*impulsive noise, myriad filter,
meridian filter, fetal heart rate,
FHR baseline*

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APPLICATION OF GENERALIZED FILTERS FOR ESTIMATION OF FETAL HEART RATE BASELINE

This paper addresses the problem of impulsive noise cancellation in digital signal area. The myriad and meridian filters are the type of robust filters which are very useful in suppressing the impulsive type of noise. The cost functions of theses filters have very similar structure. In this paper the generalized filter based on L_p norm is presented. The proposed filter operates in a wide range of impulsive noise due to the proper adjustment of p in the L_p norm. The presented filter is applied to suppress an impulsive noise in fetal heart rate (FHR) signal. Simulation results confirm the validity of the proposed filter.

1. INTRODUCTION

Cardiotocography is a widely used method of fetal monitoring, which enables evaluation of a fetal condition during pregnancy and in labour. It relies on simultaneous acquisition and analysis of three signals: fetal heart rate (FHR), maternal uterine contractions and fetal movement activity. In traditional cardiotocography the signals are recorded and processed by a bedside fetal monitor. The visual evaluation of printed waveforms is subjective and considerably depends on the experience and knowledge of clinicians. External computer-aided automated analysis allows for more accurate evaluation of signals, providing the obstetrician with a quantitative description of traces. It considerably improves the objectivity and reproducibility of signals interpretation [13,14].

Starting point of all algorithms for automated patterns detection is the estimation of so called FHR baseline which can be obtained as a result of removing the distortion from FHR signal. It is a common opinion, that just the algorithm for FHR baseline estimation determines the accuracy of quantitative analysis of the entire signal. The existing methods of FHR baseline estimation are presented in [8-10]. Event small differences from real shape of the baseline may significantly distort detection of the key patterns and thus lead to misdetection of fetal distress [6].

The paper is organized as follows. Section 2 contains definition of cost function as well as definitions of weighted myriad and weighted meridian. Section 3 introduces the generalized cost function. The section 4 shows obtained results. Conclusions complete the paper.

2. FAMILY OF M-ESTIMATOR FILTERS

One of the popular robust method is the method based on the maximum likelihood estimators (M-estimators) [3-5]. The principle of M-estimators can be formulated in the following way. Assume that a set of N data samples $x_1, x_2, \ldots, x_N$ is given, where $x_i=\beta_i+v_i$ and $1 \le i \le N$. The problem is to estimate the location parameter β_i under noise component v_i. This parameter identifies the position of the probability density function (pdf) on the real line of data samples. The distribution of v_i is not assumed to be exactly known. The only basic assumption is that $v_1,\ldots,v_N$ obey a symmetric, independent and identical distribution (symmetric i.i.d).

The generalized form of M-estimator is defined as follows. If the cost function $\rho(x, \beta)$ is chosen as

$$\rho(x,\beta) = -\log f(x,\beta) \tag{1}$$

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then the M-estimate gives the ordinary maximum likelihood estimate, where $\boldsymbol{x}$ is the observed random variable with the $f(x)$ probability density function, and β is the parameter to be estimated. In practical situations, the underlying noise *pdf* is difficult to estimate, and $\rho(x, \beta)$ is usually chosen as a fixed function of residual $\boldsymbol{z}=\boldsymbol{x}-\beta$, that is $\rho(x, \beta)=\rho(x-\beta)$.

The M-estimate was originally proposed to improve robustness of statistical estimators subject to small deviations mentioned above. The M-estimate of $\hat{\beta}$ is defined as the minimum of a global energy function

$$\hat{\beta} = \arg\min_{\beta\in\Re} \sum_{i=1}^{N} \rho(x_i - \beta). \tag{2}$$

The function $\rho(\bullet)$ is called the penalty or the cost function. An M-estimator of location is defined as the parameter $\hat{\beta}$ that minimizes the expression (2). The behavior of the M-estimator is completely characterized by the shape of $\rho(\bullet)$ function [5].

2.1. WEIGHTED MYRIAD AND MERIDIAN COST FUNCTIONS

For the Cauchy distribution, the location parameter is called the sample myriad. For a given set of N independent and identically distributed samples each obeying the Cauchy distribution with common scale parameter, the sample myriad is a value that minimizes the following expression [1,15]

$$\hat{\beta}_K = \arg\min_{\beta\in\Re} \sum_{i=1}^{N} \log\left[K^2 + (x_i - \beta)^2\right], \tag{3}$$

where: β is the location parameter, and K is the scale parameter. By assigning non-negative weights to the input samples, the weighted myriad is derived as a generalization of the sample myriad. For the N i.i.d. observations $\{x_i\}_{i=1}^{N}$ and the assigned weights $\{w_i\}_{i=1}^{N}$, the weighted myriad can be computed from the following expression

$$\hat{\beta}_K = \arg\min_{\beta\in\Re} \sum_{i=1}^{N} \log\left[K^2 + w_i(x_i - \beta)^2\right]. \tag{4}$$

The value of the weighted myriad depends on the dataset $\boldsymbol{x}$, the assigned weights $\boldsymbol{w}$ and the scale parameter K. Two interesting cases may occur. First, when the K value tends to infinity, then the value of weighted myriad converges with the weighted mean. This property is called myriad linear property [8]. Second interesting case called modal property, occurs when the value of K parameter tends to zero. In this case the value of the weighted myriad is always equal to one of the most frequent values in the dataset [1,4,11].

The random variable formed as the ratio of two independent zero mean Laplacian distributed random variables is referred to as the Meridian distribution. For the given set of N i.i.d. samples $\{x_i\}_{i=1}^{N}$ each obeying the Meridian distribution with the common scale parameter δ (called medianity parameter), the sample meridian is given by [2]

$$\hat{\beta}_\delta = \arg\min_{\beta\in\Re} \sum_{i=1}^{N} \log\left[\delta + |x_i - \beta|\right]. \tag{5}$$

The sample meridian can be generalized to the weighted meridian by assigning non-negative weights to the input samples, then the weighted meridian is given by

$$\hat{\beta}_\delta = \arg\min_{\beta\in\Re} \sum_{i=1}^{N} \log\left[\delta + w_i|x_i - \beta|\right]. \tag{6}$$

The behavior of the weighted meridian significantly depends on the value of its medianity parameter δ. Again, two interesting cases may occur. The first case occurs when the value of medianity parameter tends to infinity, the weighted meridian is equivalent to weighted median. This property is called the median property. The second interesting case, called the modal property, occurs when the medianity parameter δ tends to zero. In this case, the weighted meridian $\hat{\beta}_\delta$ is equal to one of the most repeated values in the input dataset [2,16].

3. GENERALIZED COST FUNCTION

Comparing the properties of the M-estimators presented in previous sections, a common features can be found. One of them is the behavior of the estimators when the spare parameter (e.g. K or δ) tends to zero. Then, for the same dataset $\boldsymbol{x}$, the value of the weighted myriad is equal to the value of the weighted meridian. Another common feature of the both M-estimators is similar form of the cost functions. The weighted myriad cost function uses the L_2 norm while the weighted meridian cost function uses the L_1 norm.

Let the L_p norm be defined as follows [12]:

$$\|\mathbf{z}\|_p = \left(\sum_{l=1}^{s} |z_l|^p \right)^{1/p}, \tag{7}$$

where: $\boldsymbol{z}$ is an s-dimensional real valued vector (i.e. $\boldsymbol{z} \in \Re^s$). Applying the L_p norm to the weighted myriad cost function (4) or weighted meridian cost function (6), the generalized cost function can be expressed in the following form

$$\rho_\xi^{(p)}(\beta) = \sum_{i=1}^{N} \log\left[\xi + w_i \|x_i - \beta\|_p\right], \tag{8}$$

where: $\|\cdot\|_p$ is the L_p norm to the pth power, and the parameter ξ corresponds to the medianity parameter δ for $p=1$, and to the linearity parameter K for $p=2$. It should be mentioned, that for $p=1$ the ξ parameter is equal to the medianity parameter δ, but for $p=2$ the ξ parameter is equal to the square root of the linearity parameter K (i.e. $\xi = \sqrt{K}$).

The objective function $\rho_\xi^{(p)}(\beta)$ for fixed value of ξ and given dataset $\{x_i\}_{i=1}^{N}$ the order statistics of $\boldsymbol{x}$ has the following properties:

1. $\rho_\xi^{(p)}(\beta)$ is strictly decreasing for $\beta < x_1$, and strictly increasing for $\beta > x_N$,
2. all local extrema of $\rho_\xi^{(p)}(\beta)$ lie in the interval $[x_1,\ldots,x_N]$,
3. if $0 < p \leq 1$, the solution is one of the input samples (selection type filter),
4. if $1 < p < \infty$, the objective function has at most ($pN-1$) local extrema points and therefore a finite set of local minima.

Figure 1 shows a typical shape of the $\rho_\xi^{(p)}(\beta)$ cost function for different values of p and for ξ=5.

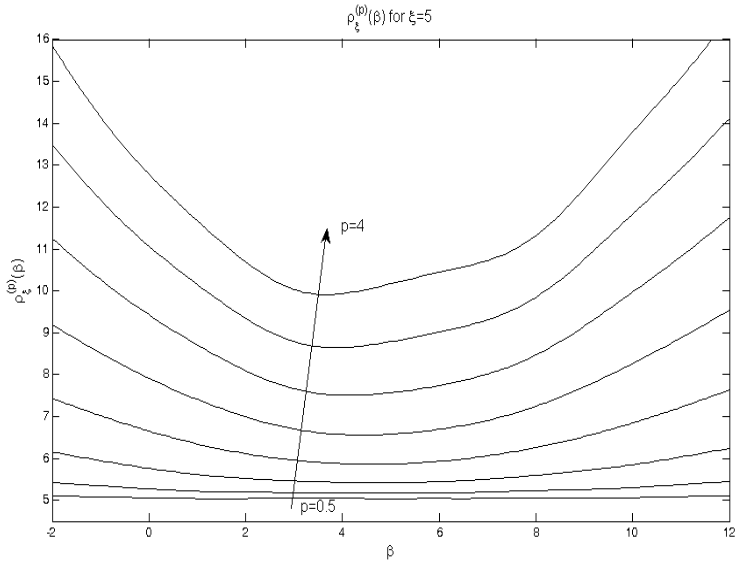

Fig. 1. Typical $\rho_{\xi}^{(p)}(\beta)$ functions for different values of p∈{0.5, 1, 1.5, 2, 2.5, 3, 3.5; 4} and ξ=5. Input samples are x={4.9, 0, 6.5, 10, 9.5, 1.7, 1}.

For the given dataset $\{x_i\}_{i=1}^{N}$ and the assigned weights $\{w_i\}_{i=1}^{N}$, let the $\hat{\beta}_{\xi}$ be the value minimizing the generalized cost function (6), i.e.:

$$\hat{\beta}_{\xi} = \arg\min_{\beta\in\Re} \sum_{i=1}^{N} \log\left[\xi + w_i \left\| x_i - \beta \right\|_p\right]. \tag{9}$$

Equation (9) defines the output of the proposed generalized filter. For *p=1* a weighted meridian is a special case of $\hat{\beta}_{\xi}$, and for *p=2* the weighted myriad is a special case of $\hat{\beta}_{\xi}$. Properties of the $\hat{\beta}_{\xi}$ value are presented in Table 1. It can be noted that the proper adjustment of ξ for Cauchy-based cost functions is well known and in the literature exists a few recipes to solve it (for example [4][17]).

The fundamental problem of the weighted robust filters is generally the proper selection of weights and the window length. The obvious fact is that special optimization procedures exist and in the adaptive way it is possible to find proper values of weights. But such operations are very time consuming and the optimization procedure has to be repeated when the properties of signal changes. For example, the description of the adaptive weighted myriad filter is described in [11]. In this work for simplicity and without a loss of generality that problem, the weights $w_i=1/N$ for $i=1,\ldots,N$ and further analysis of the weights value are not considered. The method for estimation the output of investigated filter is the fixed-point method presented in [1,4].

Table 1. Selected properties of the $\hat{\beta}_{\xi}$ estimator.

ξ	*p=1*	*p=2*
ξ→0	Most frequent value in the input dataset	
0 < ξ < ∝	$\hat{\beta}_{\xi} = meridian\left(w_i * x_i \mid_{i=1}^{N}; \xi\right)$	$\hat{\beta}_{\xi} = myriad\left(w_i * x_i \mid_{i=1}^{N}; \sqrt{\xi}\right)$
ξ → ∝	$\hat{\beta}_{\xi} = median\left(w_i * x_i \mid_{i=1}^{N}\right)$	$\hat{\beta}_{\xi} = mean\left(w_i * x_i \mid_{i=1}^{N}\right)$

4. SIMULATION RESULTS

In our experiment, fetal heart rate signals were used. All these signals were recorded using the computer aided fetal monitoring system MONAKO [7]. The signals were obtained from fetal monitor which provided every 250 ms the consecutive digital measurements of FHR signal. Such samples have value in the range from 50bpm to 210bpm, but in case of signal loss, due to bad measurement conditions, its value is set to zero. So, all the epochs with zero amplitude in the FHR signals can be regarded as impulsive noise in the signal and undergo filtration process.

Figure 2 shows the FHR signal. The left part of figure presents a good quality FHR i.e. without signal loss. The right part presents a bad quality FHR signal with signal loss marked as zero value samples. Figure 3 shows the obtained output signal for *p=2.5* and different window lengths, and figure 4 presents the obtained output signal for *p=3.0*. Figure 5 shows the output signal for *p=3.0*, when the input signal contains zero amplitudes.

It can be noticed, that for both cases (good and bad quality FHR signals), the impulsive noise was suppressed. For the window length greater than *N>71*, short loss of signal does not affect the output signal. In such a case, the signal loss is regarded as outliers and does not affect the filter output.

5. CONCLUSIONS

In a real biomedical signal analysis we deal with a noise. In case of the cardiotocography signal, the noise amplitude is relatively high. The impulsive noise decreases the signal quality and makes it difficult to analyze the signal without filtering. In this paper, the idea of the generalized filter was presented. Depending on the filter parameters, the output of the proposed filter can be: weighted meridian, weighted myriad, weighted median or weighted mean. An application of the proposed filter for the estimation of the fetal heart rate confirms its usefulness in real applications. The proposed method significantly increases the accuracy of the FHR baseline estimation in these episodes with respect to previously presented method. The present work solves the problem of performance and estimation of the ξ value.

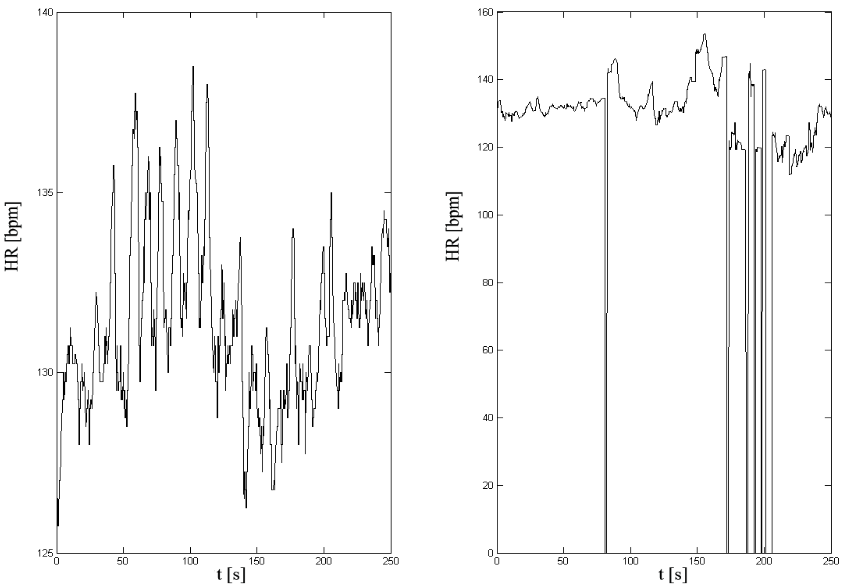

Fig. 2. Two examples of fetal heart rate signals. The left fragment presents a good quality FHR i.e. without signal loss episodes. The right one presents a bad quality FHR signal with signal loss episodes marked as the FHR samples with zero value.

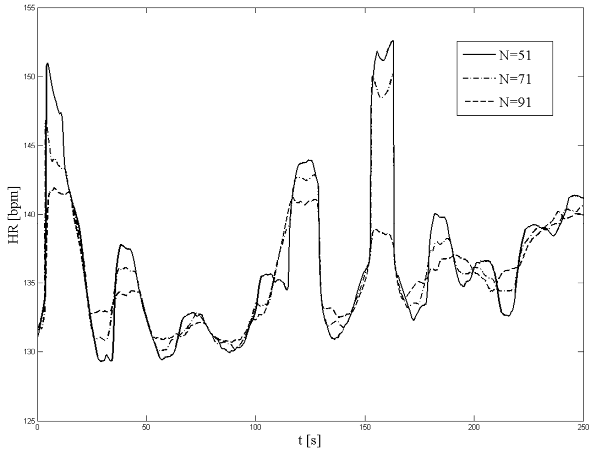

Fig. 3. Output signal of the proposed filter obtained for *p=2.5* and for the good quality FHR signal.

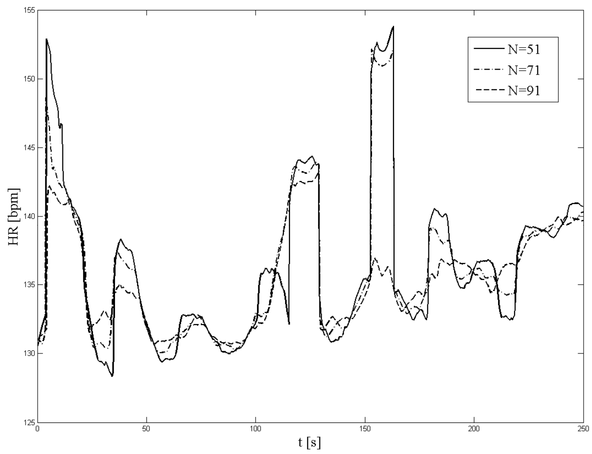

Fig. 4. Output signal of the proposed filter obtained for *p=3.0* and for the good quality FHR signal.

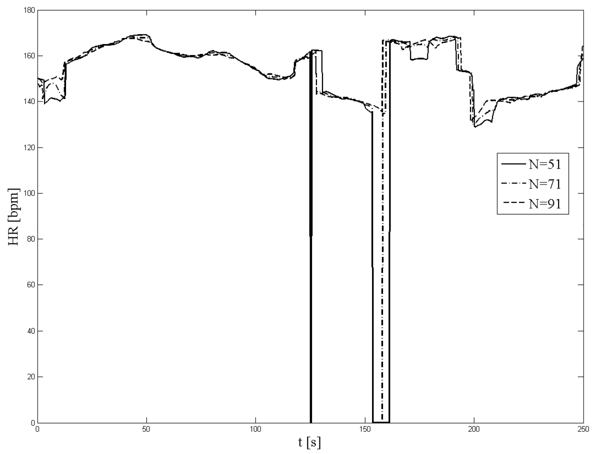

Fig. 5. Output signal of proposed filter obtained for *p=3* when the input is the bad quality FHR signal contained zero value FHR samples.

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