robust filtering, ECG, EMG,
mean-median filtering

Tomasz PANDER*

AN APPLICATION OF ROBUST FILTERS IN ECG SIGNAL PROCESSING

Robust filtering is a very promising area in application of biomedical signal processing. Signals are usually recorded with noise, which has various characteristics of baseline wander to very impulsive nature. The robust technique has been recently proposed as the tool to eliminate outliers in data samples. The main purpose of this paper is to present mean-median filters in application of ECG signal processing. The presented filter is evaluated in the presence of real muscle noise and simulated impulsive noise as a Gaussian-Laplace mixture. In order to suppress a noise with the best possible means, the special expression is proposed. The measure of distortions, which are introduced to a signal after operation of filtering, is estimated using the normalized mean square error. This factor is used to compare a quality of considered filters. Experimental results show improved performance according to the reference filters.

1. INTRODUCTION

Linear filtering technique is commonly used in various scopes of digital signal processing. The main assumption of this technique is that a noise is characterised by Gaussian distribution. Such approach is justified by the Central Limit Theory. Moreover, the analytical form of solution is often obtained [11]. But this assumption can result with too optimistic conclusions. Non-gaussianity often results in significant quality degradation for systems optimised under the Gaussian assumption [11]. Such systems are very sensitive to the presence of outliers. For example the mean filter is an optimal filter for Gaussian noise in the sense of a mean square error, but performs poorly in the noise which is described by heavy-tails distributions. These reasons motivate to investigate non-linear filtering alternatives [3]. The development of non-linear filtering techniques in the recent years brings interesting results. The main effort of investigations is placed on removing outliers from a signal without destroying fine details of a signal. Non-linear filters are characterized by their robustness to an impulsive noise. One of the most interesting groups of filters is M-filters. Such filters are sliding window filters and the output of the window is estimated as the maximum likelihood estimation of location. The fundamental M-estimators are the sample mean, the sample median and the sample myriad [2, 6].

Biomedical signal processing requires the use of filters to shape the frequency content of the signal. Signal smoothing, enhancing or shape preserving, in interval of the impulsive noise appearance, means that the only alternative for these applications is robust methods implementation. Linear filters tend to blur sharp edges, destroy lines and other fine image or

* Silesian University of Technology, Institute of Electronics, Akademicka St. 16, 44-100 Gliwice, Poland

signal details in the presence of a heavy tailed noise. On the other hand, there is an important class of smoothing applications that require careful treatment and preservation of signal edges [3]. These requirements are satisfied by robust filtering methods.

The biomedical signals are recorded in noisy environment. There are many factors which lead to occurrence of disturbances. The sources of noises are various kinds of operating devices in the human environment, and also a man is a source of the noise. In the biomedical systems the first step of the processing of biomedical signals is very important. All later activities depend on the quality of the initial step [8]. There exists many different biomedical signals, but for the purpose of this work the electrocardiogram (ECG signal) was chosen. The ECG signal is almost always disturbed by noise. Examples of noises are: 50 Hz power line interference, the baseline wander, the muscle noise, the motion artefacts. In fact, most types of noises are not stationary, it means, that the noise power measured by the noise variance features some variability. The muscle disturbances contaminations in ECG signals distort low-amplitude ECG wave components and hence lower the accuracy of computer-aided measurements of various morphological characteristics. This situation appears in applications such as an exercise ECG or an ambulatory ECG, where computer-aided measurement and interpretation are very often used [4]. The muscle noise is the most difficult noise that need to be suppressed, because the spectra of EMG signal overlap for a wide range of frequency the spectrum of ECG signal [12]. A white Gaussian noise is usually used to model EMG signal, but the muscle noise shows frequently an impulsive nature, and it means that the Gaussian model may disappoint. Another model which very likely describes some cases of the muscle noise is an application of the symmetric α-stable distribution [9].

The main aim of this paper is to present the mean-median robust filter (MEM filter) which effectively suppresses a muscle noise and an impulsive type of noise. The second aim is to check the possibility of using the Gaussian and Laplacian mixture noise to model a muscle noise. The paper is organized in the following way. In the next section the mean-median filter in application of ECG signal processing is presented. Finally, in Section III, the method of evaluation and some results are presented. Final conclusions are presented in the last section. The reference filters are the moving averaging filter, the myriad filter and the median one.

2. THE ROBUST FILTER

The aim of robust statistics is to develop solution of problem of finding the best fit of a model $\mathbf{f}=\{\mathbf{f_0}, \mathbf{f_1}, \ldots, \mathbf{f_{N-1}}\}$ to a set of data measurements, $\mathbf{g}=\{\mathbf{g_0}, \mathbf{g_1}, \ldots, \mathbf{g_{N-1}}\}$, in cases where the data differs statistically from the model assumptions. In fitting a model, the aim is to find the values of **f** that minimize the size of the residual error **(g-f)** [10]. This minimisation can be written as:

$$\min \sum \rho((g - f), \sigma), \tag{1}$$

where σ is a scale parameter, and $\rho(\cdot)$ is an estimator function, also known as the cost function. The cost function plays the fundamental role in the robust filtering. The robustness of an estimator refers to its tolerance to outliers, i.e., insensitivity to deviations from the assumed statistical model [10].

Let us consider the desired signal *s*(*n*) disturbed with noise components *v*(*n*). Then the input signal *x*(*n*) can be written as:

$$x(n) = s(n) + v(n)\,. \tag{2}$$

The main aim of filtering is to estimate the signal samples *s*(*n*) by using the noisy samples *x*(*n*). The class of M-filters is the running window filter outputting the M-estimator (maximum likelihood estimator) of location of the elements in the moving window. Assume that the measurement errors are distributed according to no Gaussian distribution. The maximum-likelihood formula for the estimated parameter $\hat{\beta}$ which predicts value of *s*(*n*), can be written as:

$$P = \prod_{i=1}^{N} \exp\left[-\rho(x_i - \beta)\right], \tag{3}$$

where the ρ function is the negative logarithm of the probability density $\rho(z) = -\log f(z)$ of the additive noise within the samples and it is monotonic no decreasing on $[0,\infty)$ [2]. The properties of M-estimators depend on properties of the cost function. Taking the logarithm of (3), we obtain an expression that need to be minimized:

$$\hat{\beta} = \arg\min_{\beta} \sum_{i=1}^{N} \rho(x_i - \beta)\,, \tag{4}$$

where $\arg\min_{\beta}(\cdot)$ denotes the value of β that minimizes the expression in parenthesis [2,6] and the $\rho(z)$ is a function of a single variable $z \equiv (x_i - \beta)$. Let the function $\psi(z)$ is the derivative of $\rho(z)$:

$$\psi(z) = \frac{d\rho(z)}{dz}\,. \tag{5}$$

The $\psi(z)$ (called the *influence function*) function is some odd, continuous, and sign-preserving function [7,10].

The special cases of M-filters are the mean filter and the median one. When errors in measurements are normally distributed, i.e.,

$$Prob\{x_i - \beta\} \sim exp[-(x_i - \beta)], \tag{6}$$

Then optimal estimator has the form $\rho(z) = 0.5 \cdot z^2$ and $\psi(z) = z$. The last dependences lead to the sample mean filter which is optimised under the normal distributed errors and reduced to the standard least-squares estimation. When errors in measurements are distributed as a double or two-sided exponential, i.e.,

$$Prob\{x_i - \beta\} \sim exp[-|x_i - \beta|], \tag{7}$$

then $\rho(z) = |z|$ and $\psi(z) = \mathrm{sgn}(z)$. This expression denotes the median filter. Properties of the median filter are described in [7,13].

Robust estimation is the means to solve the problem when the distribution function is in fact not precisely known. In this case, an adequate approach is to assume, that the density function is a member of some set, or some family of parametric families, and to choose the best estimator for the least factorable member of that set [3]. Using these facts, let us assume that the noise probability distribution is scaled version of a known member of the P_ε family of ε - contaminated normal distributions proposed by Huber [5]:

$$P_\varepsilon = \{(1-\varepsilon)\Phi + \varepsilon H : \quad H \in S\}, \tag{8}$$

where: Φ is the standard normal distribution, S is the set of all probability distributions symmetric with respect to the origin (i.e., such that $H(-x) = 1 - H(x)$), and $\varepsilon \in [0,1]$ is the known fraction of "contamination". The presence of outliers in a nominally normal sample can be modelled by a distribution H with tails that are heavier than that of normal distribution. Now let Φ denotes Gaussian distribution $N(0,\sigma_G^2)$ with variance σ_G^2 and H is Laplacian (or double-exponential) $L(0,\sigma_L^2)$ with variance σ_L^2 [1,3]. The most commonly used form in modelling outliers for detection and robustness studies is the two-component mixture, where both distributions are zero mean, but one has greater variance than the other [3].

The proposed set of distributions, which has the worth property that maximizes the asymptotic variance (or, equivalently, minimizes Fisher information), is Gaussian in the centre and Laplacian in the tails. It switches from one to the other at a point whose value depends on the fraction of contamination ε. Larger fractions corresponding to smaller switching points, and vice versa [1,3]. Another method which is frequently applied in digital signal processing to model the impulsive noise is the family of the symmetric α-stable distributions (SαS). The impulsiveness in SαS is controlled only by one parameter α which is called the characteristic exponent. The main difficulty with applying the SαS is the fact that there is no-closed form of a probability density function. Only the characteristic function exists in the analytical form [11]. This model is not used in this work.

As a consequence of above study, a convex combination of the mean and the median filters (MEM) can be defined as [3]:

$$y(n) = (1-\lambda)x_{\mathrm{ave}}(n) + \lambda x_{\mathrm{med}}(n), \qquad \lambda \in [0,1] \tag{9}$$

where $x_{\mathrm{ave}}(n)$ is the output of mean filter and $x_{\mathrm{med}}(n)$ is the output of median filter calculated in moving window of size $N = 2k + 1$. The output of mean filter $x_{\mathrm{ave}}(n)$ is defined as:

$$x_{\mathrm{ave}}(n) \equiv \hat{\theta} = \arg\min_{\theta} \sum_{i=n-k}^{n+k} (x(n+i) - \theta)^2 , \tag{10}$$

and the output of standard median filter $x_{\text{med}}(n)$ is defined as:

$$x_{\text{med}}(n) \equiv \hat{\theta} = \arg\min_{\theta} \sum_{i=n-k}^{n+k} \left| x(n+i) - \theta \right| . \tag{11}$$

As a useful quality factor for a robust estimator, Huber suggests its asymptotic variance since the sample variance is strongly dependent on the tails of the distribution. The asymptotic variance is defined as:

$$V(z,F) = \int (\psi(z))^2 \, dF(z) , \tag{12}$$

where: $\psi(z)$ is the influence function from (5) and $F(z)$ is the common distribution function of the input with corresponding $f(\theta)$ as the probability density function. Using the influence functions for the mean and the median filter, the resultant influence function for the MEM filter is given in the following form:

$$\psi(z) = (1-\lambda)z + \lambda \,\text{sgn}(z) . \tag{13}$$

As was proven in [3] the asymptotic variance for MEM filter is defined as:

$$V(\text{MEM}, F) = (1-\lambda)^2 \mu_2 + \frac{\lambda^2}{4 f(\theta)} + \lambda(1-\lambda) \frac{\mu_1}{f(\theta)} , \tag{14}$$

where: $\mu_k = E\left| X - \theta \right|^k, \quad k = 1, 2$ are the central moments. Using (14) the expression for optimal value of $\lambda_{\min}$ is given as [3]:

$$\lambda_{\min} = \left(\mu_2 - \frac{\mu_1}{2 f(\theta)} \right) \Bigg/ \left(\mu_2 + \frac{1}{4 f(\theta)^2} - \frac{\mu_1}{f(\theta)} \right) . \tag{15}$$

When the input noise is Gaussian, the mean filter leads to better results of filtering than the median filter and $\lambda_{\min} = 2/(2+\pi)$. Likewise if the noise is Laplacian, then median filtering tends to obtain better results of filtering than the mean filter, and then $\lambda_{\min} = 2/3$. It is worth noting that parameter λ can change the MEM filter from linear (mean filter) to non-linear, robust filter (median filter).

3. EXPERIMENTAL RESULTS

Filtering of a signal, in the time-domain results of the signal changes is an original spectral component. The change usually consists (in decreasing) of unwanted components of the input signal. The filtering process shouldn’t deform the signal, but there exists a group of filters which may introduce inadmissible deformations of the signal. The nonlinear filters belong to this group [9]. For that reasons, the presented MEM filter is evaluated using the normalized mean square error (*NMSE*) defined as:

$$NMSE = \left(\sum_{i=1}^{L} [y(i) - s(i)]^2 \Big/ \sum_{i=1}^{L} [s(i)]^2 \right) \cdot 100\%, \tag{16}$$

where: y(i) is the output of the evaluated filter, $s(i)$ is the deterministic part of signal, without a noise and $x(i)$ is the noisy signal, L is the signal's length. The *NMSE* factor allows measuring the relative power of the additive residual distortions introduced by the nonlinear filter. Signals $y(i)$ and $s(i)$ are aligned and they have the same time index [9]. For the testing requirements, the pure ECG cycles (e.g. with a high value of SNR) are generated using linear combination of Hermite functions on the base of real ECG cycles sampled at 2 kHz. The testing data set consists of 5 different shapes of ECG cycle, each of a length 1560 samples. Then the noise samples are added to ECG cycles with the known value of standard SNR factor (5, 10, 20 and 30 dB). In this work, a simulated noise and a real electromyogram samples (sampled at 2kHz) are used. The mixture of ε-contaminated ($\varepsilon = 0.4$ [1]), Gaussian $N(0,1)$ and Laplacian $L(0,\sigma_L^2)$ noise with value of $\sigma_L^2 = 1,2$ and 4 are applied as artificial noise. The *NMSE* factor is calculated for 200 different realizations of noisy ECG cycles and then average value of *NMSE* is calculated as:

$$NMSE = \frac{1}{200} \sum_{i=1}^{200} NMSE_i \,. \tag{17}$$

Values of *NMSE* factor are calculated for three values of λ. At first, the value of λ is optimal for Gaussian noise, at second, the value of λ is optimal for Laplacian noise. And at the third case for the optimal value of λ_{opt} for which *NMSE* gets the minimum value. But in this case the knowledge of clean ECG cycle is required. This is not possible in ambulatory measurements. In the process of estimating λ_{opt} two additonal parameters are calculated: the *kurtosis* and the first ordinary moment m_2 defined as:

$$m_2 = \frac{1}{N} \sum_{i=1}^{L} x(i) x(i) \,. \tag{18}$$

These parameters are calculated for different values of SNR and the data set consists of 1000 values of triples (λ_{opt}, m_2, *kurtosis*). On this base, by using the approximation method of the square smoothing, the λ'_{opt} can be expressed as the nonlinear statement which depends on *kurtosis* and m_2 in the following way:

$$\lambda'_{opt} = 0.3 - 0.03 \cdot m_2 - 0.04 \cdot \text{kurtosis} + 0.12 m_2^2 - 0.05 \cdot m_2 \cdot \text{kurtosis} + 0.007 \cdot (\text{kurtosis})^2 \,. \tag{19}$$

The approximated surface of λ'_{opt} is presented in the Figure 1. The last expression (19) is useful in real live measurement, when the "clean" ECG signal is unobtainable.

The results obtained for mixture noise and the real muscle noise are presented in Table 1 and Table 2 respectively. The reference are the mean, median and the myriad (with the linear parameter k=1) filters.

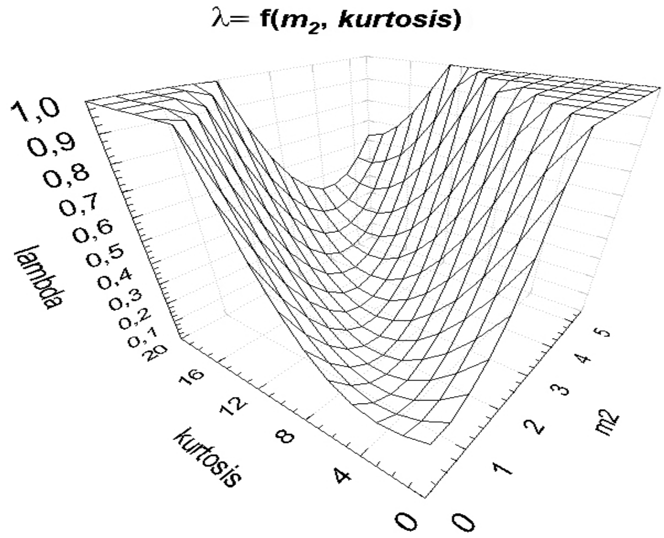

Fig. 1 The surface of nonlinear expression for estimation of λ'_{opt} on the base of kurtosis and m_2.

The best results of filtering (the smallest value of *NMSE* factor), i.e., the smallest distortion in the filtered signal are obtained (for all considered values of SNR and variance of Laplace part of noise) for MEM filter when λ is chosen optimally. However, disadvantage of such selection of λ is the requirement of acquaintance of "a pure" signal. In ambulatory measurements of ECG signal, such condition is not possible. An operation of MEM filter with estimated value of λ'_{opt} leads to a little worse results than the optimal MEM filter results and MEM filter with λ=2/3 and λ=2/(2+π). The results obtained for the myriad filter and the moving average filter are almost the same in the whole range of σ_L^2 independently from the change of SNR values. The operating of the median filter changes with the change of σ_L^2 particularly for the low values of SNR. It means that the median filter better suppresses the laplacian part of ε-contaminated noise. The same quality can be observed taking into consideration the results of MEM filtering for λ_{opt} and λ'_{opt}. An example of filtering of the ECG cycle corrupted with the ε-contaminated noise is presented in the Fig. 2.

In the case of muscle noise, the obtained results are not such optimistic. When the SNR is low, i.e., SNR=5 dB, the best results are obtained for moving average filter. For $\mathrm{SNR} \geq 10\,\mathrm{dB}$, the MEM with optimal value of λ introduces the smallest distortions in filtered signal. The results obtained for MEM filter with λ parameter estimated on the basis of m_2 and *kurtosis* are near to optimal λ except for SNR=30 dB. An example of filtering of the ECG cycle disturbed with the real muscle noise is presented in the Fig.3.

Table 1. Average NSME factor of 200 trials for a mixture ε-contaminated Gaussian and Laplacian noise (length of filter moving window $N = 21$).

	mean *NMSE* [%] for 200 trials						
SNR [dB]	myriad filter ($k = 1$)	moving average	median filter	MEM filter (λ_{opt})	MEM filter (λ'_{opt})	MEM filter λ=2/(2+π)	MEM filter λ=2/3
$\sigma_L^2 = 1$							
5	1.1635	1.3205	1.6444	**1.1488**	1.1817	1.1994	1.3422
10	0.4749	0.6734	0.6244	**0.4534**	0.467	0.4663	0.5119
20	0.1503	0.4211	0.0941	**0.0831**	0.0921	0.1031	0.0889
30	0.1328	0.3943	0.0306	**0.0303**	0.0492	0.0703	0.0431
$\sigma_L^2 = 2$							
5	1.2167	1.3728	1.3858	**1.1294**	1.1417	1.1349	1.1901
10	0.4554	0.713	0.4921	**0.4033**	0.4103	0.4078	0.4217
20	0.1404	0.4135	0.0781	**0.0712**	0.0826	0.0920	0.0761
30	0.1355	0.4379	0.0284	**0.0282**	0.0498	0.0705	0.0421
$\sigma_L^2 = 4$							
5	1.1694	1.3283	1.0861	**0.9696**	1.0132	0.9957	0.9797
10	0.4436	0.6748	0.4138	**0.3593**	0.3728	0.3729	0.3681
20	0.1473	0.3996	0.0764	**0.0694**	0.0793	0.0937	0.0755
30	0.1254	0.3759	0.0281	**0.0278**	0.0461	0.0659	0.0401

Table 2. Average NSME factor of 200 trials for a muscle noise (length of filter moving window $N = 21$).

	mean *NMSE* [%] for 200 trials						
SNR [dB]	myriad filter ($k = 1$)	moving average	median filter	MEM filter (λ_{opt})	MEM filter (λ'_{opt})	MEM filter λ=2/(2+π)	MEM filter λ=2/3
5	5.3004	**4.9837**	6.4943	5.2451	5.3536	5.4067	5.7611
10	1.7512	1.8411	2.1366	**1.7181**	1.7719	1.7579	1.874
20	0.2794	0.5105	0.2635	**0.222**	0.2286	0.2358	0.2336
30	0.1436	0.4102	0.0472	**0.0456**	0.0621	0.0818	0.0564

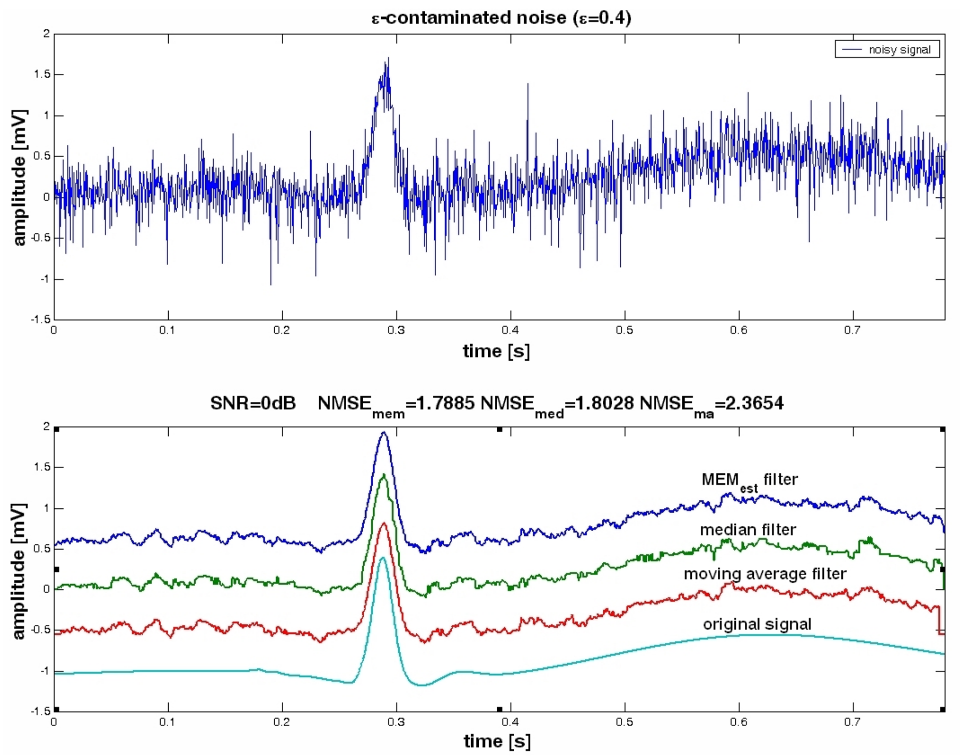

Fig. 2 An example of filtering of ECG cycle corrupted with ε-contaminated noise (upper plot) and results of filtering of various investigated filters (MEMest – MEM filter with λ'_{opt} , median – median filter, MAfilt – moving average filter, original – "clean" ECG cycle).

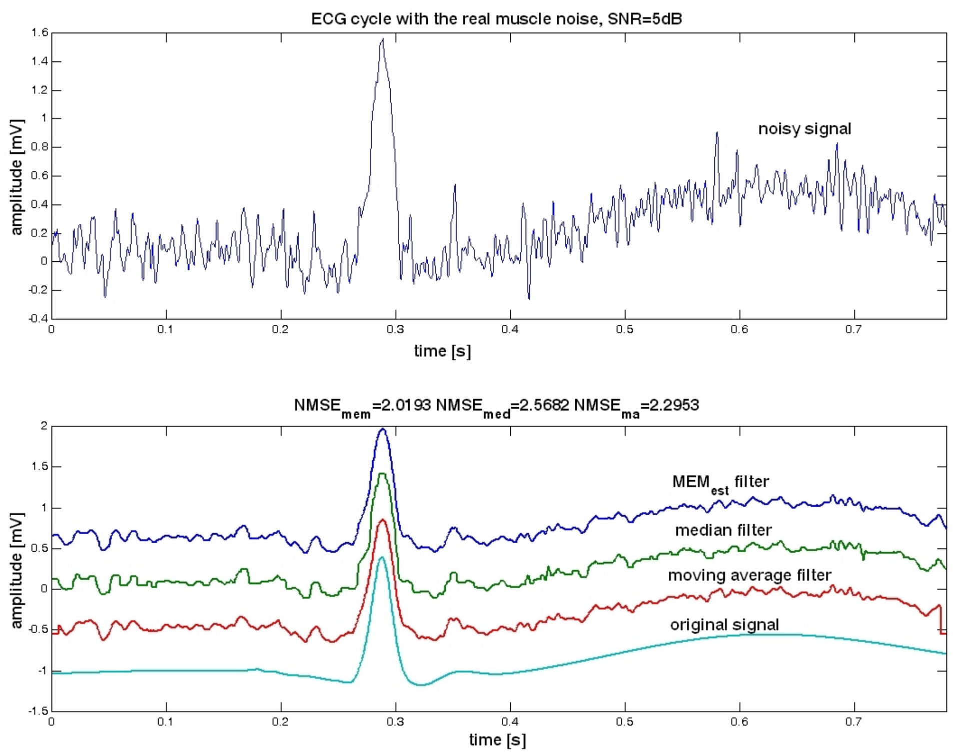

Fig.3 An example of filtering of ECG cycle corrupted with the muscle noise (upper plot) and results of filtering of various investigated filters (MEMest – MEM filter with λ'_{opt} , median – median filter, MAfilt – moving average filter, original – "clean" ECG cycle).

4. CONCLUSIONS

In this paper the mean-median filter (MEM filter) with a method of choice of λ is presented and evaluated. The analyzed filter evaluation is motivated from the robust statistics. The usefulness of an application of the MEM filter is statistically analyzed through the measurements of distortion after filtering with respect to a clean signal. The nonlinear combination of m_2 and *kurtosis* is proposed to obtain value of λ parameter that is nearly optimal for filter action. In all investigated cases, the proposed MEM filter leads to better results than the reference filters. For that reason the proposed filter can be used to suppress an impulsive type of disturbances.

Because the muscle noise has very difficult nature (for example an impulsive nature) and there does not exist one accurate model of such noise, the possibility of model the muscle noise with the mixture ε-contaminated Gaussian and Laplacian noise is tested in this paper. The muscle noises are non-stationary and non-linear by nature, but a part of an EMG signal in a finished time interval has sufficient stationary features. The proposed model is only a little step to find out more about the muscle noise.

The evaluation procedure is constructed for detection of distortions in a signal after filtering. Such assumption requires knowledge about a signal and disturbances. The full control on the level of noise and other features (for example ECG cycles with late potentials) of a signal has only artificial generated signal. Such approach also permits to investigate a noise model. This is the main and important advantage of artificial signals. The first step of the biomedical signal processing system is an application of a noise reduction method. Artificial signals allow checking the quality of the de-nosing algorithms. Using a real life signal such conditions are unavailable. Even if SNR is 30 dB, there still exists possibility for comparing filtered signal with a “pure” signal in spite of the fact that such the level of noise permits to make automatic or manual interpretation in real live measurement.

ACKNOWLEDGMENT

The author would like to thank the anonymous referees for their valuable suggestions.

BIBLIOGRAPHY

[1] AYSAL T.C., BARNER K.E., Robust frequency-selective filtering using weighted sum-median filters, in Proceedings of the 40th Annual Conference on Information Sciences and Systems (CISS2006),(Princeton, NJ), Mar. 2006.

[2] GONZALEZ J.G., ARCE G.R., Statistically-efficient filtering in impulsive environments: weighted myriad filters, EURASIP Journal on Applied Signal Processing, 2002:1, pp.4-20.

[3] HAMZA B.A., KRIM H., Image denoising: a nonlinear robust statistical approach, IEEE Transactions on Signal Processing, vol. 49, No. 12, pp. 3045-3054, 2001.

[4] HU X., NENOV V., A single-lead ECG enhancement algorithm using a regularized data-driven filter, IEEE Transactions on Biomedical Engineering, vol. 53, No. 2, pp.347-351, 2006.

[5] HUBER P., Robust Statistics, John Wiley & Sons, Inc., 1981.

[6] KALLURI S., Nonlinear Adaptive Algorithms for Robust Signal Processing and Communications in Impulsive Environments, Ph.D. Thesis (1998), University of Delaware.

[7] LEE Y.H., KASSAM S.A., Generalized Median Filtering and Related Nonlinear Filtering Techniques, IEEE Transactions on Acoustics, Speech, and Signal Processing, 1985, 33, 672-683.
[8] ŁĘSKI J., Robust Weighted Averaging, IEEE Transactions on Biomedical Engineering, vol. 49, No. 8, pp. 796-804, 2002.
[9] PANDER T., An application of a weighted myriad filter to suppression an impulsive type of noise in biomedical signals, TASK Quartarly, 2004.
[10] RABIE T., Robust estimation approach for blinding denoising, IEEE Transactions on Image Processing, vol. 14, No. 11, pp. 1755-1765, 2005.
[11] SHAO M., NIKIAS Ch.L., Signal processing with fractional lower order moments: stable processes and their applications, Proceedings of IEEE, 1993, 81, 986-1009.
[12] TOMPKINS W.J., Ed., Biomedical Digital Signal Processing, Englewood Cliffs, NJ: Prentice-Hall, 1993.
[13] YIN L., YANG R., GABBOUJ M., NEUVO Y., "Weighted Median Filters: a Tutorial", IEEE Trans. On Circuits and Systems - II: Analog and Digital Signal Processing, 1996, vol. 43, pp. 157-192.