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SWITCHING MEDIAN FILTER WITH A LOCAL ENTROPY CONTROL

This paper presents a new switching median filter utilising local contrast entropy of the samples inside the filtering window. The proposed method is fully adaptive, it requires no optimisation and eliminates the main disadvantages of the local contrast probability based switching median. Excellent performance of the proposed method is a result of the successful analysis of input samples, as the local contrast entropy concept is able to efficiently differentiate between outliers and desired edge samples.

1. INTRODUCTION

In the past, many non-linear filtering techniques [1,9,11] have been proposed for impulsive noise removal. Efficient non-linear filters should be able to remove image noise and highlight the edges, while being computationally efficient. The first two requirements correspond to optimal trade-off between the noise attenuation and the edge preservation. It means that it is necessary to analyse high-frequency components and divide the atypical image samples into two classes: edges (desired samples) and noisy elements.

In noisy environments, the success of searching for an original depends on the complexity of the original image, the nature of the corruption process and also on the adopted measure of the solution accuracy [1],[13]. It is also necessary to respect conditions and assumptions of the problem and statistical and deterministic aspects of noise process and designed filters [10].

If the noise process is characterised by random impulsive changes of image elements [1],[2], such as bit errors and random-valued impulsive noise, the most popular and efficient filtering tool is provided by a class of median based filters [11],[12],[14]. These nonlinear filters based on ordering operation provide robust estimates necessary in real applications with varying image and noise statistics. Moreover, these filters can be simplified to binary operations [1],[7],[8], which are fast and easy to implement [4],[8].

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2. MEDIAN FILTERS AND THEIR EXTENSIONS

Let $\{x_1, x_2, ..., x_N\}$ be a discrete-time continuous-valued input set determined by a filter window positioned at $x_{(N+1)/2}$ and

$$x_{(1)} \le x_{(2)} \le ... \le x_{(N)} \tag{1}$$

be an ordered set, so that $x_{(i)} \in \{x_1, x_2, ..., x_N\}$, for $i = 1, 2, ..., N$.

The well-known median filter (MF) [12] is defined by

$$y = med\{x_1, x_2, ..., x_N\} = x_{((N+1)/2)} \tag{2}$$

where med is the median operator, N is a the number of samples in the filtering window and $x_{((N+1)/2)}$ is the central sample from the ordered sequence.

Because the median filter introduces too much smoothing into the image, which results in a blurring, a number of median extensions have been developed [1],[9],[11],[12].

In this paper we will make use of the lower-upper-middle (LUM) smoothers [1],[6],[9] and weighted median (WM) filters [1],[9],[12],[13],[14] which are characterised by improved preservation capability and will develop a filter design with more degrees of freedom than that of the MF.

The LUM smoother of the size N is defined as

$$y = med\left\{x_{(k)}, x_{(N+1)/2}, x_{(N-k+1)}\right\} \tag{3}$$

where y is the filter output dependent on the parameter $k = 1, 2, ..., (N+1)/2$. The LUM smoothing function is created by the comparison of the lower $x_{(k)}$ and upper $x_{(N-k+1)}$ order statistic with the central input sample $x_{(N+1)/2}$.

Let each input sample x_i be associated with the real valued weight w_i, for $i = 1, 2, ..., N$. The WM output is the sample $y \in \{x_1, x_2, ..., x_N\}$ minimising the expression:

$$L(y) = \sum_{i=1}^{N} w_i \left| y - x_i \right| \tag{4}$$

In order to adapt the behaviour of the WM filters to varying signal and noise statistics, in [14] optimisation algorithms based on linear (LWM) and sigmoidal approximations (SWM) of the sign function have been summarised.

3. LOCAL CONTRAST PROBABILITY APPROACH

Another way to restrict the filter effect only to corrupted samples is related to the adaptive median filtering. In this way [2],[3],[5] the median filter is applied to noisy samples, and noncorrupted (desired) samples remain unchanged. In order to detect impulses, the local contrast probability approach have been introduced in [3].

Let $\{x_1, x_2, ..., x_N\}$ be the set of grey-scale samples inside the filter window of a finite size N. Let us consider that each input sample x_i, for $i = 1, 2, ..., N$, is associated with its contrast C_i derived by the Weber-Fechner law [3]:

$$C_i = \frac{|x_i - \mu|}{\mu} = \frac{\Delta_i}{\mu} \tag{5}$$

where Δ_i is the gradient level and μ is the mean of the input set $\{x_1, x_2, ..., x_N\}$.

Thus, the local contrast probability (LCP) [3] is given by

$$P_i = \frac{C_i}{\sum_{i=1}^{N} C_i} = \frac{C_i}{C_S}\ ;\ or\ \ P_i = \frac{\Delta_i}{\sum_{i=1}^{N} \Delta_i} \tag{6}$$

Any sample x_i, for $i = 1, 2, ..., N$, inside the filter window is considered as noise, if its associated local contrast probability is greater than or equal to the threshold value. In terms of the probability measure P_i related to the difference of the input sample x_i to other samples inside the filter window (all the local contrasts are equally distributed), the threshold contrast probability P_C corresponding to a filter window has been introduced in [3].

Referring to the sample under consideration, i.e. the central sample $x_{(N+1)/2}$ of the filter window, the central sample $x_{(N+1)/2}$ is considered as an outlier, when the associated local contrast probability $P_{(N+1)/2}$ is greater than or equal to the threshold local probability

$$P_C = 1/N \tag{7}$$

This comparison forms a simple definition of the adaptive LCP median filter:

$$y = \begin{cases} med\{x_1, x_2, ..., x_N\} & \text{if } P_{(N+1)/2} \geq P_C \\ x_{(N+1)/2} & \text{otherwise} \end{cases} \tag{8}$$

If the inequality $P_{(N+1)/2} \geq P_C$ is satisfied, the central sample is considered as an outlier and the output y of the adaptive median filter will be the median of the input set $\{x_1, x_2, ..., x_N\}$. Otherwise, the central sample $x_{(N+1)/2}$ has the desired features and will remain unchanged.

4. PROPOSED ENTROPY SWITCHING MEDIAN FILTER

The no adaptive threshold probability P_C represents the main disadvantage of the LCP approach that can fail especially at image edges, because in the case of samples lying at the image edges, these samples can be considered as noise. In general, the filtering of edge points is accompanied by some undesired effects such as blurring and edge jittering. Thus, the most valuable features may be significantly degraded by the filtering process.

Let us consider the input set $\{x_1, x_2, ..., x_N\}$, such that each input sample x_i, for $i = 1, 2, ..., N,$ is associated with the local contrast probability P_i given by (6). Using the entropy definition applied to the input set $\{x_1, x_2, ..., x_N\},$ input samples contribute to the entropy defined by

$$H = -\sum_{i=1}^{N} P_i \log P_i \tag{9}$$

where P_i is the local contrast probability associated with the input sample x_i and N is the window size.

In terms of the entropy concept (9), each input sample x_i is also associated with the local contrast entropy H_i defined as:

$$H_i = -P_i \log P_i \tag{10}$$

Our purpose is to provide the adaptive threshold control of the LCP approach. Because each local contrast probability P_i given by (6) is always constrained to be a value between 0 and 1, the adaptive threshold of our approach should be rescaled to the same interval.

Let us assume that each sample x_i, , for $i = 1, 2, ..., N$ is associated with the adaptive threshold β_i expressed as the rate of the local contrast entropy H_i defined by (10) and the overall entropy H (9) of the input set $\{x_1, x_2, ..., x_N\}.$ So, the adaptive threshold β_i is given by

$$\beta_i = \frac{H_i}{H} = \frac{-P_i \log P_i}{-\sum_{i=1}^{N} P_i \log P_i}. \tag{11}$$

In the same way as (5), the output of the entropy based median filter is given by

$$y = \begin{cases} med\{x_1, x_2, ..., x_N\} & \text{if } P_{(N+1)/2} \geq \beta_{(N+1)/2} \\ x_{(N+1)/2} & \text{otherwise} \end{cases} \tag{12}$$

where $\beta_{(N+1)/2}$ is the adaptive threshold associated with the central sample $x_{(N+1)/2}$.

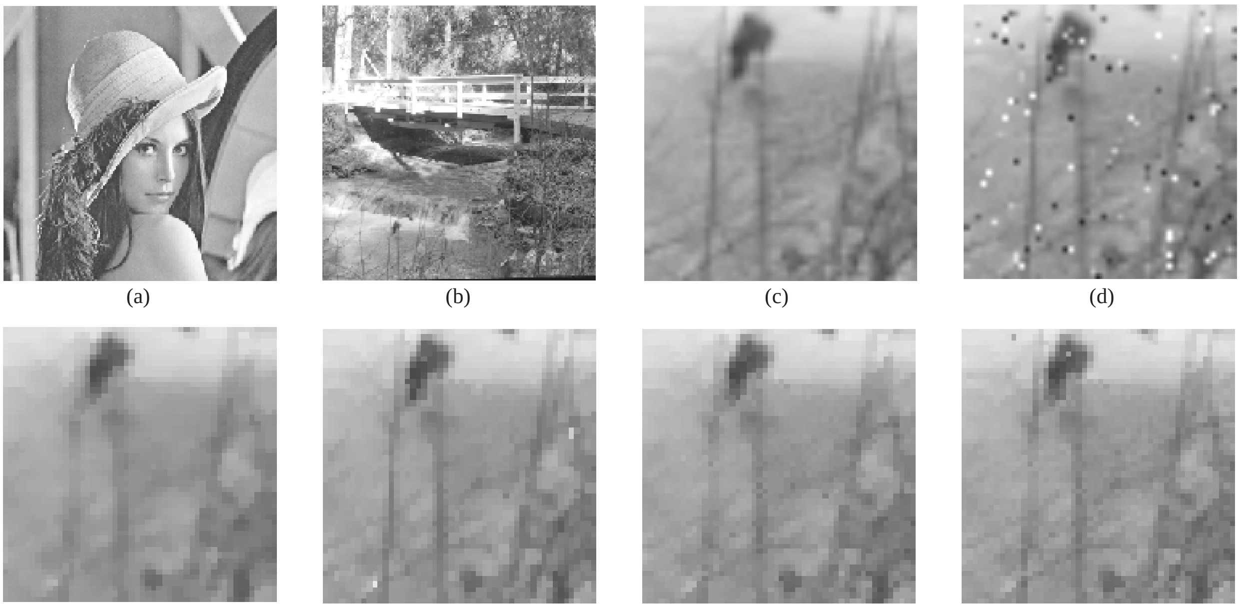


Fig.1. Illustration of the achieved results: (a) test image Lena, (b) test image Bridge, (c) detail of the test image Bridge, (d) image corrupted by 5% impulsive noise, (e) median output, (f) SWM output, (g) LCP median output, (h) proposed entropy median output

5. EXPERIMENTAL RESULTS

We tested the performance of the presented methods using the standard test images Lena and Bridge (Fig.1a,b) corrupted by random valued impulsive noise [1] (Fig. 2d and Fig. 4b) with the impulsive probability p_υ ranged from $p_\upsilon = 0$ to $p_\upsilon = 0.15$ with a step size of 0.01. This random valued impulsive noise is defined as

$$x_i = \begin{cases} o_i & \text{with probability } 1 - p_\upsilon \\ \upsilon & \text{with probability } \quad p_\upsilon \end{cases} \tag{13}$$

where x_i is the noisy image sample, o_i describes the sample original image, i denotes the sample location, υ is the random value from <0,255> and p_υ is the impulse probability. Note that impulsive noise frequently occurs as bit errors

$$^{*}k_i^m = \begin{cases} k_i^m & \text{with probability } 1 - p_\upsilon \\ 1 - k_i^m & \text{with probability } \quad p_\upsilon \end{cases} \tag{14}$$

where p_υ is the bit change probability, k_i^m and $^{*}k_i^m$ are binary values {0,1} of B-bit original sample o_i and noisy sample x_i given by

$$o_i = k_i^1 2^{B-1} + k_i^2 2^{B-2} + \ldots + k_i^{B-1} 2 + k_i^B \tag{15}$$

$$x_i = {}^*k_i^1 2^{B-1} + {}^*k_i^2 2^{B-2} + \ldots + {}^*k_i^{B-1} 2 + {}^*k_i^B \tag{16}$$

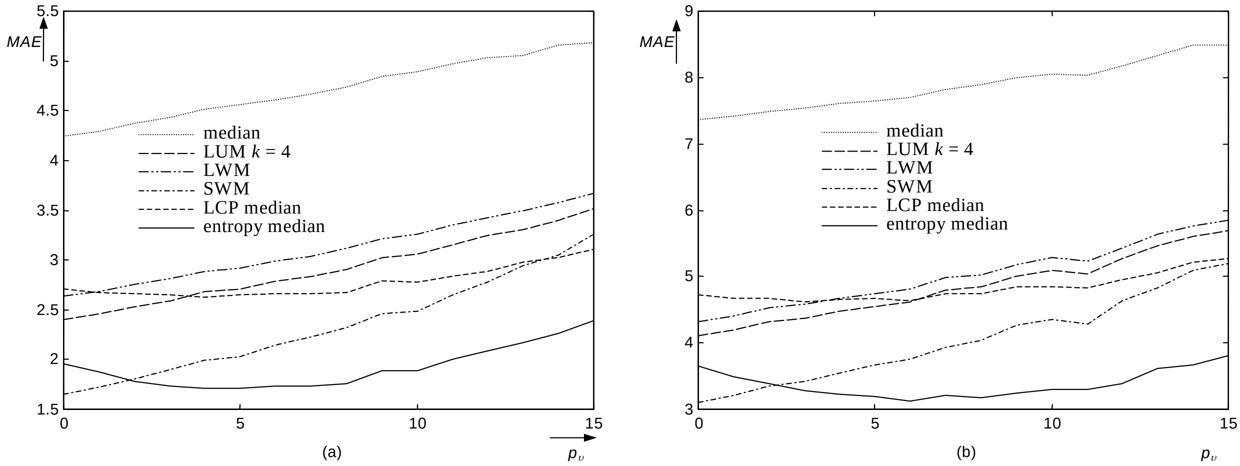


Fig.2. Dependence of the MAE criteria on the noise corruption p_υ for the test images Lena (a) and Bridge (b).

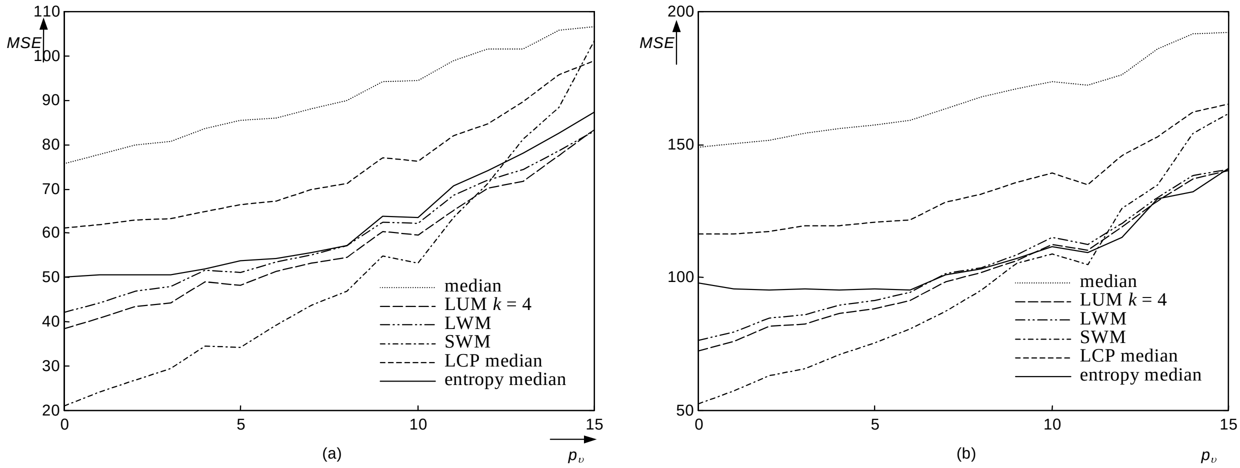


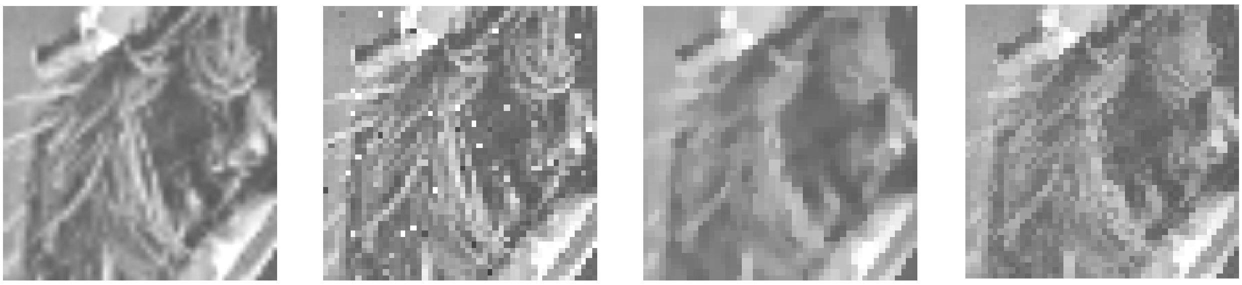
Fig.3. Dependence of the MSE criteria on the noise corruption p_υ for the test images Lena (a) and Bridge (b).

In this work, the measure of the image degradation is expressed through two objective criteria [6] the mean absolute error (MAE) and the mean square error (MSE). The MAE criteria expresses the signal-detail preservation, whereas the MSE is a measure of the noise attenuation capability:

$$\mathrm{MAE} = \frac{1}{K_1 K_2} \sum_{i=1}^{K_1 K_2} \left| o_i - x_i \right| \tag{17}$$

$$\mathrm{MSE} = \frac{1}{K_1 K_2} \sum_{i=1}^{K_1 K_2} \left(o_i - x_i \right)^2 \tag{18}$$

where o_i is the original image, x_i is the filtered (noisy) image, i, for $i = 1, 2, ..., K_1 K_2$, denotes the sample position in a $K_1 \times K_2$ digital image.



(a) (b) (c) (d)

Fig.4. Zoomed results: (a) part of the test image Lena, (b) image corrupted by 2% impulsive noise, (c) median output, (d) proposed entropy median output

Tab.1. Achieved results using the test image Lena.

Impulsive Noise	5%		10%		15%	
Method / Criteria	MAE	MSE	MAE	MSE	MAE	MSE
Noisy	3.540	374.3	7.018	759.1	10.201	1093.8
median (MF)	4.563	85.4	4.888	94.3	5.184	106.5
LUM k = 4	2.711	48.3	3.059	59.6	3.514	83.3
LWM	2.918	51.2	3.261	62.2	3.670	83.2
SWM	2.033	34.3	2.488	53.2	3.255	103.3
LCP median	2.650	66.5	2.775	76.1	3.108	98.9
entropy median	1.714	53.8	1.892	63.5	2.390	87.3

Tab.2. Achieved results using the test image Bridge.

Impulsive Noise	5%		10%		15%	
Method / Criteria	MAE	MSE	MAE	MSE	MAE	MSE
Noisy	3.568	398.9	7.221	807.6	10.646	1202.5
median (MF)	7.644	157.2	8.042	173.7	8.483	191.9
LUM k = 4	4.552	88.3	5.097	112.6	5.698	140.2
LWM	4.747	91.5	5.283	115.2	5.851	140.6
SWM	3.667	75.7	4.350	108.9	5.201	161.8
LCP median	4.667	121.0	4.856	139.1	5.268	165.3
entropy median	3.193	95.6	3.294	111.6	3.804	140.9

The new filter provides excellent improvement (Figs.1-4, Tabs. 1-2) in comparison with some relevant techniques such as standard median filter, LUM smoother and LCP median. This is also confirmed by the results depicting the zoomed parts of images (Fig.1 and Fig.4). The proposed method achieves excellent results especially in terms of MAE criteria that reflects the capability of the filter to preserve the image-details. It also provides robust noise attenuation, because the impulse detection characteristics of the proposed entropy median are sufficiently precise. In addition, the proposed method may also outperform (Fig.2 and Fig.3) the performance of optimal filters such as LWM and SWM, especially for highly corrupted images.

Although the proposed entropy median achieves worse results for small degree of the noise corruption than that of the optimised LWM and SWM filters, it does not require the optimisation procedure. Therefore, the proposed method represents more robust, useful and attractive approach than the optimised WM filters.

6. CONCLUSION

The proposed local entropy based adaptive median filter improves the preserving capability of the standard median filter while retaining its robust noise attenuation characteristics. The proposed method is computationally attractive, fast and allows optimal filtering to be performed in environments corrupted by bit errors and outliers.

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