digital filter design,
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Norbert HENZEL*

FREQUENCY AND TIME CONSTRAINED DIGITAL FILTER DESIGN

This paper describes a new method for linear phase finite impulse response (FIR) filters design. This new approach, based on the ε-insensitive loss function, allows the design process to take into account not only constraints specified in the frequency domain, but also constraints on the output, time domain, signal. The performances of the proposed approach are shortly illustrated by a high-pass filter for ECG baseline wander reduction.

1. INTRODUCTION

The first step in all ECG signal processing systems is baseline wander and power line interference reduction. The baseline wander is caused by varying electrode-skin impedance, patient's move, breath and its frequency range is placed usually under 1.0 Hz [12, 2]. Generally, methods used to reduce this kind of disturbance can be divided into three groups: methods based on baseline wander estimation, methods based on high-pass filtering and method based on nonlinear filtering [3, 9, 1, 7].

The contribution presents a new digital filter design method motivated by the baseline wander reduction problem. In order to eliminate, as much as possible, the noise from the processed signal but without distorting the useful part, there is a growing need for flexible digital filter design techniques that accept sophisticated specifications. The proposed method uses not only the constraints on filter's frequency response, but takes also into account the constraints on the output, time domain, signal. This approach exploits the ε-insensitive loss function, that plays recently an important role in a vast range of intelligent processing systems, e.g. [10, 4, 5, 6]. The performances of the filter are compared with a classic band filter used for ECG baseline wander reduction [12].

2. A NEW CONSTRAINED FILTER DESIGN METHOD

Many recent digital filters development methods depend on minimising a given error criterion. Usually, the error criterion is defined as:

* Silesian University of Technology, Institute of Electronics, Akademicka 16, 44-101 Gliwice, Poland.

$$E_c(\omega) = H\left(e^{j\omega}\right) - D\left(e^{j\omega}\right) \tag{1}$$

Where: $H\left(e^{j\omega}\right)$ and $D\left(e^{j\omega}\right)$ are the actual and the desired frequency response of the filter, respectively, and

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n} \tag{2}$$

If the impulse response, $h(n)$, has even or odd symmetry, the phase response of the considered filter is linear and the obtained solution is real-valued. In this case, the present (complex) frequency response function $H\left(e^{j\omega}\right)$ can be replaced with a real-valued function $H_0\left(e^{j\omega}\right)$, called amplitude response. The amplitude response function is related to the frequency response as:

$$H\left(e^{j\omega}\right) = e^{-j(N-1)/2\omega} e^{-j\beta} H_0(\omega) \tag{3}$$

where: $\beta \in \{0, \pi/2\}$. The desired frequency response of the filter, $D\left(e^{j\omega}\right)$, is in this case also replaced with a real-valued function $D_0\left(e^{j\omega}\right)$. For $\beta = 0$ the amplitude response is given by:

$$H_0(\omega) = \begin{cases} \sum_{n=0}^{(N-1)/2} b_n \cos(\omega n) & \text{for } N-1 \text{ even,} \\ \sum_{n=0}^{N/2} b_n \cos\left(\omega\left(n - \frac{1}{2}\right)\right) & \text{for } N-1 \text{ odd.} \end{cases} \tag{4}$$

where: the coefficients b_n are related to the impulse response, $h(n)$, as follows

$$b_n = \begin{cases} h\left(\frac{N-1}{2}\right) & \text{for } N-1 \text{ even, } n=0, \\ 2h\left(\frac{N-1}{2} - n\right) & \text{for } N-1 \text{ even, } n \neq 0, \\ 2h\left(\frac{N}{2} - n\right) & \text{for } N-1 \text{ odd.} \end{cases} \tag{5}$$

In the time domain, the input-output relation for a digital filter with $h(n) = 0$ for $0 > m > N-1$ is given by

$$y(k) = \sum_{m=0}^{N-1} x(k-m)h(m). \tag{6}$$

where: $x(k)$ is the filter's input signal and $y(k)$ is the output signal, $k = 1, \ldots, K$. This input-output relation can be represented in matrix form as

$$\mathbf{Y} = \frac{1}{2}\mathbf{X} \cdot \mathbf{b}' + \mathbf{X_0} \cdot b_0, \tag{7}$$

where:

$$M = \frac{N-1}{2}, \tag{8}$$

$$\mathbf{b}' = \left[b_1, \ldots, b_M\right]^T, \mathbf{b} = \left[b_0, \mathbf{b}'^T\right]^T, \tag{9}$$

$$\mathbf{Y} = \left[y(2M+1), \ldots, y(K)\right]^T = \left[y_1, \ldots, y_{K-2M}\right]^T, \tag{10}$$

$$\mathbf{X_0} = \left[x(M+1), \ldots, x(K-M)\right]^T = \left[x_1, \ldots, x_{K-2M}\right]^T, \tag{11}$$

$$\mathbf{X} = \left[\mathbf{x}_1^T, \ldots, \mathbf{x}_{K-2M}^T\right] = \mathbf{S_1} + \mathbf{S_2}, \tag{12}$$

and

$$\mathbf{S_1} = \begin{bmatrix} x(M+2) & \cdots & x(2M+1) \\ \vdots & \ddots & \vdots \\ x(K-M+1) & \cdots & x(K) \end{bmatrix}, \mathbf{S_2} = \begin{bmatrix} x(M) & \cdots & x(1) \\ \vdots & \ddots & \vdots \\ x(K-M-1) & \cdots & x(K-2M) \end{bmatrix}. \tag{13}$$

In the frequency domain, the relation between the filter's coefficient vector $\mathbf{b}$ and the amplitude response function can be represented as:

$$\mathbf{H}_0 = \mathbf{T}\mathbf{b} \tag{14}$$

where:

$$\mathbf{H}_0 = \left[H_0(\omega_1) \;\; H_0(\omega_2) \;\; \cdots \;\; H_0(\omega_L)\right]^T \tag{15}$$

L is the number of frequency grid points, and

$$\mathbf{T} = \begin{bmatrix} \cos(0\cdot\omega_1) & \cos(\omega_1) & \cdots & \cos(M\cdot\omega_1) \\ \cos(0\cdot\omega_2) & \cos(\omega_2) & \cdots & \cos(M\cdot\omega_2) \\ \vdots & \vdots & \ddots & \vdots \\ \cos(0\cdot\omega_L) & \cos(\omega_L) & \cdots & \cos(M\cdot\omega_L) \end{bmatrix}. \tag{16}$$

It is useful to represent the last matrix as:

$$\mathbf{T} = \left[\mathbf{t}_1, \mathbf{t}_2, \ldots, \mathbf{t}_L\right]^T, \;\; \mathbf{t}_i = \left[\cos(0\omega_i), \cos(\omega_i), \ldots, \cos(M\omega_i)\right]^T. \tag{17}$$

The constrained digital filter design can be viewed as an optimization problem, where the parameter vector $\mathbf{b}$ is obtained as a solution of the following problem

$$\underset{\mathbf{b}\in\mathbb{R}^{M+1}}{\text{minimize}} \; E_{\varepsilon,\delta}(\mathbf{b}) \triangleq \frac{1}{2}\mathbf{b}^T\mathbf{b} + C\sum_{i=1}^{L} \rceil d_i - \mathbf{t}_i\mathbf{b}\lceil_\varepsilon + V\sum_{k=1}^{K} \rceil y_k - \mathbf{x}_k\mathbf{b}\lceil_\delta, \tag{18}$$

where: d_i specifies the desired amplitude response for frequency ω_i and the ε-insensitive loss function, introduced by Vapnik in [11], is defined for a general vector argument as

$$\rceil \mathbf{g} \lceil_\varepsilon \triangleq \sum_{i=1}^{L} \rceil g_i \lceil_\varepsilon \tag{19}$$

$$\rceil g_i \lceil_\varepsilon \triangleq \begin{cases} 0, & |g_i| \le \varepsilon, \\ |g_i| - \varepsilon, & |g_i| > \varepsilon. \end{cases} \tag{20}$$

The parameters $C, V > 0$ control the trade-off between the coefficient energy and the amount of frequency and time domains bound errors.

To cooperate with (possibly) infeasible constraints, we introduce slack variables

$$\xi_i^+, \xi_i^-, \mu_k^+, \mu_k^- \;\ge 0 \tag{21}$$

and represent the preceding criterion (18) (for all impulse response points d_i, input x_k and output y_k), in the following (primal) form

$$\underset{\mathbf{b},\xi_i^+,\xi_i^-,\mu_k^+,\mu_k^-}{\text{minimize}} \quad \tfrac{1}{2}\mathbf{b}^T\mathbf{b} + C\sum_{i=1}^{L}\left(\xi_i^+ + \xi_i^-\right) + V\sum_{k=1}^{K}\left(\mu_k^+ + \mu_k^-\right),$$

$$\text{subject to} \quad \begin{cases} d_i - \mathbf{t}_i\mathbf{b}' - b_0 \le \varepsilon_i + \xi_i^+, \\ \mathbf{t}_i\mathbf{b}' + b_0 - d_i \le \varepsilon_i + \xi_i^- \\ \xi_i^+ \ge 0,\ \xi_i^- \ge 0 \\ y_k - \mathbf{x}_k\mathbf{b}' - x_k b_0 \le \delta_k + \mu_k^+, \\ \mathbf{x}_k\mathbf{b}' + x_k b_0 - y_k \le \delta_k + \mu_k^- \\ \mu_k^+ \ge 0,\ \mu_k^- \ge 0 \end{cases} \tag{22}$$

where: $i = 1,\ldots,L,\ k = 1,\ldots,K$. This kind of optimisation problem can be solved more easily in a complementary (dual) form. The key idea is to use the objective function and the constraints to construct a Lagrange function, by introducing a dual set of variables. It can be shown, that this dual formulation has a saddle point at optimal solution.

The Lagrangian function of the preceding criterion (22) is given by:

$$\begin{aligned} \mathscr{L} = {} & \frac{1}{2}\mathbf{b}^T\mathbf{b} + C\sum_{i=1}^{L}\left(\xi_i^+ + \xi_i^-\right) + V\sum_{k=1}^{K}\left(\mu_k^+ + \mu_k^-\right) + \\ & -\sum_{i=1}^{L}\alpha_i^+\left(\varepsilon_i + \xi_i^+ - d_i + \mathbf{t}_i\mathbf{b}' + b_0\right) - \sum_{i=1}^{L}\alpha_i^-\left(\varepsilon_i + \xi_i^- + d_i - \mathbf{t}_i\mathbf{b}' - b_0\right) \\ & -\sum_{k=1}^{K}\beta_i^+\left(\delta_i + \mu_k^+ - y_k + \mathbf{x}_k\mathbf{b}' + x_k b_0\right) - \sum_{i=1}^{L}\left(\eta_i^+\xi_i^+ + \eta_i^-\xi_i^-\right) \\ & -\sum_{k=1}^{K}\beta_i^-\left(\delta_i + \mu_k^- + y_k - \mathbf{x}_k\mathbf{b}' - x_k b_0\right) - \sum_{k=1}^{K}\left(\lambda_k^+\mu_k^+ + \lambda_k^-\mu_k^-\right). \end{aligned} \tag{23}$$

where: the Lagrange multipliers are equal to

$$\alpha_i^+, \alpha_i^-, \eta_i^+, \eta_i^-, \beta_k^+, \beta_k^-, \lambda_k^+, \lambda_k^- \ge 0. \tag{24}$$

The dual optimisation problem of (22) we get by setting the derivatives of (23) equal to zero

$$\begin{cases} \frac{\partial\mathscr{L}}{\partial\mathbf{b}} = \mathbf{b} - \sum_{i=1}^{L}\left(\alpha_i^+ - \alpha_i^-\right)\mathbf{t}_i - \sum_{k=1}^{K}\left(\beta_k^+ - \beta_k^-\right)\mathbf{x}_k = \mathbf{0}, \\ \frac{\partial\mathscr{L}}{\partial b_0} = \sum_{i=1}^{L}\left(\alpha_i^+ - \alpha_i^-\right) + \sum_{k=1}^{K}\left(\beta_k^+ - \beta_k^-\right)x_k = 0, \\ \frac{\partial\mathscr{L}}{\partial\xi_i^+} = C - \alpha_i^+ - \eta_i^+ = 0, \\ \frac{\partial\mathscr{L}}{\partial\xi_i^-} = C - \alpha_i^- - \eta_i^- = 0, \\ \frac{\partial\mathscr{L}}{\partial\mu_k^+} = V - \beta_k^+ - \lambda_k^+ = 0, \\ \frac{\partial\mathscr{L}}{\partial\mu_k^-} = V - \beta_k^- - \lambda_k^- = 0. \end{cases} \tag{25}$$

Substituting (25) into the Lagrangian (23) results in the following (dual) optimisation problem

$$\underset{\alpha_i^+,\alpha_i^-,\beta_k^+,\beta_k^-}{\text{maximize}} \ \mathscr{L}$$

$$\text{subject to} \ \begin{cases} \sum_{i=1}^{L} \left(\alpha_i^+ - \alpha_i^-\right) + \sum_{k=1}^{K} \left(\beta_k^+ - \beta_k^-\right) x_k = 0, \\ 0 \le \alpha_i^+, \alpha_i^- \le C, \\ 0 \le \beta_k^+, \beta_k^- \le V. \end{cases} \tag{26}$$

where: the Lagrangian is written as

$$\mathscr{L} = -\frac{1}{2}\Big(\sum_{i=1}^{L} \left(\alpha_i^+ - \alpha_i^-\right) \mathbf{t}_i + \sum_{k=1}^{K} \left(\beta_k^+ - \beta_k^-\right) \mathbf{x}_k\Big) \cdot \Big(\sum_{i=1}^{L} \left(\alpha_i^+ - \alpha_i^-\right) \mathbf{t}_i + \sum_{k=1}^{K} \left(\beta_k^+ - \beta_k^-\right) \mathbf{x}_k\Big)^T - \sum_{i=1}^{L} \left(\alpha_i^+ + \alpha_i^-\right) \varepsilon_i + \sum_{i=1}^{L} \left(\alpha_i^+ - \alpha_i^-\right) d_i - \sum_{k=1}^{K} \left(\beta_k^+ + \beta_k^-\right) \delta_k + \sum_{k=1}^{K} \left(\beta_k^+ - \beta_k^-\right) y_k. \tag{27}$$

Knowing the solution of this optimisation problem, the parameters $\mathbf{b}'$ could be determined as:

$$\mathbf{b}' = \mathbf{t}^T \left(\alpha^+ - \alpha^-\right) + \mathbf{x}^T \left(\beta^+ - \beta^-\right), \tag{28}$$

from the first condition of (25).

Finally, the last unknown parameter b_0 can be determined from the Karush-Kühn-Tucker (KKT) conditions. The KKT conditions state that at the saddle point the following relations must be satisfied

$$\begin{cases} \alpha_i^+ \left(\varepsilon_i + \xi_i^+ - d_i + \mathbf{t}_i \mathbf{b}' + b_0\right) = 0, \\ \alpha_i^- \left(\varepsilon_i + \xi_i^- + d_i - \mathbf{t}_i \mathbf{b}' - b_0\right) = 0, \\ \beta_k^+ \left(\delta_k + \mu_k^+ - y_k + \mathbf{x}_k \mathbf{b}' + x_k b_0\right) = 0, \\ \beta_k^- \left(\delta_k + \mu_k^- + y_k - \mathbf{x}_k \mathbf{b}' - x_k b_0\right) = 0, \\ (C - \alpha_i^+)\xi_i^+ = 0, \\ (C - \alpha_i^-)\xi_i^- = 0, \\ (V - \beta_k^+)\mu_k^+ = 0, \\ (V - \beta_k^-)\mu_k^- = 0. \end{cases} \tag{29}$$

So, the parameter b_0 can be determined from the following equations, derived from KKT conditions (29), by taking arbitrary condition for which the corresponding condition is met

$$b_0 = \begin{cases} d_i - \mathbf{t}_i \mathbf{b}' - \varepsilon_i, & \text{for } 0 < \alpha_i^+ < C, \\ d_i - \mathbf{t}_i \mathbf{b}' + \varepsilon_i, & \text{for } 0 < \alpha_i^- < C, \\ \left(y_k - \mathbf{x}_k \mathbf{b}' - \delta_k\right)/x_k, & \text{for } 0 < \beta_k^+ < V, \\ \left(y_k - \mathbf{x}_k \mathbf{b}' + \delta_k\right)/x_k, & \text{for } 0 < \beta_k^- < V. \end{cases} \tag{30}$$

3. EXPERIMENTAL RESULTS

The usefulness of the proposed filter design method was investigated using standards developed by the International Electrotechnical Commission (IEC) within the European project "Common Standards for Quantitative Electrocardiography". These standards were developed in order to reliably evaluate the accuracy of ECG signal processing methods. These standards are very well suited to analyze ECG system's software performance in term of baseline removal, powerline frequency suppression, waveform detection, localization of fiducial points, measurement of ECG parameters, etc. These standards establish also procedures for evaluating the hardware aspects of ECG systems, i.e., calibration, amplifier linearity, gains factors, etc.

As a possible approach to evaluate methods used for ECG baseline wander reduction, the IEC committee suggests use of an artificial signal composed of triangular waves. Each triangular wave is 1.5 mV high and has 80 ms base width. This signal shall not produce an output signal with an offset from the isoelectric line greater than 20 μV , and shall not produce a slope greater than 50 μV/s in a 200 ms region following the impulse and a slope of 100 μV/s anywhere outside the region of the impulse. On the other hand, the amplitude response of the required high-pass filter in the 0.67 – 40 Hz passband, should not have ripples greater than ±10%. In other frequency bands the constraints are less important. The described specifications represents a high-pass filter with a very narrow transition band and constraints in both frequency and time domains. Using classical filter design methods e.g. [8], it was impossible to find filter that fulfils all the constraints. Also baseline wander removing filter proposed in [12] does not met the given above requirements (see Fig. 1).

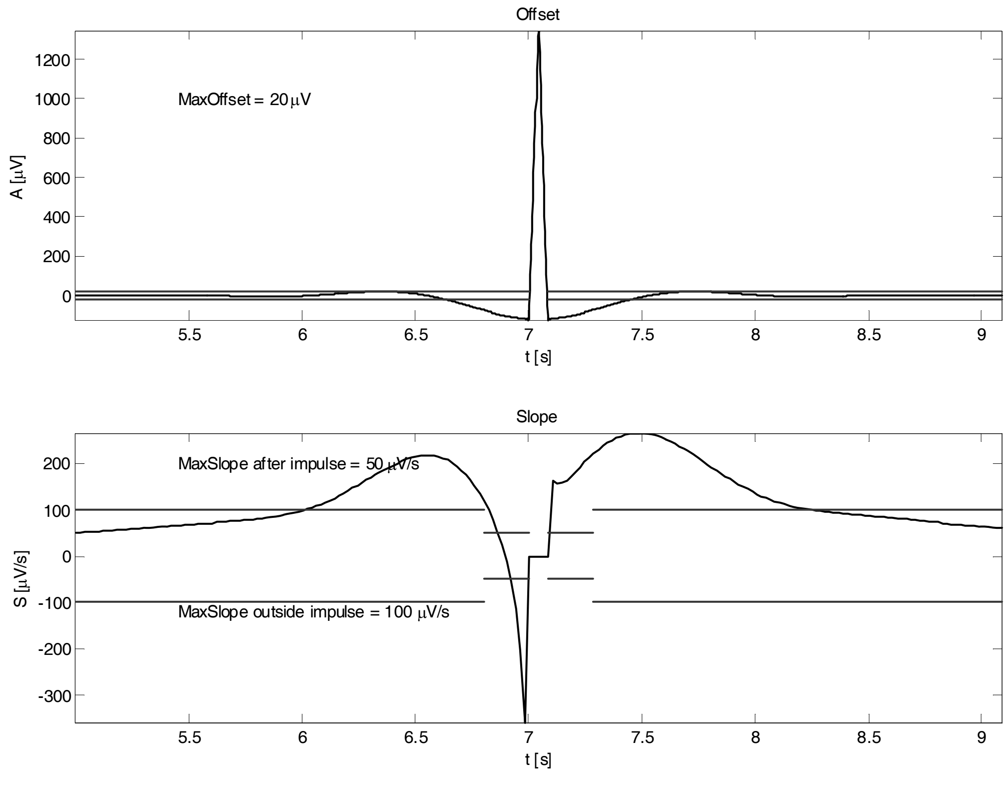

Fig. 1. Time domain response of the filter presented in [12] for a triangular-wave signal.

The proposed filter design method was used to calculate the required filter coefficients. The minimum filter order, sufficient to fulfil the specified constraints was 890. The parameters $\varepsilon_i, i = 1,\ldots,L,\ \delta_k, k = 1,\ldots,K$ was chosen according to the frequency and time domain constraints, defined above. Fig. 2 presents the time domain output signal of the designed filter for the described above, triangular-wave input signal. All constraints concerning the maximum distortion and slope were fulfilled.

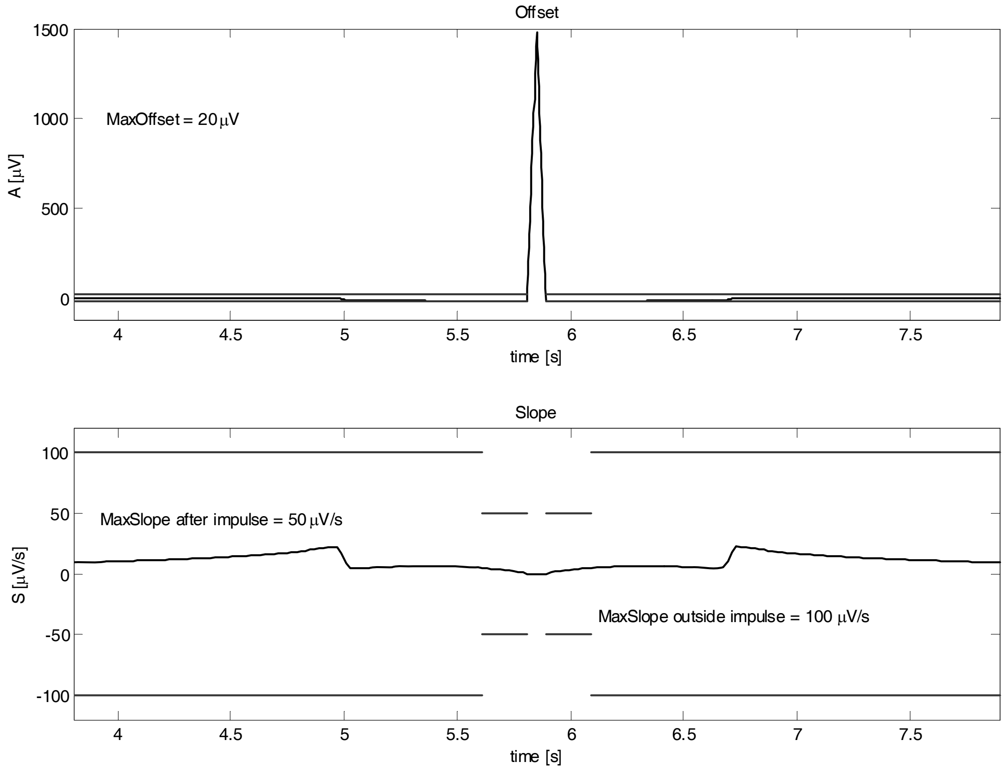

Fig. 2. Time domain response of the designed filter for a triangular-wave signal.

The next figure (Fig. 3) presents the stopband details of the amplitude response of the two considered here high-pass filters. The first one, proposed by VanAlste et al. in [12] is marked with a solid line. The filter designed with the new method presented in this paper is drawn using a dashed line. The amplitude response, in the case of the proposed filter, satisfies the desired filter specification. For the reference filter, the cut-off frequency is much higher. This results in the unacceptable distortion introduced in the test signal, as shown in Fig. 1.

The comparison of the amplitude responses shows also, that the maximum attenuation in the stopband is much higher for the new filter. These results in a better reduction of the baseline wander. Next figure, Fig. 4, shows an example of ECG baseline wander reduction capabilities for both considered FIR filters. The first signal marked with (A) consists of a ECG signal with sampling frequency 500 Hz and added sinusoidal signal with frequency 0.3 Hz and amplitude ±500 μV. Both signal and disturbance were chosen according to the IEC Committee recommendations. The second signal, (B), represents the output signal obtained by filtering the input signal (A) with the filter proposed in this paper. The last signal, (C), is obtained with the reference filter. In order to improve the clarity of this figure, a constant value of 1000 μV and 2000 μV was added to signals (B) and (A), respectively. It could be observed that the output signal (B) contains much less residual baseline noise compared to the signal (C). This is more clearly visible in the next figure, Fig. 5, which shows the difference between the output, filtered, signal and the input, noisy, signal. The first signal, marked with (A), represents the reduced part of the baseline noise eliminated by the reference filter proposed by VanAlste (difference between signal (A) and (C) in Fig. 4). The second signal, (B), represents the part baseline noise eliminated with the proposed filter (difference between signal (A) and (B) in Fig. 4). Once again, in order to improve the clarity of the figure, a constant value of 1500 μV was added to signals (A). In the first case, the difference shows some important deformations of the original sinusoidal noise, especially in the vicinity of QRS and T complexes. The maximum amplitude of the difference signal is ±551 μV. In the second case, the removed disturbance represents an almost ideal sinusoidal curve with a maximum amplitude of ±469 μV.

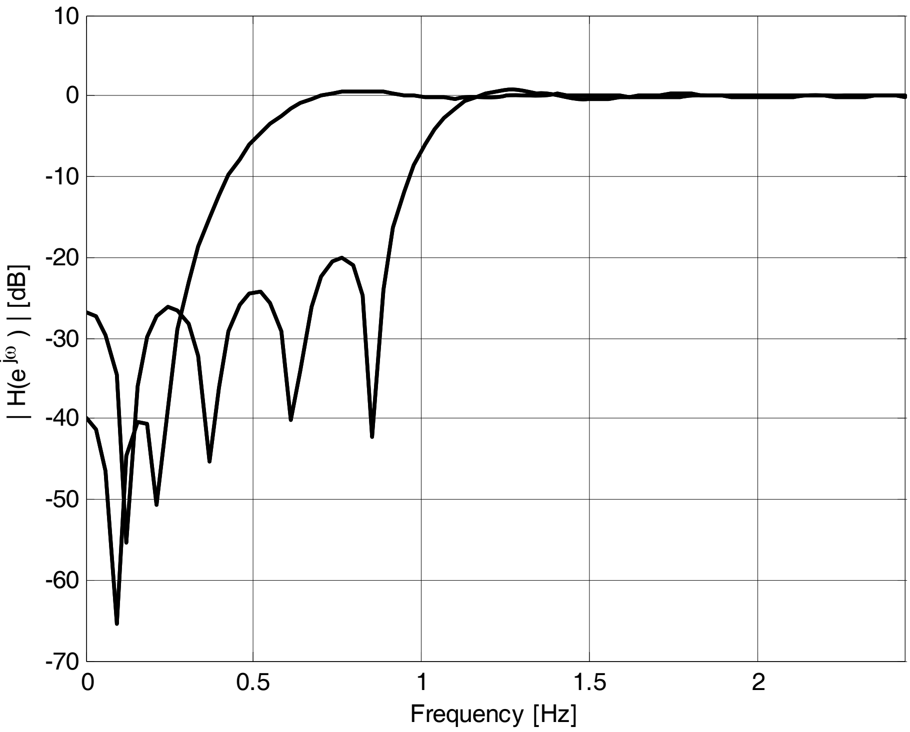

Fig. 3. Time details of the amplitude response for the reference filter (solid line) and the proposed filter (dashed line).

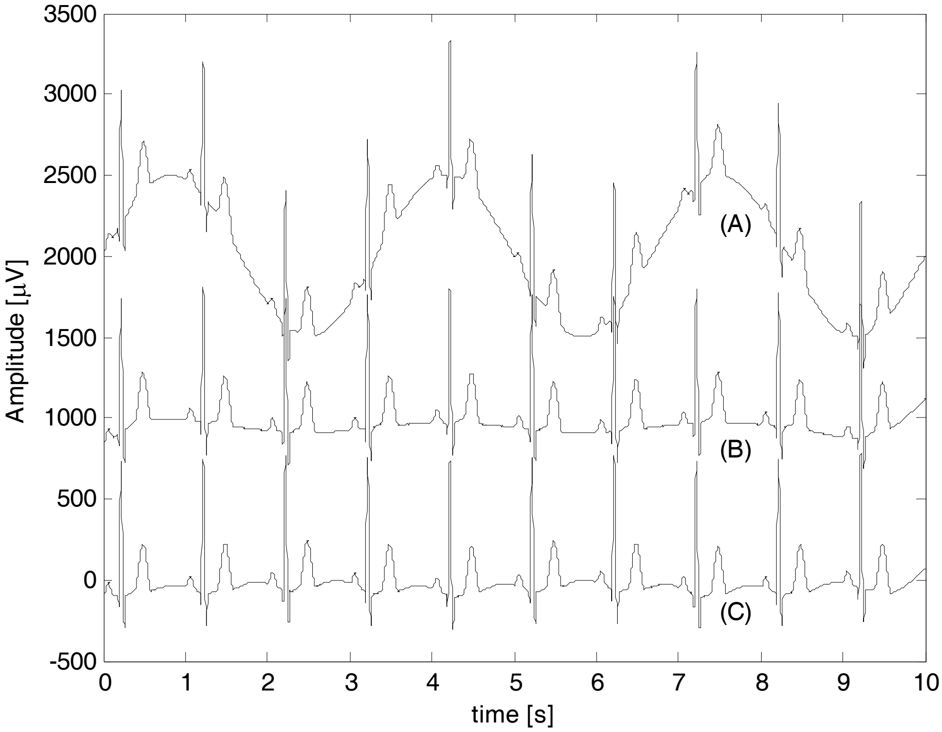

Fig. 4. Low-frequency noise reduction performances: original noisy input signal (A), output signal for the proposed filter (B) and output signal for the reference filter (C).

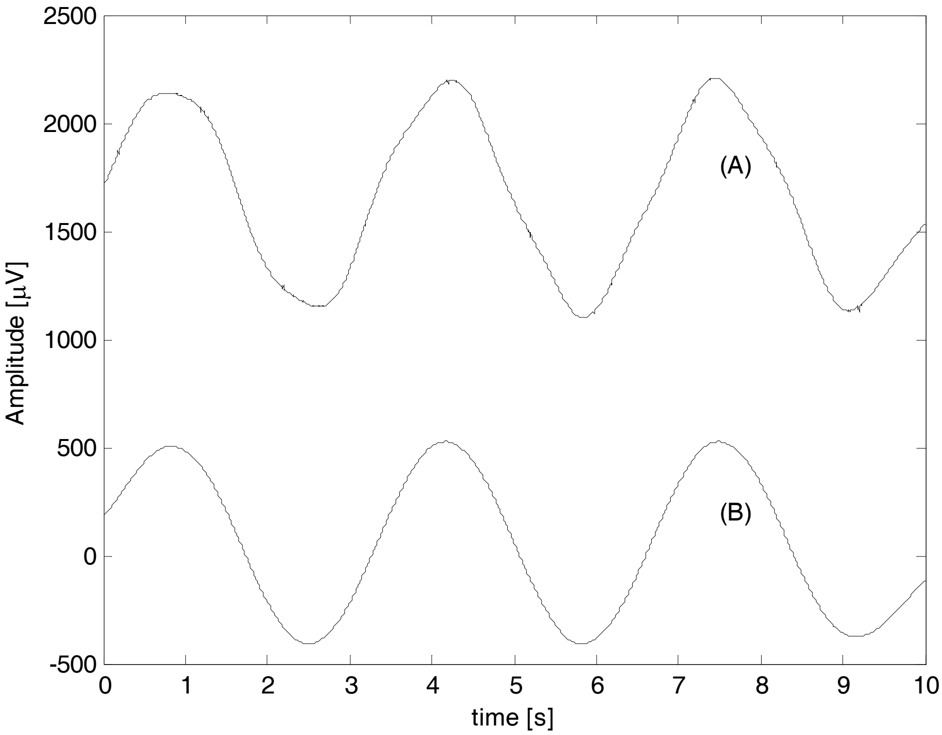

Fig. 5. The part of low-frequency disturbance eliminated by the reference filter, (A), and the proposed filter, (B).

4. CONCLUSIONS

In this paper a new method for digital filter design was presented. This method allows defining the filter's specification constraints not only in the frequency domain, but also with respect to the required output signal. The possibilities offered by this new method were shortly illustrated with design of an ECG high-pass filter for baseline wander reduction. The resulting filter shows better performances compared to the generally accepted reference filter.

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