fuzzy reasoning, fuzzy systems, fuzzy truth value

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ADVANTAGES OF AN APPROXIMATE REASONING BASED ON A FUZZY TRUTH VALUE

The approximate reasoning based on a fuzzy truth value is based on a different view of linguistic statements and comparing with the compositional rule of inference has some advantages. Benefits of the method are especially important for fuzzy expert systems with large sets of premises. The problem is very common for many applications in medicine, biology and biometry. By a short analysis of the approach and comparing to the compositional rule of inference the paper emphasizes the most important advantages of a possible implementation, which is particularly significant for the mentioned fields.

1. INTRODUCTION

Since 1973, when foundations of a fuzzy reasoning were presented by Zadeh [19]], many researchers have proposed different methods of an approximate inference based on the theory of fuzzy sets. It is important to mention the first practical approach developed by Mamdani and Assilan [10], then the system of Takagi, Sugeno and Kang [16] and the approach of Tsukamoto [18]. These solutions are commonly used because of the simplicity and efficiency of their implementation. However, the methods do not fully correspond with the Zadeh's theory because of an important simplification. The reason can be noticed only for systems with fuzzyfied input, where membership functions of facts are different than a singleton.

The compositional rule of inference, which is considered as classical, for modus ponendo ponens can be expressed by the following equation [8]:

$$\mu_{B'}(y) = \sup_{x \in X}\left(\mu_{A'}(x) *_T I\left(\mu_A(x), \mu_B(y)\right)\right), \tag{1}$$

where $\mu_{A'}(x)$, $\mu_A(x)$, $\mu_{B'}(y)$, $\mu_B(y)$ are membership functions, $*_T$ is a triangular norm representing intersection between a fuzzy fact A' and a fuzzy implication I, obtained from a fuzzy premise A and a conclusion B. Fuzzy sets are described in universes of discourse X, for a premise and a fact, and Y, for a conclusion and a result of the inference B'. Unfortunately, for compound premises computations for the approach (1) become very complex, because of a multi-dimensional analysis. This is the main reason why the method in this form is not used for rules with a compound premise containing multiple connectives, which are very common. For example data mining solutions for gene classification using microarray technique produces rules with thousands of premises [9].

Looking at the contemporary literature of the subject, one can find that an interesting approach presented in 1979 by Baldwin [1] seems to be forgotten. The most important advantage of the solution is a reasoning within the same truth space for all premises and conclusions. It is achieved by a transformation of a relation between a fact and a premise to so called truth function τ, which is also a fuzzy set. Bringing all relations between different facts and premises to one fuzzy truth space simplifies calculation of a compound truth function. That is why Baldwin's reasoning is devoid of a problem of multi-dimensional analysis and better suited for applications in medicine, biology and biometry, where the fastest approaches are desired.

Journals related to computer science for medicine and biology were always interested in improvements in fuzzy reasoning and fuzzy systems area (i.e. [3]). This paper compares Baldwin's

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solution with an approach presented by Zadeh and emphasizes advantages of the first one. The results of numerical examinations are presented at the end to confirm theoretical considerations.

2. SIMPLIFICATION OF THE CLASSICAL APPROACH

Compositional rule of inference for two premises can be expressed by the following equation

$$\mu_{B'}(y) = \sup_{x_1 \in X_1, x_2 \in X_2} \left(\mu_{A_1'}(x_1) *_T \mu_{A_2'}(x_2) *_T I\left(\mu_{A_1}(x_1) *_T \mu_{A_2}(x_2), \mu_B(y)\right)\right). \quad (2)$$

Interception of facts, as well as premises, are in this case three-dimensional. Adding a new premise causes that the compositional rule of inference work in one more dimension. It is a big disadvantage of this approach, especially from computational point of view. That is a direct reason why the solution is not used in form (2) for multiple premises.

For computationally efficient approaches [10], a relation between a fuzzy fact A' and a fuzzy premise A is in the end limited to a precise value m

$$m = \sup_{x \in X} \left(\mu_{A'}(x) *_T \mu_A(x) \right). \quad (3)$$

In such a case equation (1) can be expressed by the following

$$\mu_{B'}(y) = I\left(m, \mu_B(y)\right). \quad (4)$$

It only corresponds to (1) when a fact is in a singleton form. For other shapes of membership functions, assuming different types of triangular norms and fuzzy implications, the solution (4) is a simplification.

Value m can easily represent a relation between facts and premises in a compound case. For two premises in a rule, joined with an AND conjunction, (3) can be expressed by the following expression

$$m = m_1 *_{T_{AND}} m_2 = \sup_{x_1 \in X_1} \left(\mu_{A_1'}(x_1) *_T \mu_A(x_1) \right) *_{T_{AND}} \sup_{x_2 \in X_2} \left(\mu_{A_2'}(x_2) *_T \mu_{A_2}(x_2) \right), \quad (5)$$

where $*T_{AND}$ is a triangular norm representing a conjunction. This allows such a solution to calculate a result very efficiently, even for a compound premise with great number of conjunctions, because a computational complexity of the approach is linear.

Further analysis shows that Baldwin's idea combines two important advantages: preserves fuzzy relation between facts and premises during a reasoning process and is characterized by a linear computational complexity.

3. REASONING WITH TRUTH FUNCTIONS IN CLASSICAL LOGIC

Baldwin's fuzzy reasoning is based on an extension of classical logic. It was suggested to consider assigning truth values of a proposition using so called truth functions [1]. The three truth functions were proposed to describe three different states: true, false and undecided. Truth functions simply modify the truth values of a proposition, which for classical logic can have only two truth values assigned: 0 or 1 (false or true).

The following equations describe the three truth functions [1]

$$\psi_{true}(x) = \begin{cases} 0, & x = 0 \\ 1, & x = 1 \end{cases}, \quad (6)$$

$$\psi_{false}(x) = \begin{cases} 1, & x = 0 \\ 0, & x = 1 \end{cases}, \quad (7)$$

$$\psi_{undec.}(x) = \begin{cases} 1, & x = 0 \\ 1, & x = 1 \end{cases}, \tag{8}$$

where ψ_{true}, ψ_{false}, $\psi_{undec.}$ represent truth functions for states: true, false and undecided respectively.

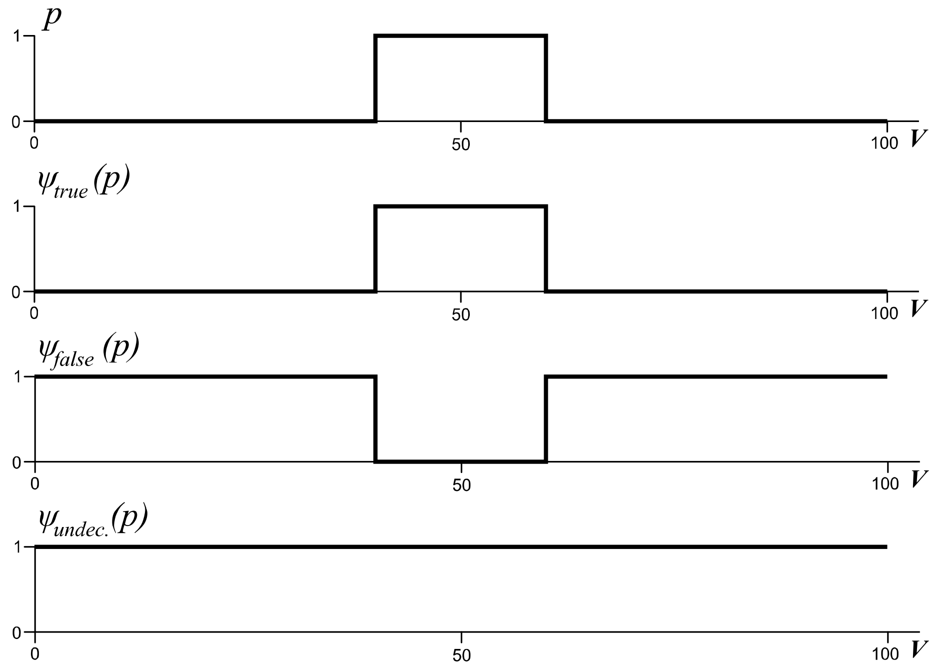

Fig. 1. Examples of truth function modification for classical logic.

The meaning of the truth functions is depicted in fig. 1, where truth values of the proposition p = *"car's velocity is medium"* are presented along with it's modification by analyzed truth functions.

Assigning a ψ_{true} function to a proposition p means that p is true, so the car being analyzed is driving at medium speed. Further example shows a modification where ψ_{false} is assigned to p, which means that p is not true. This example is equal to a case of the proposition q = *"car's velocity is NOT medium"*. In that case the truth value 1 is assigned to the set of velocities opposite to those classified as medium. Finally $\psi_{undec.}$ is used, which means that no information of a car speed is given (p is undecided). In this case any velocity of a car can be true, so the truth value 1 is assigned to the whole V space.

Presented transformations are called truth function modifications [2] applied for a two-valued logic.

The reasoning mechanism in the approach for modus ponendo ponens calculates a truth function of a conclusion (ψ_q) basing on a truth function of a premise. Subsequently, a truth function ψ_q can be used in the same way as shown in fig. 1 to calculate a conclusion. Baldwin proposed the following equation [1]

$$\underset{y \in \{0,1\}}{\forall} \quad \psi_q(y) = \max_{x \in \{0,1\}} \left[\min\left(\psi_p(x), I(x,y) \right) \right]. \tag{9}$$

ψ_q equal ψ_{true} (true) can be achieved only for true premise (ψ_p = ψ_{true}). For other cases of ψ_p, ψ_q is undecided (ψ_q = $\psi_{undec.}$), which is consistent with classical logic.

It is important to emphasize that arguments of the expression (9) do not include truth values of p and q directly. Only truth functions are present, which means that the reasoning is performed in a truth function space.

The additional stages of transforming truth values into truth functions and then truth functions into truth values in classical logic seem to be redundant. However, a reasoning within a truth function space can be useful, but real advantages of such an approach can be noticed for fuzzy logic.

4. EXTENDING THE SOLUTION FOR FUZZY LOGIC

Steps of Baldwin's mechanism for fuzzy logic are similar. However, an additional phase of calculating a truth function of a premise is needed. Assigning one of the three truth functions is not an option, because of an infinite number of truth values and, therefore, an infinite number of possible truth functions. For fuzzy logic truth functions are denoted by the Greek letter τ (τ: [0,1]→[0,1]) and are a membership functions of a fuzzy truth values [2].

Truth function of a premise τ_p, when fuzzy sets of a fact and a premise are given, can be calculated using an extension principle [1]:

$$\underset{\eta\in[0,1]}{\forall} \quad \tau_p(\eta) = \sup_{\substack{\eta=\mu_A(x)\\ x\in X}} \left[\mu_{A'}(x)\right]. \tag{10}$$

The result of (10) answers the question *"what is the truth of the expression that A' is A"* i.e. the truth of the expression *"a temperature is medium"*. The equation (10) generates a membership function (a fuzzy truth value) which is relevant to a relation between a fact and a premise. Several characteristic truth functions were named like: absolutely true, true, very true, fairly true, absolutely false, false, very false, fairly false and undecided [1]. Infinite number of possibilities represent states between the named cases. Examples of fuzzy truth functions and a truth function modification for two cases are depicted in fig. 2.

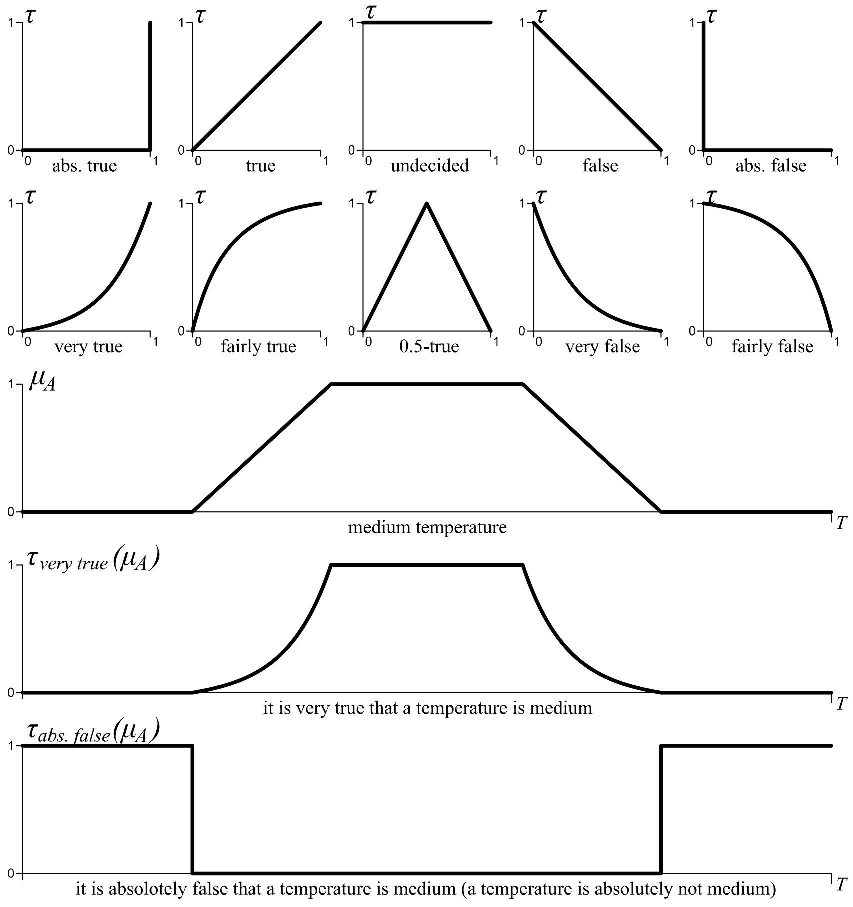

Fig. 2. Examples of fuzzy truth values and a truth function modification.

The subsequent step of Baldwin's reasoning is a generalization of (8) for fuzzy logic

$$\underset{\phi\in[0,1]}{\forall} \quad \tau_q(\phi) = \sup_{\eta\in[0,1]} \left[\tau_p(\eta) *_T I(\eta,\phi)\right]. \tag{11}$$

The only difference is the analyzed space. For classical logic it is a set {0,1} and for fuzzy logic it is a range [0,1]. Similarly to (9) the result of the equation (11) is a truth function of a conclusion.

It is very important to emphasize that a fuzzy truth function τ_p preserves a fuzzy relation between a fact and a premise, which is carried through the reasoning process.

Another very important element is the fact that the approach moves a reasoning process into a truth space only. When no universes of discourse of premises are present, only their truth functions, it is easier to deal with compound expressions like p_1 AND p_2 AND p_3, or p_1 OR p_2 OR p_3. Baldwin suggested the following method, which can be obtained from the extension principle

$$\underset{z\in[0,1]}{\forall} \quad \tau_p(z) = \sup_{\substack{x *_N y = z \\ x,y\in[0,1]}} \left\lfloor \tau_{p_1}(x) *_T \tau_{p_2}(y) \right\rfloor, \tag{12}$$

where $*_N$ indicates triangular norm, either T-norm or S-norm, depending on a type of conjunction (AND or OR respectively).

A unified space of all truth functions allows equation (12) to calculate τ_p of a compound premise with multiple connectives by joining subsequent truth functions one by one. A complexity of such a solution is linear and in comparison to Zadeh's approach it is a very big advantage, because adding a new simple premise to a rule causes only a little more cost.

5. NUMERICAL EXAMINATIONS

Possible acceleration of numerical computations for Baldwin's reasoning was tested using the FUZZLIB library [6]. A sample rule with Gaussian membership functions of fuzzy facts, premises and a conclusion were constructed. A reasoning method and a number of simple premises were parameters of an examination, where a time of a reasoning process was measured. To reduce an influence of other computer tasks each examination was reiterated (an average was calculated as one final result).

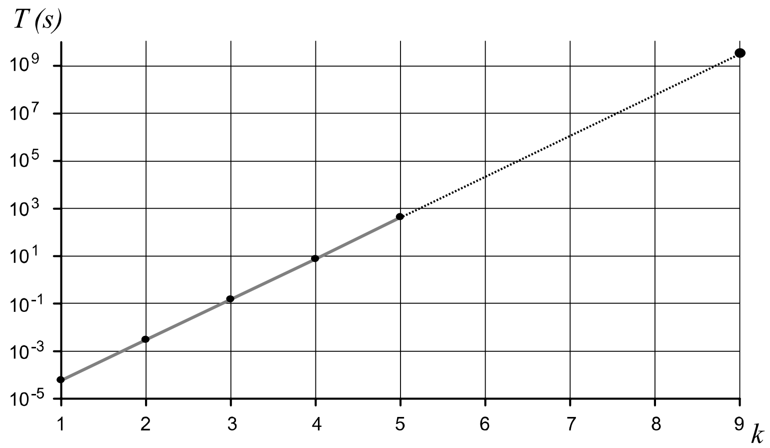

Fig. 3. Dependence of a reasoning time on number of conjunctions in a rule with a compound premise for Zadeh's compositional rule of inference (general case, without any simplification).

The first group of tests considered the general approach of Zadeh (2). The results are depicted in fig. 3. Computations were continued to 5 premises only, because of an exponential complexity of the problem (extending a rule by one simple premise adds one dimension to the analysis). A dotted line from 5 to 9 is an extrapolation.

It is necessary to emphasize that such a complexity characterizes only a general case, where any T-norms and fuzzy implications are considered. In some cases, such as T-norm minimum, the complexity for simplified equation (2) is linear [8].

The second group of tests considered Baldwin's reasoning. The results depicted in fig. 4 confirm a linear time complexity of the problem.

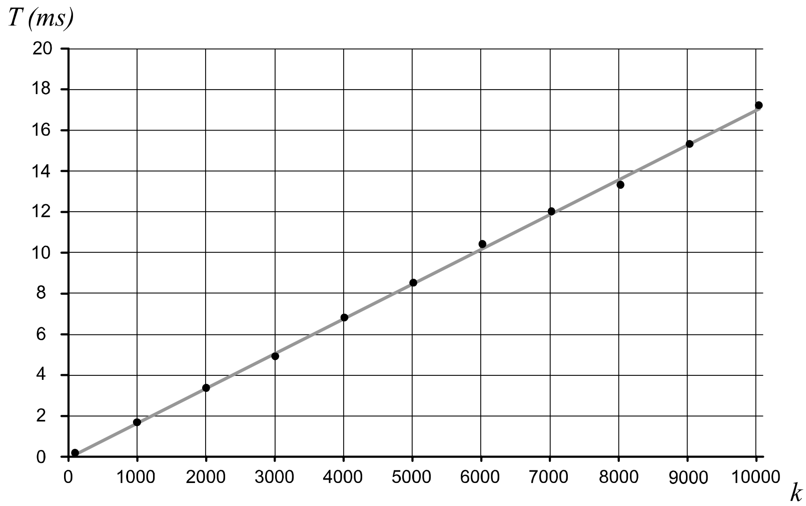

Fig. 4. Dependence of a reasoning time on number of conjunctions in a rule with a compound premise for Baldwin's reasoning based on a fuzzy truth value.

6. SUMMARY

By a brief analysis of two reasoning methods (based on a fuzzy truth value and the compositional rule of inference) a couple of important advantages of Baldwin's approach can be found, in spite of its more complex theory.

The most important advantage is preserving a fuzzy relation between a fact and a premise in a reasoning process with linear computational complexity. All universes of discourse of different premises are transformed to fuzzy truth values representing a unified relation between a fact and a premise. That allows the mechanism to build conjunctions within the same truth space and implies computation in only three dimensions. In a general case Zadeh's approach is susceptible to extending the set of premises because of an exponential computational complexity. Although the simplified version, described by (3)(4)(5), is less complex than Baldwin's solution, it reduces a relation between a fact and a premise to one precise value and loses a fuzziness.

A linear computational complexity allows the considered method to be applied even in very complex expert systems like applications in biology, genomics and medicine (i.e. [9]). The efficiency is proofed by numerical examinations performed for rules with thousands of connectives in a compound premise (the results depicted in fig. 4).

Baldwin's approach seems to be closer to classical logic because the proposed extension, in a form of truth functions and the reasoning mechanism, is easily applied (almost directly) to fuzzy logic. Moreover, providing an additional level of abstraction, by describing a truth of statements with linguistic (fuzzy) truth values, seems to be closer to human reasoning as well.

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