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SPATIAL FREQUENCY FILTERING OF IMAGES USING THE DIFFRACTION IN ACOUSTO-OPTIC CELL

The paper presents the optical spatial frequency filter that allows to filter in real time by using a 0th order diffraction in the acousto-optic cell, which operates in the Bragg regime.

Keywords: acousto-optics, spatial frequency filtering.

1. INTRODUCTION

Discovered in the last century, the phenomenon of light diffraction on ultrasonic waves was and is the subject of numerous research papers examining various aspects of the interaction of photons with phonons [1, 2]. This interaction gives great possibilities for its practical use in many fields of science and technology. I mention here, for example, only some of the most common applications, such as controlled changing of light frequency, intensity or direction of the light beam. This work, however, concerns other than the above-mentioned applications. Shown here, using calculations based on Fourier optics, how the image is modified, if in the zone of its formation the light travels through a medium in which the acoustic wave propagates. More precisely, it can be say that the work demonstrates, by means of some examples, in which way the transition of light through acousto-optic cell in which the light undergone diffraction in the Bragg regime affects the image formed by lens.

2. OUTLINE OF THE THEORETICAL BASIS UNDERLYING THE ACOUSTO-OPTIC PHENOMENON

There are two complementary formalisms describing the interaction of light with the sound of acousto-optic cell. One is based on the corpuscular concept of matter i.e. photons interact with phonons. And another, which assumes that ultrasonic wave moving in the medium modifies the refractive index of the medium. Light moving in such regularly modified medium undergoes diffraction. In general, one can observe many orders of diffraction, as shown in Fig. 1.

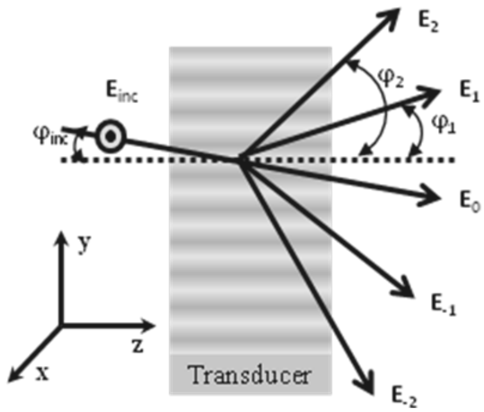

Fig. 1. E_{inc} denotes magnitude of electric field strength of incident light. E with other subscripts indicates magnitude of electric field of diffracted light. Circle with a dot points in the direction of light polarization. φ_{inc} stands for angle which the incident light beam makes with plain of constant phase of sound wave. Transducer produces the ultrasound.

Sound wave can be described by the expression:

$$S = Re(Aexp(i(\Omega t - Ky))) \tag{1}$$

Where A is complex amplitude of sound, i, Ω, t, K, y are: imaginary unit, frequency of sound, time, propagation constant of sound and y coordinate, respectively. Propagation constant $K = 2\pi/\Lambda$. Λ stands for sound wavelength.

Change in the relative permittivity $\in'$ due to the sound wave we take in the form:

$$\in' = \in \cdot C \cdot Re(A\,exp(i(\Omega t - Ky))) \tag{2}$$

Where $\in$ is the relative permittivity in undisturbed medium, C is the acousto-optic interaction constant. It is obvious that the change in $\in$ produces variation in refractive index of light.

Each order of diffraction, shown in Fig. 1, is coupled with its nearest neighbour. This coupling is described in many sources [1, 3]. But if the interaction length is long enough (the transducer is broad enough) and the angle of incidence φ_{inc} is equal to Bragg angle φ_B only two orders are left e.g. 0 and 1 or -1. Configuration in which appear only two diffraction orders is shown in Fig. 2. The Bragg angle can be determined from the following expression.

$$sin\,\varphi_B = \frac{\lambda}{\Lambda} = \frac{K}{k_0}$$

Where λ is wavelength of light, $k_0 = 2\pi/\lambda$ stands for propagation constant of light.

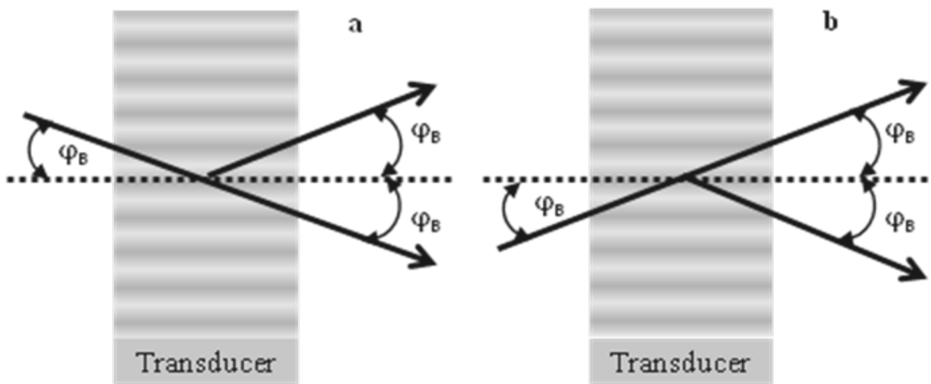

Fig. 2. On the left side is prezented so called upshifted Bragg diffraction. In this configuration the light frequency of oscillations ω_0 is increased by the sound frequency Ω ($\omega_1 = \omega_0 + \Omega$). On the right side is presented downshifted Bragg diffraction. In this configuration the light frequency of oscillations ω_0 is diminished by sound frequency Ω ($\omega_1 = \omega_0 - \Omega$).

In this configuration both amplitudes *E* of diffraction orders can be expressed as follows [1]:

$$\frac{\partial E_0}{\partial z} = i\frac{f_y^2 + 2 \cdot f_y \cdot k_1}{2k_2}E_0 - i \cdot F \cdot E_{-1} \tag{3}$$

$$\frac{\partial E_{-1}}{\partial z} = i\frac{f_y^2 - 2 \cdot f_y \cdot k_1}{-2k_2} E_0 - i \cdot D \cdot E_{-1} \tag{4}$$

Where $k_1 = k_0 \sin\varphi_B$, $k_1 = k_0 \sin\varphi_B$ and f_x which elsewhere is denoted as k_x is spatial frequency in y direction. $D = k_0CA^*/4$, $F = k_0CA/4$.

3. RESULTS OF NUMERICAL CALCULATIONS AND CONCLUSIONS

In this paper the optical system shown in Fig. 3 is analyzed.

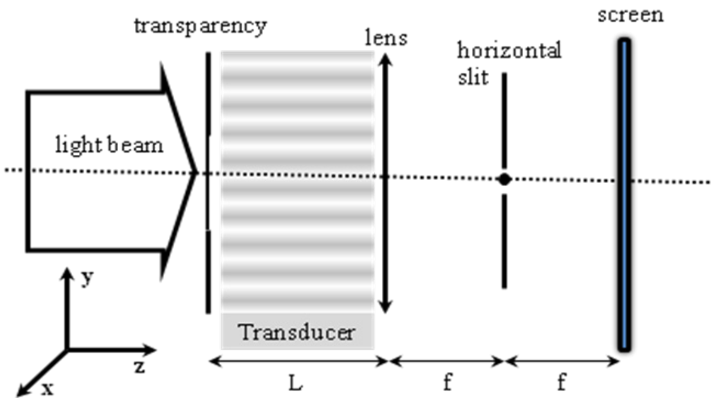

Fig. 3. The monochromatic ($\lambda = 0.5 \cdot 10^{-6}$ m), parallel light beam travels along z-axis and passes through the transparency, which is an object for the lens. Horizontal slit cuts out the -1st diffraction order which exists when transducer works ($\Lambda = 0.1 \cdot 10^{-6}$ m). The real image is formed on the screen. The distance of L = 6 cm, the focal length of the lens f = 4.5 cm. The refractive index of the acousto-optic cell is equal to 1.5. Light beam has the unit amplitude.

The transverse size of transparency was always chosen in this way that the diffracted angle, caused by its size, was many times smaller than Bragg angle. Slight deviation of the acousto-optic cell from the vertical direction in Fig. 3 was intentionally omitted for clarity. In calculations this deviation was taken into account. The actual position the acousto-optic cell is shown in Fig. 4.

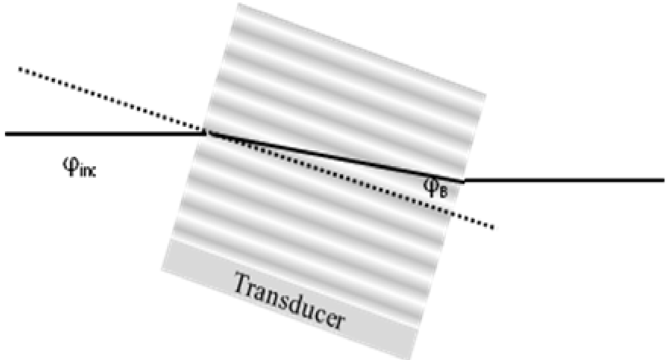

Fig. 4. The actual position of the acousto-optic cell. The sizes of angles are exaggerated for clarity. The 0th diffraction order of light undergoes a little displacement.

The calculation of the images on the screen performed using Fourier optics. The initial stages of the calculation procedure were as follows. At the beginning the phase of the transparency was modified linearly due to slope of the acousto-optic cell. Then the Fourier transform of the outcome was calculated. In the next step, the result from previous stage of calculation was multiplied by the relevant transfer function. The choice of transfer function depended on whether the sound wave was traveled or not in the cell. If sound does not exist in the acousto-optic cell only spatial frequency transfer function for propagation was used [4] otherwise it was multiplied by transfer function H_0 [1].The remaining stages of the calculations will not be discussed because they are typical for this type of calculations [4].

The H_0 can be expressed as:

$$H_0\left(f_x,L\right)=exp\left(i\cdot f_y^2\cdot L\right)\cdot\left(cos\,\beta+\frac{i\cdot f_x\cdot k_1\cdot L}{k_0}\cdot\frac{sin\,\beta}{\beta}\right) \tag{5}$$

Where

$$\beta=\sqrt{\left(\frac{f_x\cdot k_2\cdot L}{k_0}\right)^2+\left(\frac{\alpha}{2}\right)^2} \tag{6}$$

$$\alpha = k_0\cdot C\cdot|A|\cdot L/2 \tag{7}$$

To show that the 0th diffraction order works as high pass filter simple but characteristic example will be demonstrated. The transparency in the optical system will be aperture in shape of square. The calculation was repeated for distinct values of α parameter. This parameter for the conditions adopted in this paper varies with the change of the sound intensity inside acousto-optic cell, as it results from Eq. (7).

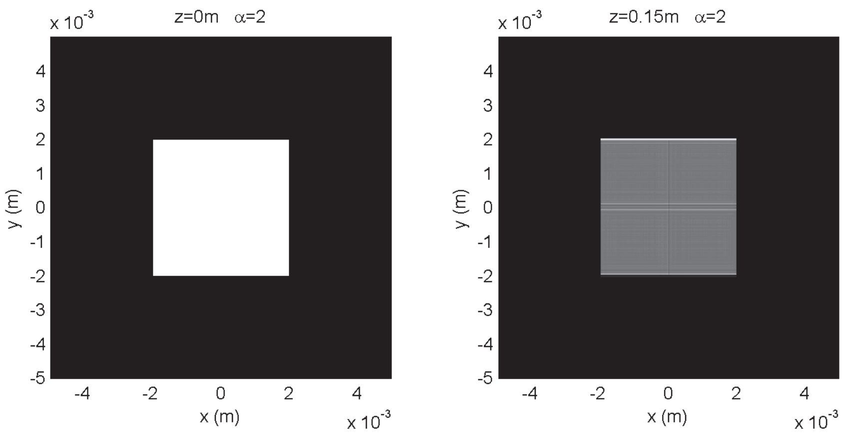

Fig. 5. On the left side is visible the transparency which is located in the object plain. On the right side the real image (on the screen) is presented. α = 2.

As it is seen in Fig. 5 and better in Fig. 6 the optical system formed real image with magnification equal to -1, however low spatial frequencies were damped.

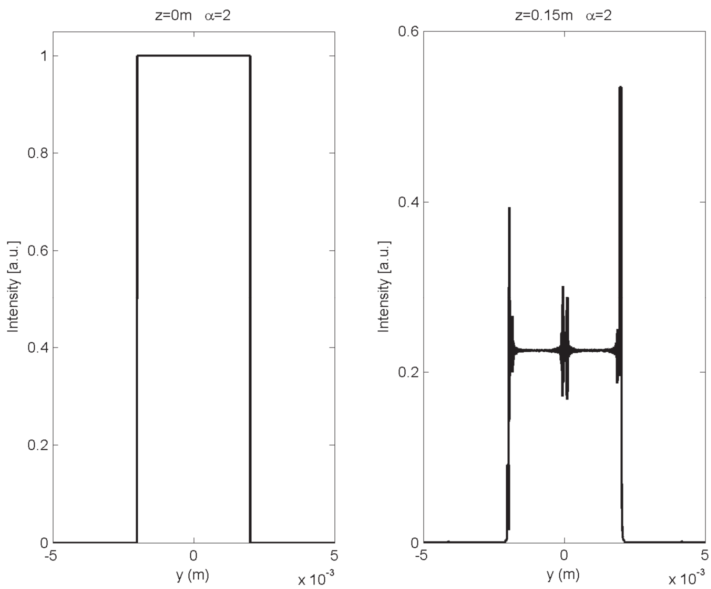

Fig. 6. On the left side the intensity profile of transparency along y-axis is presented. On the right side is plotted the same profile along y-axis but in the plane of the screen. α = 2.

Figs. 7-8 demonstrates similar pictures as Figs. 5-6. But in this case α equals 3.

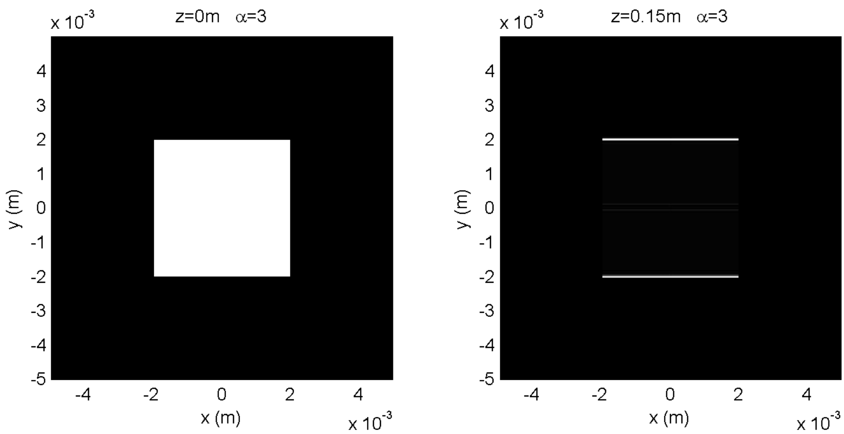

Fig. 7. On the left side is visible the transparency which is located in the object plain. On the right side the real image (on the screen) is presented. $\alpha = 3$.

Now we can see how strongly the low spatial frequencies were damped.

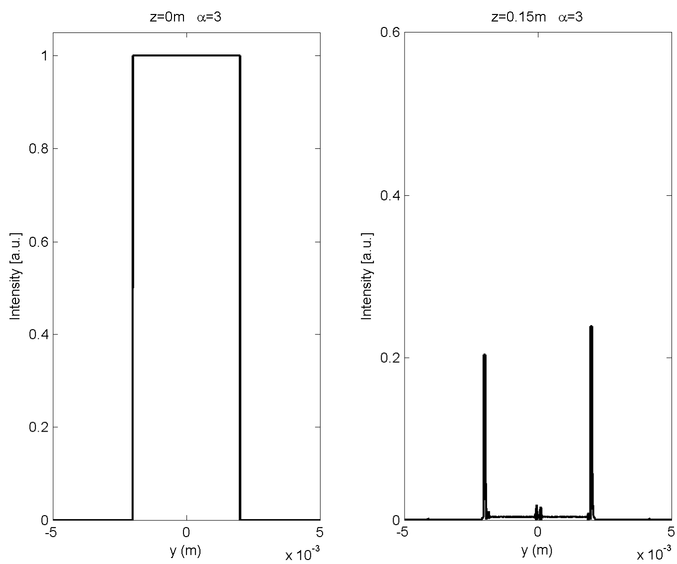

Fig. 8. On the left side the intensity profile of transparency along y-axis is presented. On the right side is plotted the same profile along y-axis but in the plane of the screen. $\alpha = 2$.

Above results of calculation proves that 0th order Bragg diffraction can be used to the spatial frequencies filtering during a process of formation of optical images in real time [5, 6]. But this technique has very important drawback because one side of image is filtered stronger than the other what is visible in each figures. To overcome this problem more sophisticated optical system should be analyzed.

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FILTROWANIE CZĘSTOTLIWOŚCI PRZESTRZENNYCH OBRAZU Z WYKORZYSTANIEM DYFRAKCJI W KOMÓRCE AKUSTOOPTYCZNEJ

Streszczenie

W pracy przedstawiono układ optyczny umożliwiający filtrowanie często-tliwości przestrzennych w czasie rzeczywistym, korzystając z zerowego rzędu dyfrakcji w komórce akustooptycznej, która pracuje w reżimie Bragga.