entropy and energy measures of fuzziness,
FCM with median estimator

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THEORY OF FUZZY SETS IN EDGE LOCATION OF THE POSTERIOR CRUCIATE LIGAMENT REGION

An approach to location of a region including the posterior and anterior cruciate ligament in the MR knee images has been developed. The proposed method of the PCL location in T1-weighted MR knee images is based on entropy (or energy) measure of fuzziness and fuzzy C-means (FCM) algorithm. Then, edges of a region of interest containing the ligament are found. The procedure has been tested on clinical T1- and T2- weighted MR knee images resulting in a 3D visualisation.

1. INTRODUCTION

Posterior and anterior cruciate ligament (PCL/ACL) are anatomical structures which are (together with collateral ligament) responsible for the knee stability. Together with the shape of articular surface, muscles and contact forces ensure proper arthrokinematics. During passive motion of the knee cruciates help to change rolling into sliding movements and during active motion they resist translations and reduce shear forces. Cruciates control rotational movements in the flexed knee and together with collateral ligaments ensure rotational stability of the extended knee. For that reason cruciate ligaments belong to the group of anatomical structures, which are frequently susceptible to injuries, especially in the case of athletes. Much has been written regarding general treatment and various surgical procedures of the destabilized knee. The success of ligaments reconstructive procedure depends on many factors, mainly accurate diagnosis based on the location and visualization of the anatomical structures of the cruciate ligament.

2. THE ENTROPY AND ENERGY MASURES OF FUZZINES

In order to introduce measures of fuzziness in image quantitative description, let us first discuss a concept of a fuzzy image. This concept is based on the idea of a fuzzy signal, introduced by Czogala and Leski in [3,8]. Let us consider an image *X(N,M)* at the size of NxM with pixels *I(n,m)* and n=1,2,…,N; m=1,2,…,M. Moreover, the image is scanned

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within a window at the size of $(2k+1)x(2k+1)$. The concept of a fuzzy image derived from the original image is based on two assumptions: if no fuzzy uncertainty is considered in the image, a fuzzy image pixel $I(n,m,k)$ is reduced to a real number $I(n,m)$, which is referred to as a singleton. The measure of fuzziness is equal to zero and the information is maximal; the measure of fuzziness increases if the original image changes, as does the dynamic, as smaller amount of information is conveyed by the image. In order to construct a fuzzy image from crisp image, the pixels within the window are sorted into an increasing order is shown in the Fig. 1a. The median value is located at the (n,m) position within the window, ie. $I_{MED}(n,m,k)=I(n,m,k)$. Based on the median operation performed in windows scanned over the image, the membership function μ is defined. First we assume, that $\mu_{n,m,k}(I_{min}(n,m,k))=0$, $\mu_{n,m,k}(I_{max}(n,m,k))=0$ and $\mu_{n,m,k}(I_{MED}(n,m,k))=1$. The remain elements are defined according to the following formula

$$\mu_{n,m,k}(I_{i,j}(n,m,k)) = (2k - d_{i,j} + 1)/(2k+1) \quad (1)$$

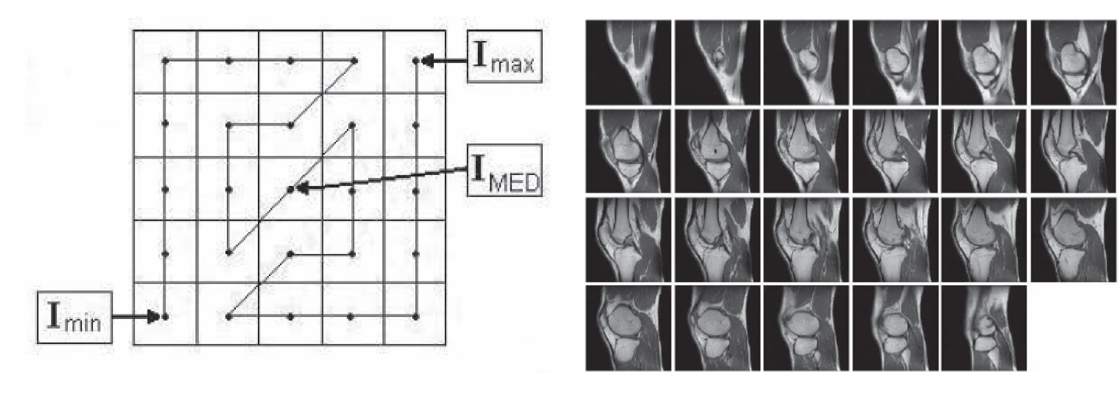
where: $d_{i,j}=|n-i|+|m-j|$. In order to discriminate against a certain level of membership $\lambda \in [0,1]$, the Heaviside pseudofunction $\mathbf{1}(.)$ is introduced. Implementation of the Heaviside function is defined as

$$\mu^{\lambda}_{n,m,k}(I_{i,j}(n,m,k)) = \mu_{n,m,k}(I_{i,j}(n,m,k))\mathbf{1}(\mu_{n,m,k}(I_{i,j}(n,m,k)) - \lambda). \quad (2)$$

The introduction of the λ level into the measurement of fuzziness allows the insignificant membership degree to be removed. A concept of the entropy measure of fuzziness has been introduced in [4] and implemented to biomedical signals in[3,8,9]. The entropy measure of fuzziness (Fig. 2a) is a mapping from the set of all fuzzy subsets of a base set X into the nonnegative reals. It can be expressed as

$$H(A,\lambda) = \int_X h_{\lambda}(A(x))dv \quad (3)$$

where: $A:X \rightarrow [0,1]$ is any v-measurable function, $dv=dx$ or $dv=p(x)dx$, where $p(x)$ stands for a



(a) (b)

Fig.1 Sorted pixels within the window (a), the signal from data base clinical hospital - MR knee T1-weighted (b) probability density function; h:[0,1]→R_+ is an increasing function in [0, 0.5], a decreasing function in [0.5, 1], and $h(0)=h(1)=0$; $F:R_+\to R_+$ is an increasing function and $F(z)=0$ if $z=0$. Substituting (2) into (3) we obtain

$$H\left(\mu_{n,m,k}^{\lambda}\left(I_{i,j}\right)\right)=F_1\left(\sum_{i=1}^{2k}\sum_{j=1}^{2k}h_{\lambda}\left(\mu_{n,m,k}^{\lambda}\left(I_{i,j}\right)\right)\cdot p\left(I_{i,j}\right)\cdot\Delta I_{i,j}\right), \tag{4}$$

where: $I_{i,j}$ denote $I_{i,j}(n,m,k)$, and $h_\lambda(z)$ is defined as

$$h_{\lambda}(z)=\begin{cases} h(z) & if\ \ z\in(\lambda,1-\lambda) \\ 0 & otherwise \end{cases}, \tag{5}$$

and $\Delta I_{i,j}(n,m,k)$ is a gradient between neighbour pixel values as marked in the Fig. 1a, and the probability density function $p(I_{i,j}(n,m,k))$ is obtained from a histogram, by dividing the number of pixels by $2k+1$. The membership functions, as defined above, serves also as a basis for the energy extraction. The energy measure is expressed as

$$E(A,\lambda)=\int_X f_{\lambda}(A(x))dv,\ where\ f_{\lambda}(z)=\begin{cases} f(z) & if\ z\in(\lambda,1) \\ 0 & otherwise \end{cases}. \tag{6}$$

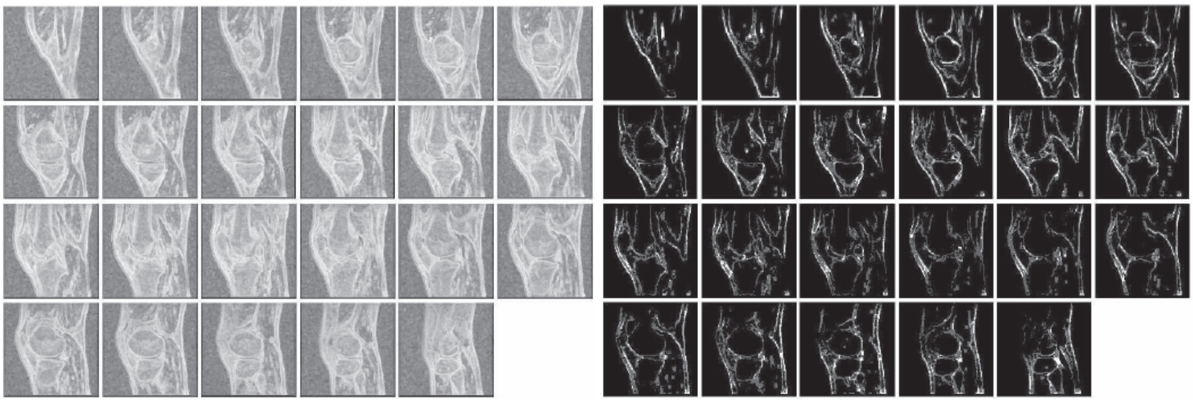
The final formula for the energy measure can be written as

$$E\left(\mu_{n,m,k}^{\lambda}\left(I_{i,j}\right)\right)=F_2\left(\sum_{i=1}^{2k}\sum_{j=1}^{2k}f_{\lambda}\left(\mu_{n,m,k}^{\lambda}\left(I_{i,j}\right)\right)\cdot p\left(I_{i,j}\right)\cdot\Delta I_{i,j}\right). \tag{7}$$

3. MEDIAN FILTERING

Standard Fuzzy C-Means (FCM) algorithm [1] is widely used in many clustering approaches. Its advantages include a conceptual and computational simplicity and the ability to model uncertainty within the data. FCM has also several weaknesses. It does not incorporate spatial context information which makes it sensitive to noise and image artifacts. In this paper a modified method of FCM is used for the clustering. The FCM objective function is modified by adding a second term, which formulates a spatial constraint based on the median estimator. Median of an ordered data set $A=\{x_1',...,x_N'\}$ is defined as follows

$$Median(A)=\begin{cases} x_{(n+1)\cdot 0.5}' & n=1,3,5,... \\ 0.5\cdot(x_{n\cdot 0.5}'+x_{n\cdot 0.5+1}') & n=2,4,6,... \end{cases}. \tag{8}$$



(a) (b)

Fig.2 MR knee T1-weighted (a) entropy measure of fuzziness, (b) FCM median modified (3 classes)

In image processing approaches an implementation of median filtering replaces each data sample by its spatial neighbourhood function. Neighbourhood function is defined as $MEDF(x,Z) = median(S)$, where $S = neighborhood(x,Z)$ and Z determines the size of the mask. Standard FCM is derived to minimize the objective function with respect to the membership function u_{in} and the prototype $\mathbf{v}_i$ for a given fuzzyfication level m (where $1 \le m < \infty$).

$$M(U,V) = \sum_{i=1}^{c}\sum_{n=1}^{N} u_{in}^{m} \left\| \mathbf{x}_n - \mathbf{v}_i \right\|^2 \tag{9}$$

where $\mathbf{x}_n = \{x_i ,...,x_k\}$ and $\mathbf{x}_n, \mathbf{v}_i \in F^k$, and F^k is a feature space. Chen and Zhang [2] have modified the objective function of FCM by introducing an element which depends on the mean value of neighbouring pixels. Since the modification propagates features of a mean filtered image into the clustering results, blurred edges is one of the most important disadvantages of the method. In order to reduce the drawback, median estimator has been added into the objective function [5, 6]

$$M(U,V) = \sum_{i=1}^{c}\sum_{n=1}^{N} u_{in}^{m} \left(\left\| \mathbf{x}_n - \mathbf{v}_i \right\|^2 + \alpha \left\| MEDF(\mathbf{x}_n, Z) - \mathbf{v}_i \right\|^2 \right) \tag{10}$$

In the Fig.2 the processed clinical MR knee T1-weighted is shown, with entropy measures of fuzziness (Fig. 2a) and entropy measures of fuzziness with the FCM median modified (Fig. 2b).

4. REGION OF INTEREST DETECTION

On the basis of the analysis of each profile in every slice of the MR knee T1-weighted after fuzzyfication and fuzzy clustering (Fig. 2b), a main axis running along thighbone and tibia is determined. The main axis is determined according to following formula

$$th = \min_k \left[L_{pr}(k) \right], \; for \; all \; L_{pr}(k) \neq 0, \tag{11}$$

where: $L_{pr}(k)$ denote number of nonzero values of the pixel intensity in the k profile. When several profiles meet condition (11), the profile of the lowest nonzero pixel value is chosen. In the next step the centering and superposition operation is implemented. Profile analysis after the centering and superposition operation permits the membership function to be created. This stage gives the information, which are indispensable for the formation of the membership function (location and size of the stripes in the main axis). On the basis of this analysis a membership function (Fig. 3a) has been determined

$$\mu(i) = \sum_{n=1}^{3} f_{Gn}(i, \sigma_n, k_n), \tag{12}$$

where: f_G denotes a Gaussian function. The Gaussian function depends on two parameters σ (the standard deviation in the specific range) and k (the average column number in the specific range) as given by

$$f_{Gn}(i, \sigma_n, k_n) = \exp\left(\frac{-(i-k_n)^2}{2\sigma_n^2}\right). \tag{13}$$

In the current study the standard deviation and average number of column take the values: $\sigma_1 = 7$, $k_1 = 25$, $\sigma_2 = 19$, $k_2 = 132$, $\sigma_3 = 5$ $k_3 = 222$, respectively.

The ROI edges including the posterior cruciate ligament satisfy the following rules:

(1) Left edge (axis I in the Fig. 4b) meets the condition

$$\min_k \left[\min_j \left[\overline{\mu}(i), pr_{i(k)}^j \right] \right], \tag{14}$$

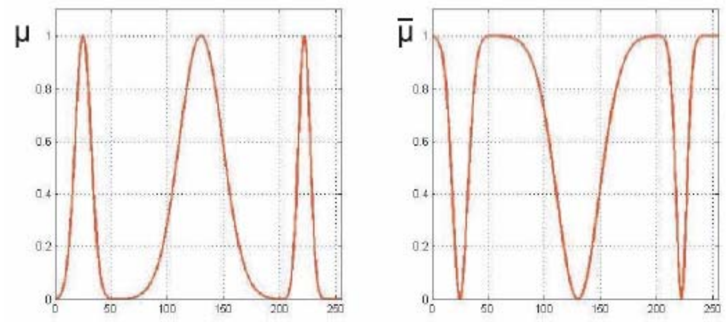
where: k denotes a profile number in the slice, j is a slice number in the image, $\overline{\mu}$ denotes a complement of the membership function (Fig. 4b), $pr_{i(k)}^j$ is a i-value of a k-profile.

(2) Upper edge (axis II in the Fig. 4b) is indicated by a horizontal profile, which meets the condition

$$\max_u [d_u], \tag{15}$$

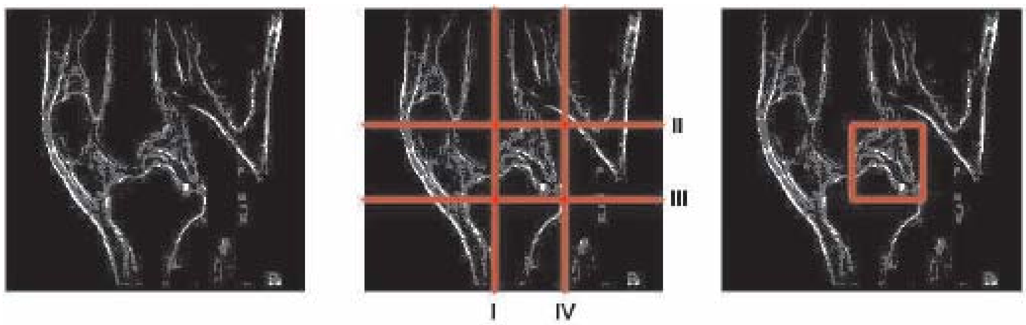
where: d_u denotes a distance between I axis (Fig. 4b) and right edge of thighbone for u-profile.

(3) Lower edge (axis III in the Fig. 4b) is marked by a horizontal profile, which meets the condition



(a) (b)

Fig.3 Membership function (a) and complement of the membership function (b)



(a) (b) (c)

Fig.4 Location of the PCL in the knee, (a) slice no.12 (Fig.3b), (b) marked axis, (c) ROI

$$\max_p \lfloor d_p \rfloor, \tag{16}$$

where: d_p denotes a distance between I axis (Fig. 4b) and right edge of tibia for p-profile.

(4) Right edge (axis IV in the Fig.4b) has been determined on the basis of the axis I shifted by (16).

There four edges locate a ROI, which includes the posterior cruciate ligament (Fig.4c).

5. CONCLUSION

The methodology has been tested on 58 clinical T1-weighted MR knee studies. In 84 % cases the region of interest (ROI) including the PCL has been extracted correctly. The remaining regions, mostly with a severe ligament injury, require a manual enlargement. Introduced application software allows for an automatic extraction of the ROI extracted from T1- and T2-weighted MR knee studies. Location of the posterior cruciate ligament on the T1- and T2-weighted MR knee images allows also the three-dimensional visualization in the process of the computer aided diagnosis of the cruciate ligament.

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