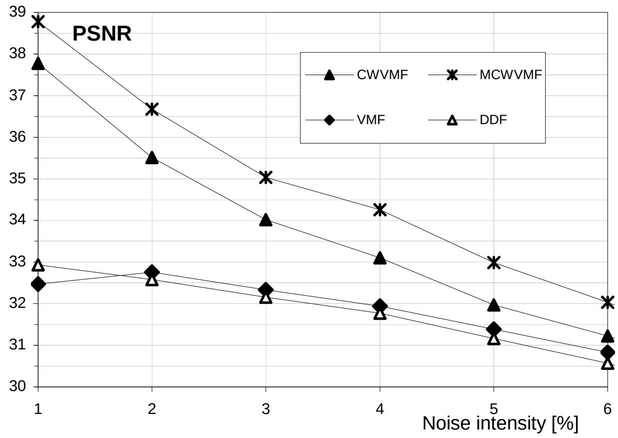

Fig.4. Comparison of the efficiency of the standard central weighted vector median with the new filter and the vector median (LENA with 4% impulsive noise).

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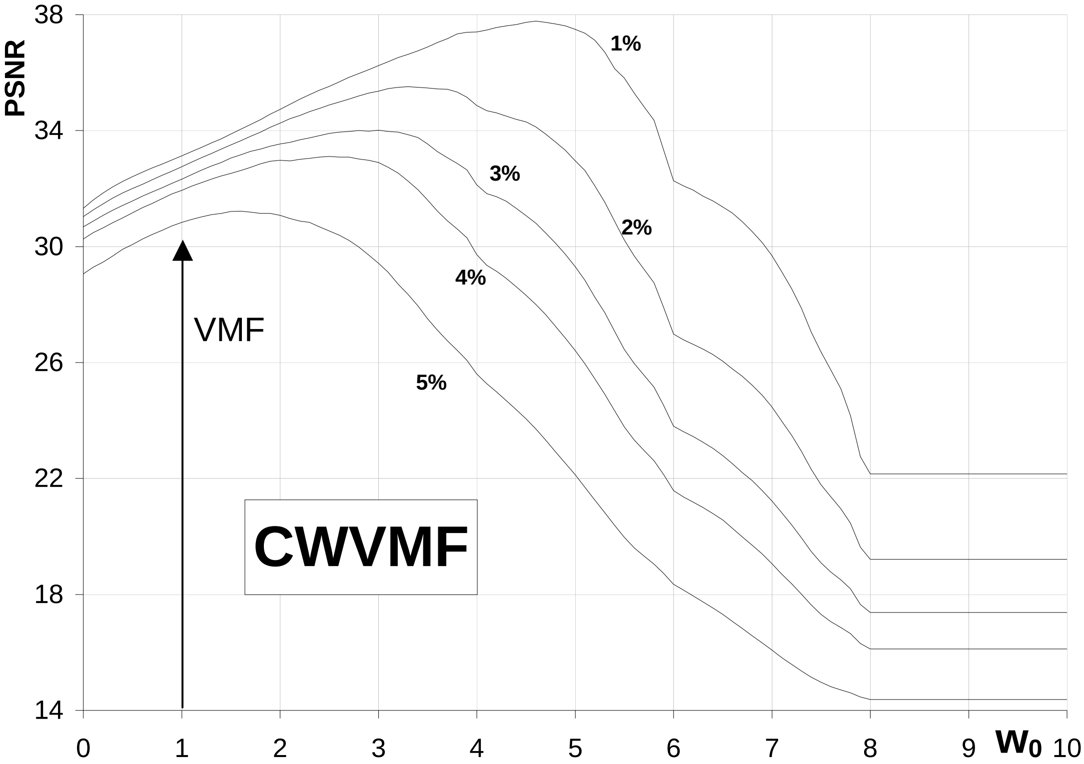
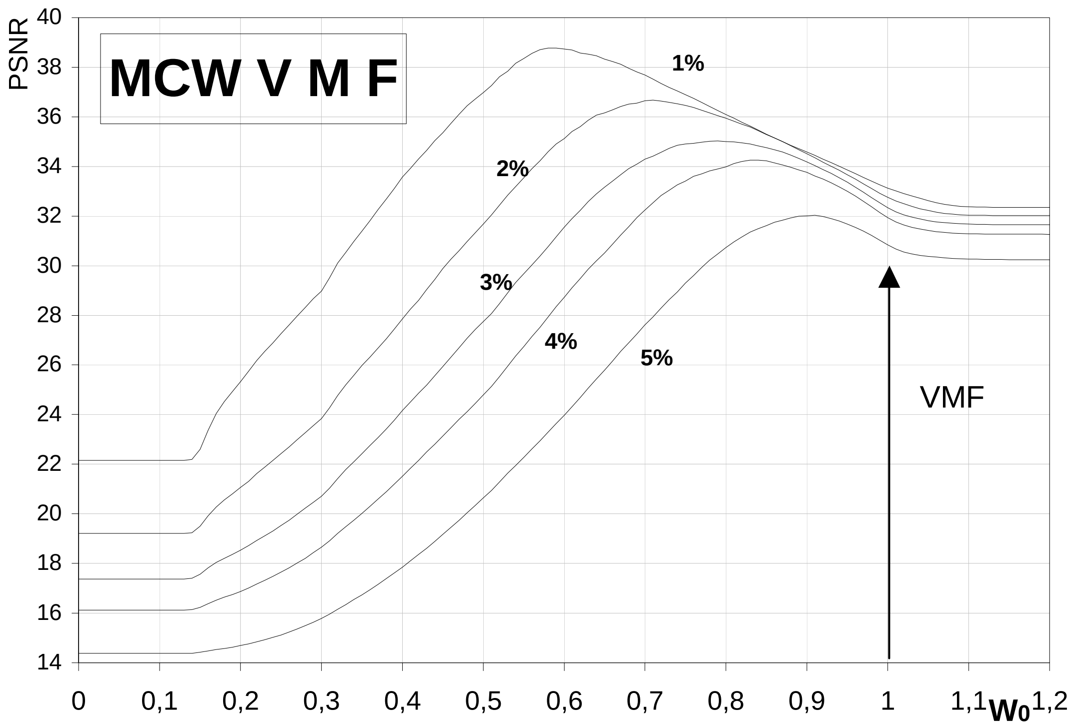


a)



b)

Fig.5. Dependence of the CWVMF and MCWVMF filter efficiency (**a)** and **b)** respectively) on the w_0 weighting coefficient for the *LENA* color image contaminated with impulsive noise (each RGB channel was distorted by salt and pepper noise with probability ranging from 1% to 5%).

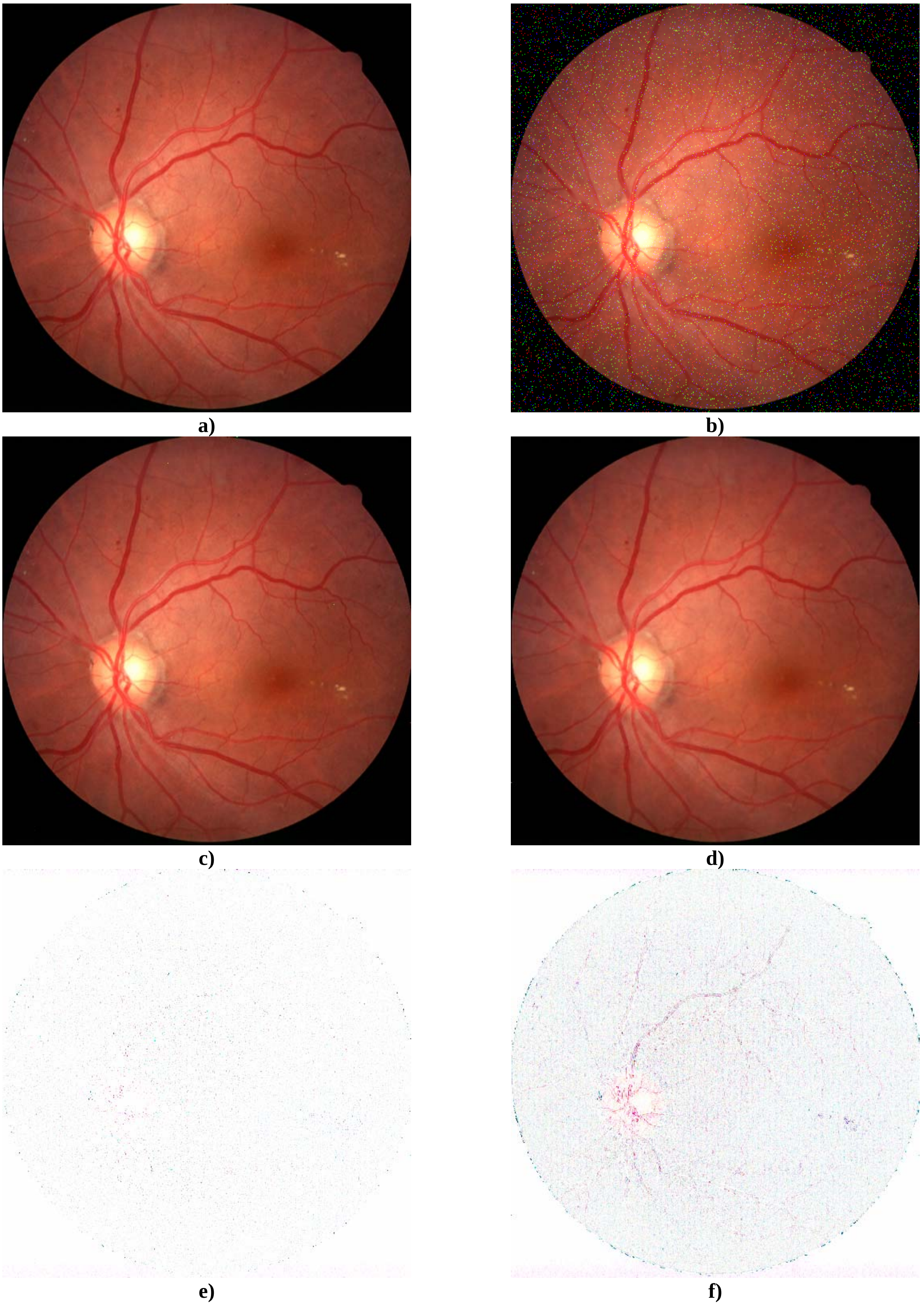


Fig.6. Illustration of the efficiency of the new filtering technique: a) color image of the human retina, b) image corrupted by impulsive noise of 2%, c) the new technique, d) VMF, e) and f) the difference between the original image a) and images c) and d) respectively