ECG signal,
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WEIGHTED AVERAGING OF ECG SIGNALS BASED ON PARTITION OF INPUT SET IN TIME DOMAIN

The paper presents new approach to problem of signal averaging which is commonly used to extract a useful signal distorted by a noise. The averaging is especially useful for biomedical signal such as ECG signal, where the spectra of the signal and noise significantly overlap. In reality can be observed variability of noise power from cycle to cycle which is motivation for using methods of weighted averaging. Performance of the new method, based on partition of input set in time domain and criterion function minimization, is experimentally compared with the traditional averaging by using arithmetic mean, weighted averaging method based on empirical Bayesian approach and weighted averaging method based on criterion function minimization.

1. INTRODUCTION

The electrocardiogram (ECG) is the recording of the heart's electrical potential versus time. The underlying physiological process, the electrochemical excitation of cardiac tissue, is non-linear and the signals show both fluctuations and remarkable structures which are not explained by linear correlations. These structures make the ECG useful to cardiologists as a diagnostic tool [6].

While the predominant QRS complexes which reflect the electrical depolarisation of the ventricle are usually visible even in the presence of rather strong noise, more subtle features like the atrial P-wave (which normally occurs 120-200 ms before the peak of the QRS complex) may be concealed by errors which are due the imperfect transmission of the signal from the heart through different kinds of tissue, the electrode and the electronic equipment. These errors are particularly serious for ECG signal taken during exercise (sweaty skin, muscle activity) and on long-term ambulatory (Holter) recordings where the experimental conditions can not be controlled that well.

The peculiar twofold nature of ECG signal (a pronounced pattern repeated with irregular intervals) makes it difficult to filter these signals with Fourier methods. The continuous part of the spectrum is due to both the measurement noise (which should be removed) and to the irregular interbeat intervals (which should be preserved).

Noise reduction is one of the main problems to overcome in order to obtain a correct interpretation of ECG signal. However, improvement the signal-to-noise ratio usually is

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very intractable because noise usually overlapping the signal in both time and frequency domains [1]. Using the fact that certain biological systems produce repetitive patterns (for example electrocardiographic signal, which is the main case of interest in this work), an averaging in the time domain may be used for noise attenuation. Traditional averaging technique assumes the constancy of the noise power cycle-wise, however, the most types of noise are not stationary. In these cases a need for using weighted averaging occurs, which reduces influence of hardly distorted cycles on resulting averaged signal (or even eliminates them). Therefore, many recently developed noise removal techniques involve weighted signal averaging [2], [4], [5].

The paper presents new method for resolving of signal averaging problem which incorporates partition of input set in time domain. By exploiting criterion function minimization it can be derived an algorithm of weighted averaging which application to electrocardiographic (ECG) signal averaging is competitive with alternative methods as will be shown in the later part of the paper.

2. SIGNAL AVERAGING METHODS

Let us assume that in each signal cycle $y_i(j)$ is the sum of a deterministic (useful) signal $x(j)$, which is the same in all cycles, and a random noise $n_i(j)$ with zero mean and variance for the *i*th cycle equal to σ_i^2. Thus, $y_i(j) = x(j) + n_i(j)$, where *i* is the cycle index $i = 1,2,\ldots,M$, and the *j* is the sample index in the single cycle $j = 1,2,\ldots,N$ (all cycles have the same length *N*). The weighted average is given by

$$v(j) = \sum_{i=1}^{M} w_i y_i(j), \tag{1}$$

where w_i is a weight for *i*th signal cycle and $v(j)$ is the averaged signal.

2.1. TRADITIONAL ARITHMETIC AVERAGING

The traditional ensemble averaging with arithmetic mean as the aggregation operation gives all the weights w_i equal to M^{-1}. If the noise variance is constant for all cycles, then these weights are optimal in the sense of minimizing the mean square error between *v* and *x*, assuming Gaussian distribution of noise. When the noise has a non-Gaussian distribution, the estimate (1) is not optimal, but it is still the best of all linear estimators of *x* [3]

2.2. NEW WEIGHTED AVERAGING METHOD

Presented new weighted averaging method is based on partition input set $Y = [y_1, y_2, \ldots, y_M]$ (where $y_i = [y_i(1), y_i(2), \ldots, y_i(N)]^T$) into two disjoint subsets Y_1 and Y_2. The weights vectors w_1 and w_2 are calculated as minimum of the following scalar criterion function

$$I(w_1, w_2) = (Y_1 w_1 - Y_2 w_2)^T (Y_1 w_1 - Y_2 w_2), \qquad (2)$$

with constraints $w_1{}^T \mathbf{1} = 1$ and $w_2{}^T \mathbf{1} = 1$, which means that sum of weights for each vector is equal to one.

The Lagrangian of (2) with the constraint is

$$L(w_1, w_2, \lambda_1, \lambda_2) = (Y_1 w_1 - Y_2 w_2)^T (Y_1 w_1 - Y_2 w_2) + \lambda_1 \left(w_1{}^T \mathbf{1} - 1\right) + \lambda_2 \left(w_2{}^T \mathbf{1} - 1\right), \qquad (3)$$

where λ_1 and λ_2 are the Lagrange multipliers. Setting the Lagrangian's gradient with respect to w_1 and w_2 to zero, we obtain

$$2Y_1^T (Y_1 w_1 - Y_2 w_2) - \lambda_1 \mathbf{1} = \mathbf{0}\ ,$$

$$-2Y_2{}^T (Y_1 w_1 - Y_2 w_2) - \lambda_2 \mathbf{1} = \mathbf{0}\,, \qquad (4)$$

which yields

$$w_1 = \left(Y_1{}^T Y_1\right)^{-1} Y_1{}^T Y_2 w_2 + \tfrac{1}{2} \lambda_1 \left(Y_1{}^T Y_1\right)^{-1} \mathbf{1}\,,$$

$$w_2 = \left(Y_2{}^T Y_2\right)^{-1} Y_2{}^T Y_1 w_1 + \tfrac{1}{2} \lambda_2 \left(Y_2{}^T Y_2\right)^{-1} \mathbf{1}\,. \qquad (5)$$

Setting the Lagrangian's gradient with respect to λ_1 and λ_2 to zero, we obtain

$$w_1{}^T \mathbf{1} - 1 = 0\,,$$

$$w_2{}^T \mathbf{1} - 1 = 0 \qquad (6)$$

Combining (5) and (6) yields

$$w_1 = \left(Y_1{}^T Y_1\right)^{-1} Y_1{}^T Y_2 w_2 + \frac{1 - \mathbf{1}^T \left(Y_1{}^T Y_1\right)^{-1} Y_1{}^T Y_2 w_2}{\mathbf{1}^T \left(Y_1{}^T Y_1\right)^{-1} \mathbf{1}} \left(Y_1{}^T Y_1\right)^{-1} \mathbf{1} \qquad (7)$$

and

$$w_2 = \left(Y_2{}^T Y_2\right)^{-1} Y_2{}^T Y_1 w_1 + \frac{1 - \mathbf{1}^T \left(Y_2{}^T Y_2\right)^{-1} Y_2{}^T Y_1 w_1}{\mathbf{1}^T \left(Y_2{}^T Y_2\right)^{-1} \mathbf{1}} \left(Y_2{}^T Y_2\right)^{-1} \mathbf{1}\,. \qquad (8)$$

Therefore the proposed new weighted averaging algorithm can be described as follows, where ε is a preset parameter:

1. Determine partition of input set into two disjoint subsets Y_1 of size M_1 and Y_2 of size M_2. Initialise $w_1^{(0)} \in \mathbf{R}^{M_1}$ and $w_2^{(0)} \in \mathbf{R}^{M_2}$. Set the iteration index $k = 1$.
2. Calculate $w_1^{(k)}$ using (7) and $w_2^{(k)}$ using (8), assuming respectively $w_2 = w_2^{(k-1)}$ and $w_1 = w_1^{(k-1)}$.
3. If $\left\| w_1^{(k)} - w_1^{(k-1)} \right\| + \left\| w_2^{(k)} - w_2^{(k-1)} \right\| > \varepsilon$ then $k \leftarrow k+1$ and go to 2.
4. Calculate averaged signal $v = \alpha Y_1 w_1 + (1-\alpha) Y_2 w_2$, where $\alpha = M_1 (M_1 + M_2)^{-1}$

3. NUMERICAL EXPERIMENTS

Performance of the new method was experimentally compared with the traditional averaging by using arithmetic mean, weighted averaging method based on empirical Bayesian approach [5], and weighted averaging method based on criterion function minimization [4]. In all experiments using weighted averaging calculations were initialised as the means of disturbed signal cycles. The parameter ε was equal to 10^{-6}.

For a computed averaged signal the performance of tested methods was evaluated by the maximal absolute difference between the deterministic component and the averaged signal (MAX). The root mean-square error (RMSE) between the deterministic component and the averaged signal was also computed. All experiments were run in the **R** environment for **R** version 2.4.0 (http://www.r-project.org).

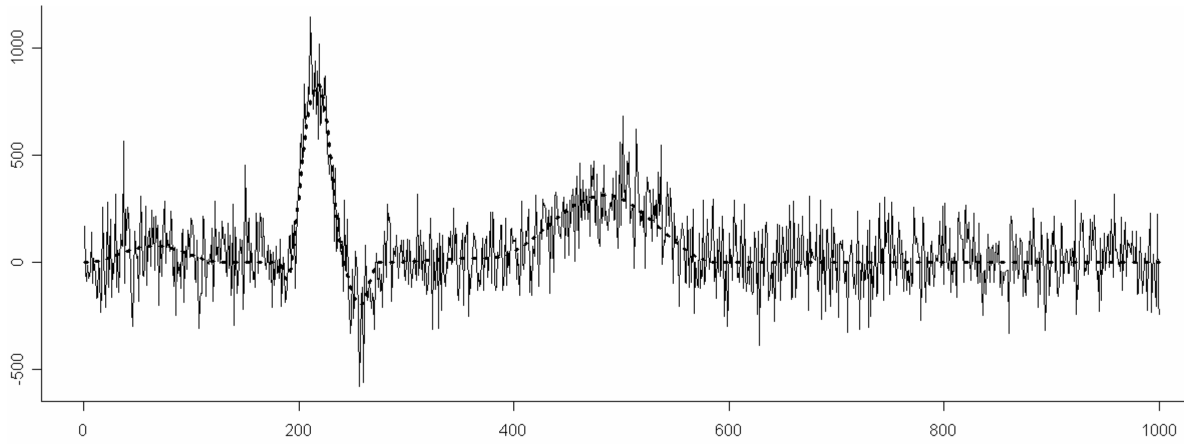

Fig.1. The example of ECG signal and this signal with Gaussian noise

The simulated ECG signal cycles were obtained as the same deterministic component with added independent realizations of random noise. The deterministic component was ANE20000, taken from database CTS [7]. A series of 100 ECG cycles was generated with the same deterministic component and zero-mean white Gaussian noise with four different standard deviations. For the first, second, third and fourth 25 cycles, the noise standard deviations were respectively 0.1*s*, 0.5*s*, 1*s*, 2*s*, where *s* is sample standard deviation of the deterministic component. Figure 1 presents the ANE 20000 ECG signal (bold dashed line) and this signal with Gaussian noise with standard deviation equal sample standard deviation of the deterministic component.

Table 1 presents the RMSE and the absolute maximal value (MAX) of residual noise for all tested methods such as traditional Arithmetic Averaging (AA), Weighted Averaging method based on Empirical Bayesian approach (WAEB), Weighted Averaging method based on Criterion Function Minimization (WACFM) with parameter m equal 2 and presented new method of Weighted Averaging based on Partition of input set in time domain and criterion function Minimization (WAPM). Using presented algorithm the input set was divided into two equal in number of elements subsets (i.e. $M_1 = M_2 = 50$) Y_1 and Y_2, where Y_1 contains elements with even numbered indexes of the input set Y and Y_2 contains all the remaining elements. For the reason of randomness of noise, the experiment described above was performed several times and the results of two realizations are presented respectively in second and third rows as well as in fourth and fifth rows. The best result in each row is bolded.

Table 1. RMSE and absolute maximum errors for averaged ECG signals.

Type of error	AA	WAEB	WACFM	WAPM
RMSE	15.93941	**2.791202**	2.866539	2.882829
MAX	55.40676	10.864082	10.923617	**9.974265**
RMSE	16.36139	**2.786436**	2.867156	2.839169
MAX	52.48568	**8.985509**	10.872320	9.135329

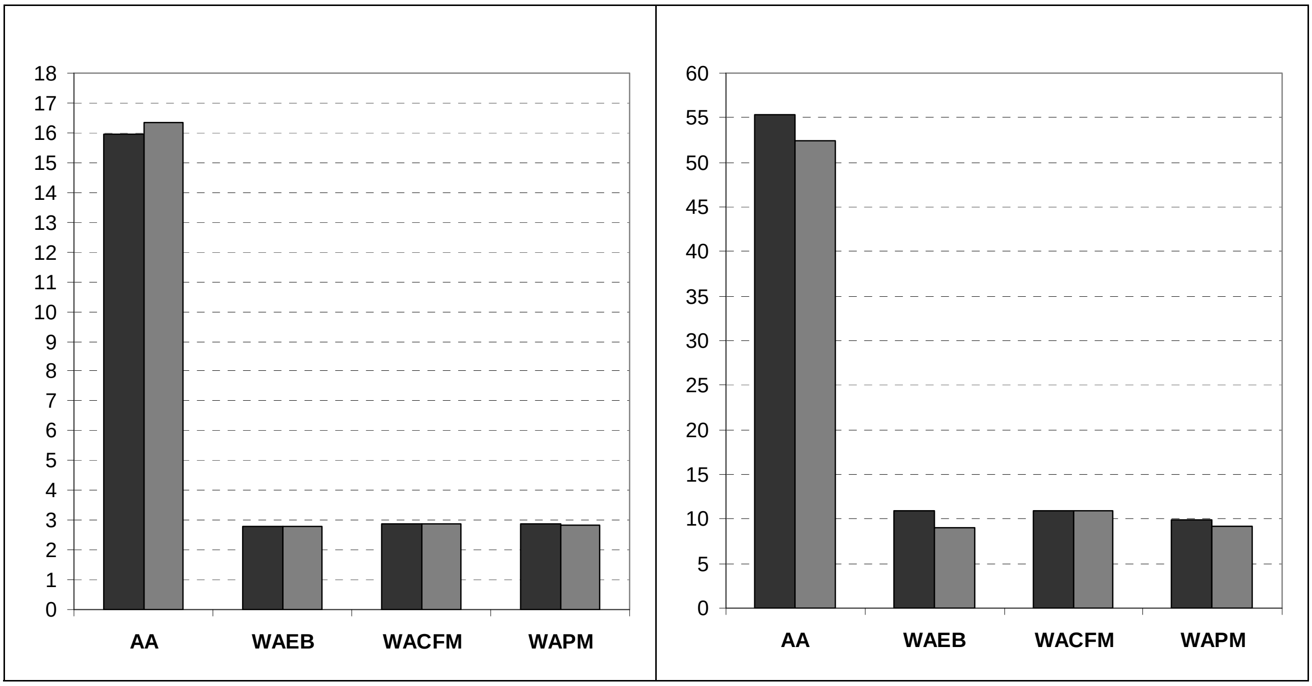

Fig.2. RMSE (left) and MAX errors (right) for averaged ECG signals

Figure 2 shows the RMSE and the absolute maximal value (MAX) of residual noise for all tested methods, presented in Table 1, in graphic form. Figure 3 presents the ANE 20000 ECG signal (bold dashed line) and the averaged signal obtained by using presented method (as can be seen in Table 1, the other results are similar except traditional arithmetic averaging).

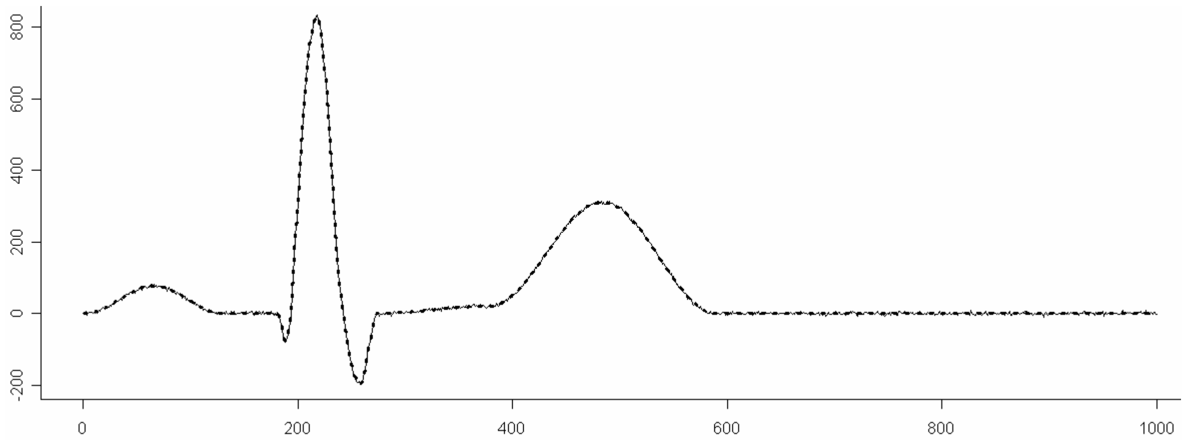

Fig.3. The example of ECG signal and its averaging using proposed method

As can be seen the results of numerical experiments show usefulness of the presented method in the noise reduction in ECG signal competitively to existing algorithms.

4. CONCLUSIONS

In this work the new approach to weighted averaging of biomedical signal was presented along with the application to averaging ECG signals. Presented new method, which incorporates partition of input set in time domain, resolves the weighted signal averaging problem. By exploiting criterion function minimization it can be derived an algorithm of weighted averaging which application to electrocardiographic (ECG) signal averaging is competitive with alternative methods such as weighted averaging method based on empirical Bayesian approach or weighted averaging method based on criterion function minimization.

It is worth noting that the new iterative algorithm requires setting only one additional parameter M_1, which describes size of Y_1 set (Y_2 contains all the remaining elements). The algorithm also requires determining the method of partitioning of input set. One of the possible method is random, another is based on cycle index, for example interlaced, like in described experiment.

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