optimal training loads,
mathematical model

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APPLICATION OF A MATHEMATICAL MODEL IN CONTROL OF SPORT TRAINING

The value of each state variable (physical condition, sport scores) at the end of certain period is a function of general state of the athlete, his scores described at the beginning of this period of time and adapted training, that is implemented intensity of separate training means (controlling variables) in the analyzed time period. The aim of this work was to determine optimal values of control in swimmer training, targeted at achieving the best score in 25 m and 800 m swimming. For this purpose a model was developed, beginning with its shape (it has been defined, what is state variable and what is control, how the state variables and control variables influence the increase) to determining numerical values of all parameters. The basis for the development of the detailed model was pedagogic experiment conducted on the group of 14-year old boys. Conclusions were drawn on the practical possibility of using a mathematical model in control of sport training.

1. INTRODUCTION

The developments of sport seen in recent years significantly stressed the need of close control of training loads used. The problem of relation between training stimulus and reaction of the individual to this stimulus is one of most significant problems of sport training. The fast increase of sport scores forces the athletes to continuously increase the volume and intensity of sport training. However, this trend has approached the limits of the adaptability of human body. The use of too high training loads leads to overtraining. The contusions appear resulting from physical limitations of the human body. As a result of constant overload, the states of apathy, fatigue and indifference appear. On the other hand, the use loads too low does not bring the expected results [2,14,17].

Taking the above into account, an attempt is needed for determination, how the volume and structure of the load should change to give the result optimizing the current abilities of the athlete and characteristics of training.

The number of authors investigating modelling of training loads is relatively small. Among the few are papers of Gordon et al. [4], Witt [23], Ryguła [13], Mester, Perl [8], Perl et al. [10]. In spite of the attempts, no algorithm has so far been found to enable precise control of training. Taking into account the fact of approaching the limits of the adaptation ability of human body [3,7,17,22], the fundamental problem of to-day sports seem to be developing the tool to enable non-verbal (non-intuitive) implementation of training control.

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THE QUESTIONS AND RESEARCH HYPOTHESES

With a view to meet the urgent needs of sports theory and practice, the aim of this work is to respond to the following research questions:

1. Do we have mathematical tools able to point the ways of planning optimum training loads?
2. How to determine the parameters of training loads?
3. How to use the developed mathematical model?

In this work, the following research hypothesis was adapted:
On the basis of training experience, contained in collected and precisely described experimental data, it is possible to plan, compute and implement the optimal training.

2. MATERIAL AND METHODS

The investigation covered 40 seven-graders aged 14, of comparable biological age [21], from the sport school in Mysłowice.

The main experiment was preceded by the efficiency preparation. In the first year of training, the subjects had training sessions twice in a week in swimming pool (45 min). In the second year additionally once per week in gymnasium (90 min). Both in the first and second year, the training was general and supplementary in character. During the proper experiment, the subjects were divided into four groups. They had three times a week swimming pool sessions (45 min) and additionally two times per week in gymnasium (1.5 hour).

In the investigation was developed a simplified catalogue of training means used, that were logged both from the quantitative and qualitative point of view. The quantity was expressed by time of exercise, while the qualitative point of view was described using the concept of "intensity range" [18].

During the experiment, the following training means were used, playing the role of decision variables (control variables):

The means used on land:

1. General development exercises: elements of gymnastics, athletics, strength exercises (int.4).
2. Continuous running, running plays, team games, motion games (int.3).

The means used in water:

3. Speed exercises: distances to 25 m., starts, turns, dives (full break) (int.5).
4. Exercises in the anaerobic milk acid area: distances 25 to 50 m, swimming with arms only or legs only and in coordination of these elements. Full rest breaks (int.4).
5. Exercises in the mixed area: distances 100 - 200 m, swimming AA, LL and in coordination medium break. (range int.3)
6. Exercises in the anaerobic area: distances 400 to 800 m, short break (range int.2)
7. Easy swimming, free swimming. (range int.1)
8. Exercises improving swimming technique. (range int.2)
9. Exercises teaching technique. (range int.1)

10. Competition starts.

The training loads were changed in monthly cycles both qualitatively and quantitatively. The loads were chosen randomly. All adapted training means had their limits.

For development of a model, the quality indices were alternative scores on 25 m (X1) or 800 m (X2) distance. The measurements were made under the competition conditions.

During the experiment, the following features (variables) were measured:

1. State variables: body height [cm] X3, body mass [kg] X4, the length of lower limbs [cm] X5, the length of upper limbs [cm] X6, oxygen efficiency evaluated on the basis of Vo_{2max} [l/kg * min] X7, anaerobic efficiency evaluated by maximum power obtained in Wingate test [W/kg] X8, breathing capacity [cm^3] X9, static wide jump [cm] X10, score in 10 m run [s] X11, pulls on the rod [N cycles] X12 ("Eurofit"), swimmer step [number of full cycles at the distance of 25 m] X13.
2. Decision variables: summary time of exercise in the separate training cycles in 10 training cycles described above.

The state variables were measured in semiannual cycles, during the initial training and before the proper experiment and after each monthly training period.

Table 1. Static characteristics of state variables of the investigated athletes before the start of experiment.

State variables	$\bar{x}$	S	A_s	Ku-3	V
Heigth [cm]	164,2	7,37	1,456	2,824	4,48
Body mass [kg]	50,73	7,76	-1,081	2,591	15,20
Length of upper limbs [cm]	73,49	3,94	0,952	2,171	5,36
Length of lower limbs [cm]	79,15	4,19	1,259	2,651	5,29
Vo_{2max} [L/min]	3,15	0,52	-1,721	2,981	16,50
Maximum power [W/kg]	9,21	1,49	1,081	2,191	16,20
Breathing capacity [cm^3]	3027,5	243,59	2,001	-1,852	8,04
Static wide jump [cm]	199,15	24,46	1,061	2,856	12,30
10 m run[s]	1,50	0,21	-1,536	-2,896	42,00
Pulls on the rod [number of repeats]	4,40	1,31	1,826	1,009	29,80
Swimmer step [number of full cycles]	14,66	2,66	1,081	2,007	18,10

$\bar{x}$ - mean, S - standard deviation, A_s - asymmetry index, Ku - kurtosis, V - variability index

3. CONSTRUCTION OF THE MATHEMATICAL MODEL FOR SWIMMERS TRAINING

The value of each state variable (physical condition, scores) at the end of certain unit of time is a function of general condition of the athlete, his scores, described by the state variables at the beginning of this unit and adapted training, that is implemented intensity of separate training means (control variables) in the analyzed unit of time. This may be written in symbolic notation:

$$X^{(l+1)} = X^{(l)} + \Delta X^{(l)}$$

where

$$\Delta X^{(l)} = \Phi(X^{(l)}, U^{(l)})$$

l denotes the number of training periods, l=0,...,N-1.

The aim of the optimization of swimmers training is to determine such a set of controls $U^{(0)}$,… $U^{(N-1)}$, to obtain the extremum of the value of certain state variable (the score, physical fitness).

$X^{(N)}$=max (min)

In a general case determination of the Φ is not possible. Its approximation may be implemented for certain group of athletes, for which we know the measurement results of both state variables and control variables. To be able to use optimization methods and automatic the construction of a model, the Φ function should be expressed with possibly simple formula. The following formula is suggested:

$$\Phi(X,U)=aX+cU+bUX+d$$

This is a part of Taylor series for the Φ function. In the above formula, the first term at the right hand side (aX) may be interpreted as a description of the influence of the condition of the athlete on his scores, the second term (cU) - as the influence of training, and the third (bUX) - the influence of condition and training together.

To be able to use the methods of determination of optimum control according to Pontriagin principle, the model should be transformed into continuous model and replace finite increments in the training periods with derivatives.

For further analysis, we have assumed to have at our disposal the measurement of n state variables and m training means. Henceforth, the state variable vector is denoted as $X(t)=(x_1(t),...,x_n(t))^T$, vector of control variables as $U(t)=(u_1(t),...,u_m(t))^T$. The upper index ^{T} denotes transposition.

In accordance with earlier considerations, as a model of sport training has been adapted a set of differential equations of the form:

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} a_{ij}x_j + \sum_{j=1}^{n}\sum_{k=1}^{m} b^i_{jk}x_j u_k + \sum_{j=1}^{m} c_{ij}u_j + d_i + h_i(t,U) \quad i=1,...,n \qquad (1)$$

where:

x_i is the i-th state variable, i=1,...,n,

u_j is the utilization of the j-th training means, j=1,...,m.,

$u_j \in [0,1]$, 0 - none, 1 - maximum possible use of the j-th training means.

This equation was considered in the interval [0,T], where 0 is the conventional beginning and T - the end of the training period. It is also possible to use in the model the absolute values of the training means, such as the time of doing a certain exercise during the training. In this case, defining the upper and lower limit for such control is also required.

The quality of the adaptation of the model to the measured data was determined from the formula:

$$\delta_i = \frac{H_i}{\sum_{l=1}^{z}\sum_{s=0}^{N-1}((\xi_i^l(s+1)-\xi_i^l(s))^2} \quad i=1,\ldots,n \tag{2}$$

$\xi_i^l(s)$ - value of i-th state variable for l-th athlete in s-th time unit,
H_i - partial derivatives Hamilton function.

The quantity δ_i will be henceforth called coefficient of fitting.

Because of the space limitations at this paper, the description of the method of model solving and determination of its parameters has been omitted. These results may be checked by the authors.

4. THE RESULTS OF MODELING

We have decided to use the first seven state variables and all control variables for the construction of the model. The smaller number of variables will facilitate the interpretation of results. The simplification of the model brings material shortening of computation, which enabled to make a greater number of different optimizations. To confirm the correctness of this approach, in two cases (maximizing the score on 25 m and 800 m), the computations were made for the full and abridged model. The optimum controls and values for best score were almost identical.

Using the formula (2) the coefficients of fitting model to data were calculated; they form multidimensional equivalent of the relative error (ration of difference between the exact value and approximated to the exact value). The results are shown in table below.

Table 2. The coefficients of fitting for separate equations

Equation for variable	δ_i [%]	Equation for variable	δ_i [%]
X1	5.5	X5	4.9
X2	4.0	X6	1.2
X3	6.8	X7	4.4
X4	5.5		

The values of the coefficients of fitting are very small, which indicates a great accuracy of the model. It was therefore assumed that the model sufficiently approximates the experimental data.

After construction of the model, one athlete was chosen and his initial conditions were introduced to the model. Two optimal controls were determined, one for maximization of point score at the distance of 25 m and the second for 800 m distance. Table 3 shows the results theoretically possible to obtain in case of using the optimal training. The graphs 1,2 illustrates the course of controls for optimization of 25 m score.

Table 3. Comparison of the scores: real and calculated from the model at different optimizations for the athlete X_E.

Maximum score	State variable						
	X1	X2	X3	X4	X5	X6	X7
25 m	368	121	196	14	143	30	2506
800 m	315	139	196	13	143	32	2790
Real	**191**	**105**	**190**	**16.0**	**140.0**	**30.0**	**2500**

Because in the numerical calculation of the model equations, the step equal to half of the observation period was assumed, the optimal control was determined with the frequency of two weeks. The values of the separate control variables therefore define the total time of doing given exercise during two weeks.

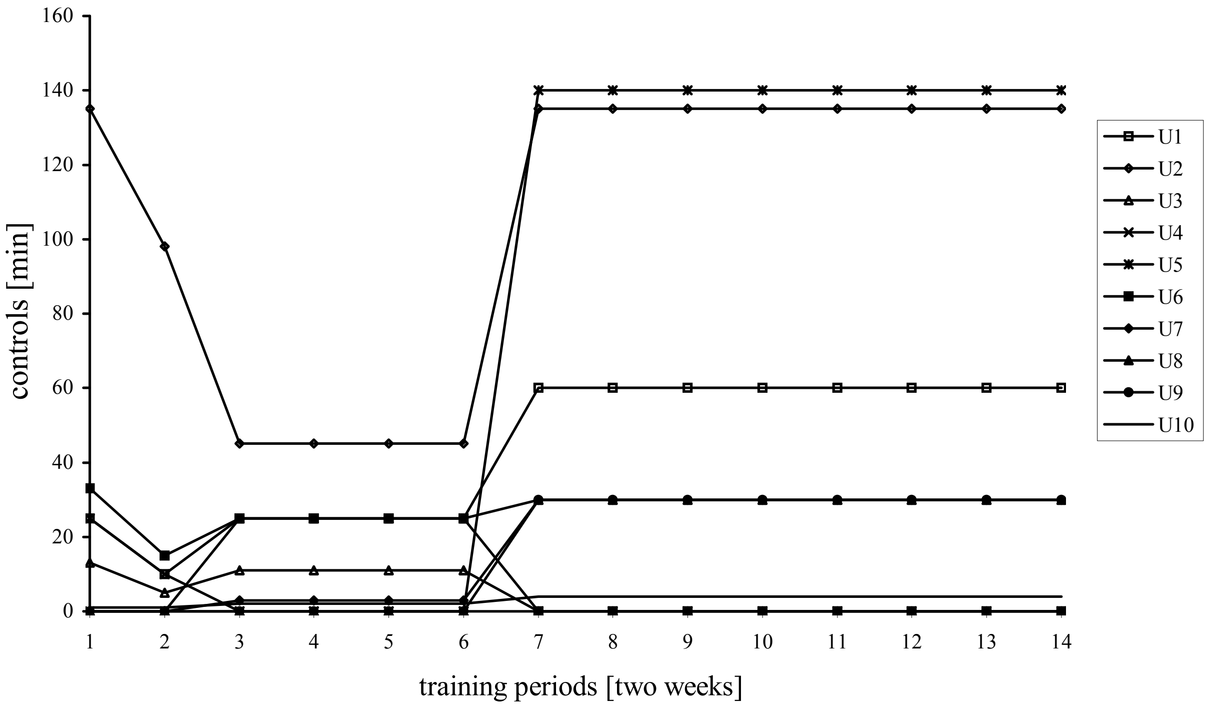

Fig. 1. Optimum controls of 10 training means.

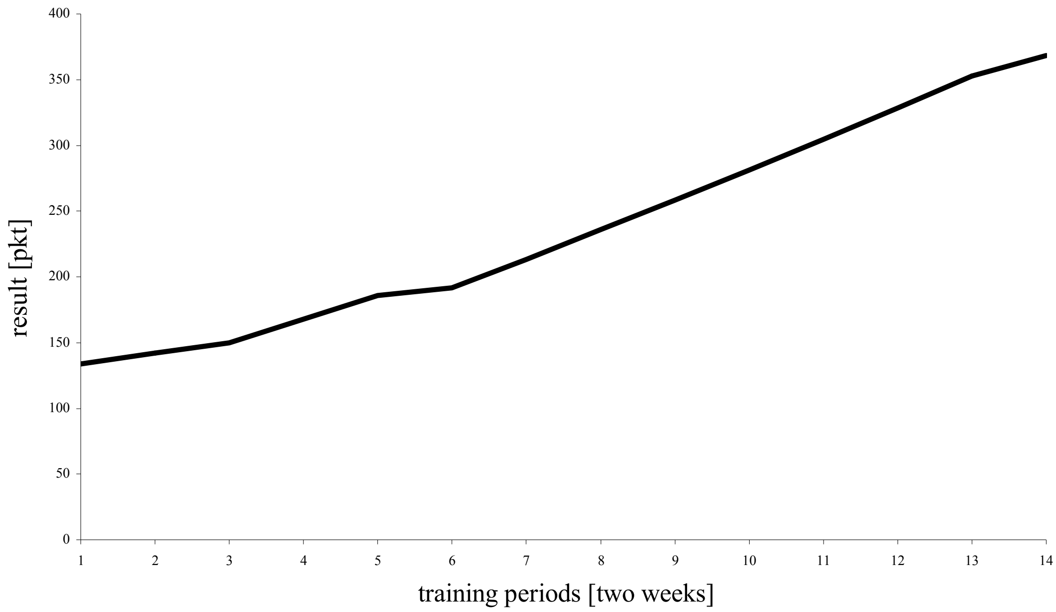

Fig. 2. Value of the quality coefficient (25 m score) of chosen swimmer, applying optimum controls**.**

5. DISCUSSION OF RESULTS

The analysis of the optimal control functions has shown that they are time variable. It is worth noticing that each period of training is characterized by variable controls. The variability range is very large, and the proportions between the controls used significantly differ from each other depending on the training period.

The theorists of sports have long represented the opinion that controlling at the training process is possible and even should be implemented by systematic comparison of the real state of the athlete's body and his score with the so called model criterion [13]. A help was obtained from the mathematicians and physicists, who have created methods of mathematic formulation of this problem [5,9,19]. The basic problem however was in the application of using the knowledge concerning mathematic modelling in theory of sports, describing the relationship between the state variables (i.e. indices describing changes in the athlete body), decision variables (training loads used) and quality indices (results of the optimized feature or sport score). The investigation has proved that it is possible to control the sport training through the continuous flow of information on the parameters of controlled objects (athletes), as well as the value of the target function. In other words, the meaning of controlling at the sport training reduces to the adaptation of the controlled object (athlete) to constantly varying endogenous conditions and the environment of the athlete.

The results of the investigation have confirmed the theoretical considerations of Szczotka [19,20], indicating the possibility of controlling at the training, when the improvement of score is dependent on the time of using separate training means. As was the case in the investigation of the mathematical model of high jump training, Ryguła and Wyderka [15] have found a great practical importance of the model developed.

Comparing the results of the investigation of determination of the optimal control with the work of Gordona et al. [4] it should be said that the construction of the mathematical model is a helpful tool of non-intuitive control of training process. It enables computation of the optimal strategy for the athletes preparing for the competition. Summing up, we may say that with the use of training experience, contained in the collected and precisely described experimental data, it is possible to plan, compute and implement the optimal training. This enables to accept the research hypothesis formulated in this work.

6. FINAL CONCLUSIONS

The measurement results collected in the experiment and analyzed authorize to formulate the following conclusions:

- on the basis of the computed mathematical model, such sequence of changes of the control parameters may be determined, that the final effect of improving the score was maximal;
- mathematical tools are available to show the way to plan the optimal training loads;
- on the basis of the value of the fitting error of the experimental values of the quality index to maximum values of the target function (computed from the mathematical model), it may be said that the developed optimization model of maximization of the sport score has a great practical value.

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